LAST Time

Center-of-Mass coordinates

↳ mass weighted Average coordinates

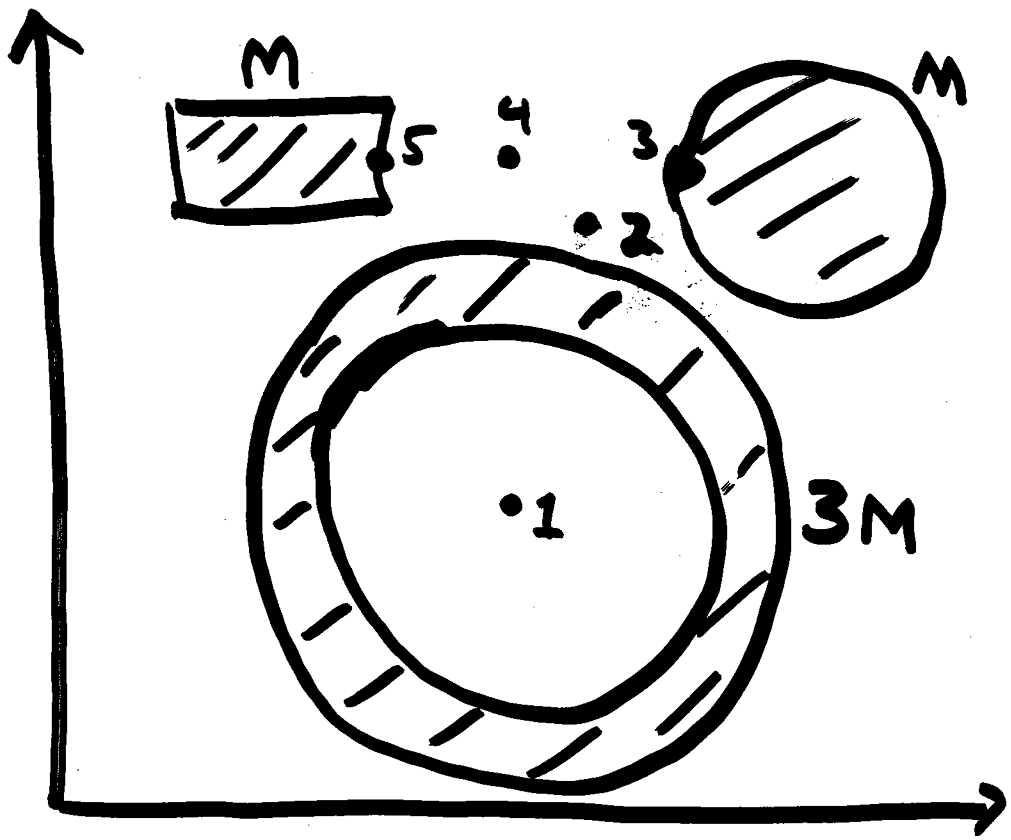
$$\vec{R}_{cm} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i} \quad \left\{ \begin{array}{l} x_{cm} = \dfrac{\Sigma m_i x_i}{\Sigma m_i} \\ y_{cm} = \dfrac{\Sigma m_i y_i}{\Sigma m_i} \\ z_{cm} = \dfrac{\Sigma m_i z_i}{\Sigma m_i} \end{array} \right.$$

Why do you care?

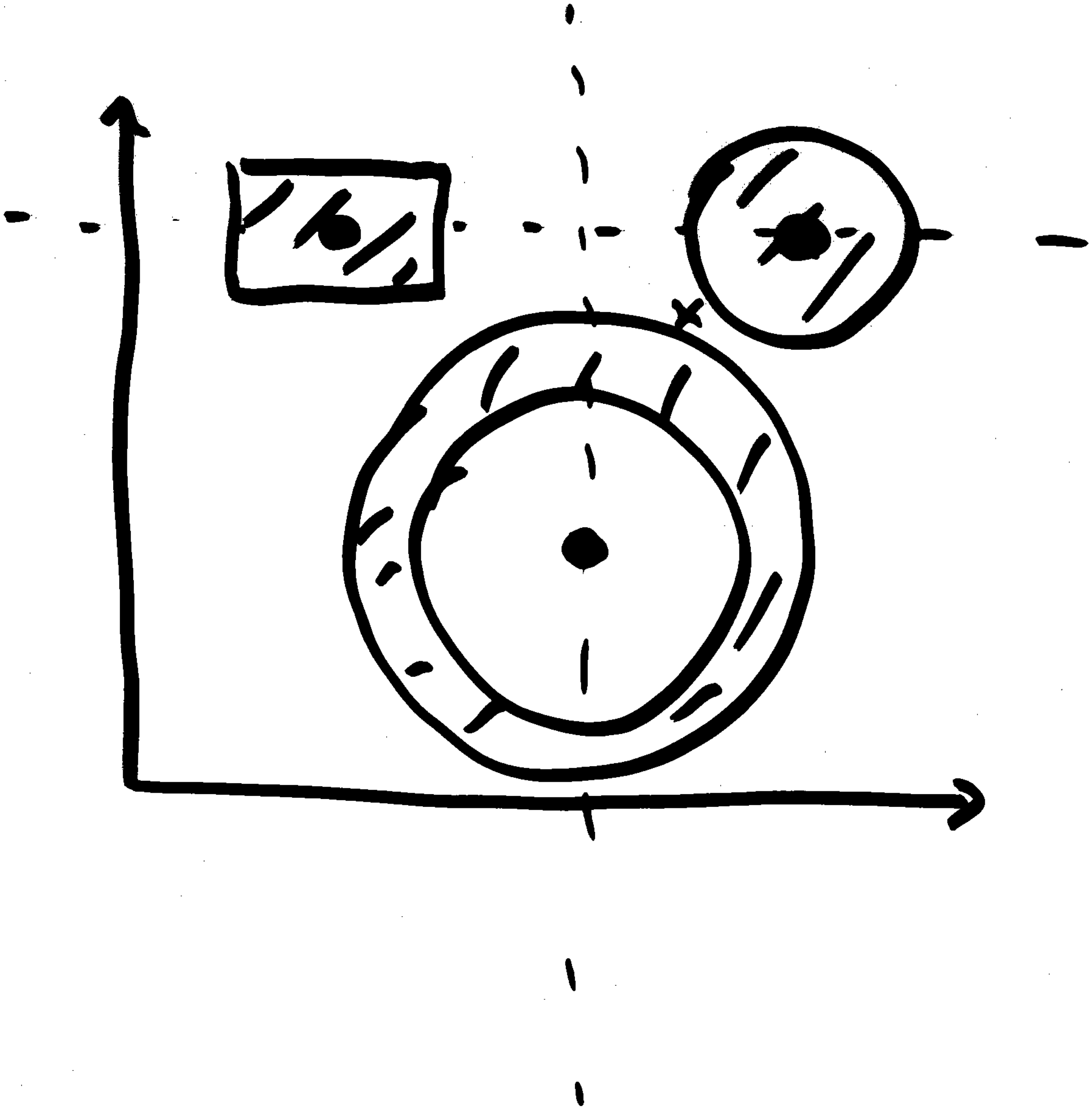
The motion of an extended body or system of bodies acted on by an external force can be described as the Motion of the C.M. superimposed w/ Motion About the center of Mass

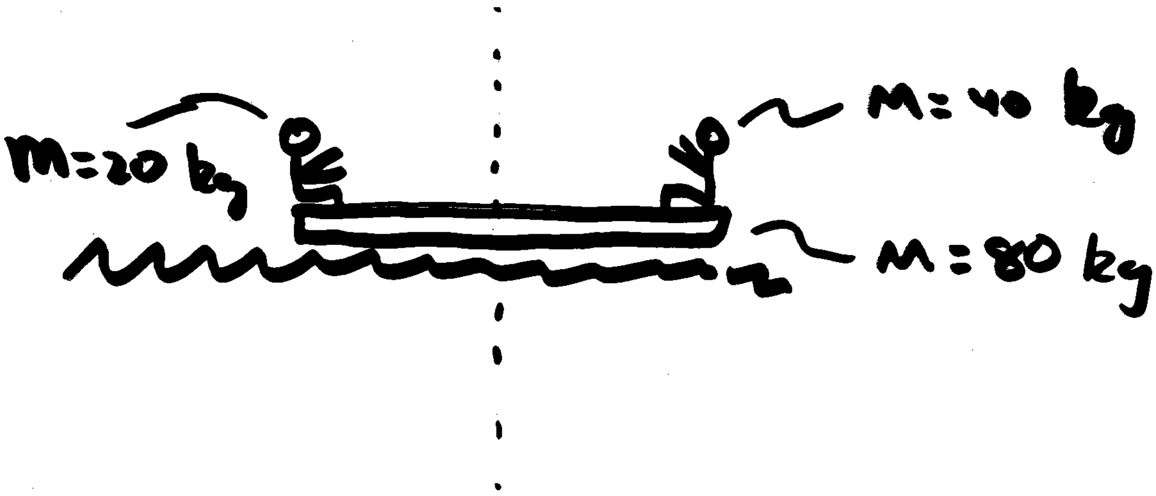
—AND—

The center-of-Mass moves as if all Mass were concentrated at that point and Acted on by a Net force equal to the (vector) sum of external forces!



The center of Mass of system Most likely located at which point?





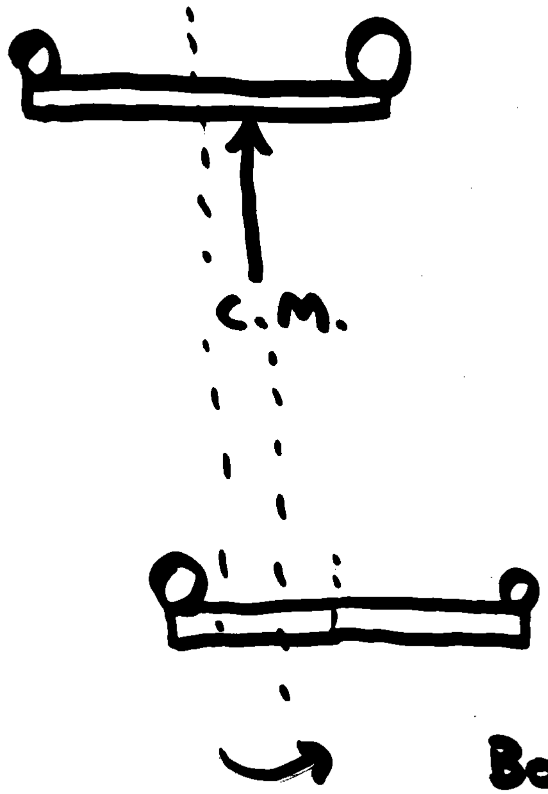
2 kids in a boat (glides frictionlessly on water) exchange places. The boat moves . . .

① To the right

② To the left

③ The boat does NOT move

Before

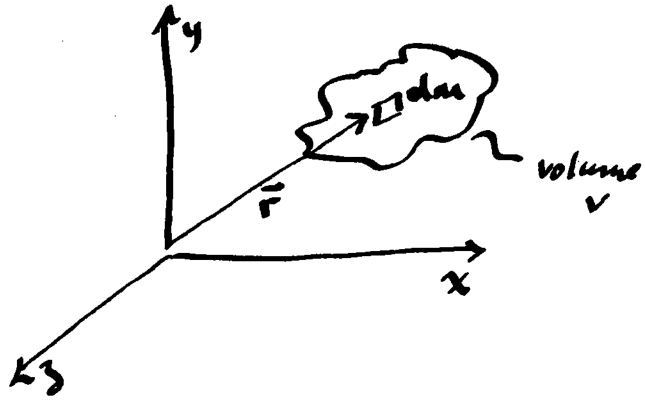


C.M. does NOT Move

Boat Moves To Right

$\Rightarrow$ 2

Suppose we go to the continuous limit



$$x_{c.m.} = \int_V x\,dm \Big/ \int dm = \frac{\int_V x\,dm}{M}$$

$$y_{c.m.} = \int_V y\,dm \Big/ M$$

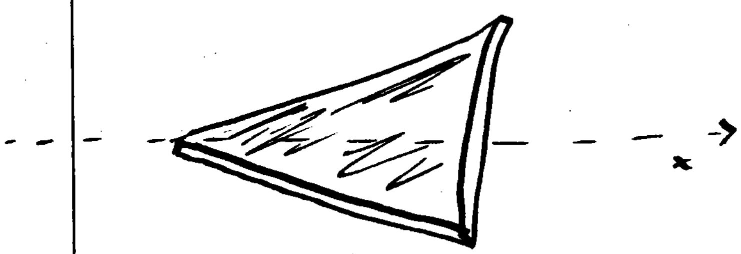
$$z_{c.m.} = \int_V z\,dm \Big/ M$$

- or -

$$\vec{r}_{c.m.} = \frac{\int_V \vec{r}\,dm}{M}$$

What do I mean by center-of-mass?

What do I mean by center-of-weight?



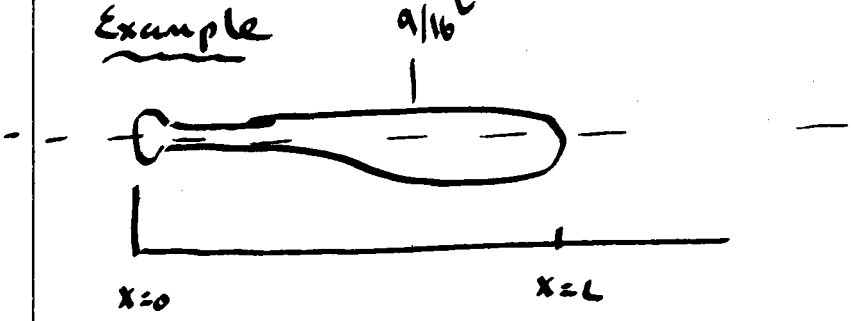
Suppose I had a triangular piece of metal (heavy) you had to hold it up in this orientation w/ one finger ... where would you put your finger?

- or -

baseball bat

Discuss how "Mass weighting" allows one to find such a position

Example



do demos
Mp-2
Mp-3

Suppose a bat has a length L and a Mass/unit length $\equiv \lambda$

$$\lambda \equiv \lambda_0 \left(1 + \frac{x^2}{L^2}\right) \qquad 0 \le x \le L$$

(λ larger by factor of two at the thick end of the bat)
Find the c.o.m of the bat as a fn of L.

by symmetry ... lay bat along x ... can ignore y, z

-or- $y_{c.m.} = 0$
$z_{c.m.} = 0$

$$M = \int dm = \int_{x=0}^{x=L} \lambda dx$$

$$M = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) dx = \lambda_0 \left(x + \frac{x^3}{3L^2}\right)\Big|_0^L$$

$$M = \lambda_0 \left(L + \frac{L}{3}\right) = \frac{4L}{3} \lambda_0$$

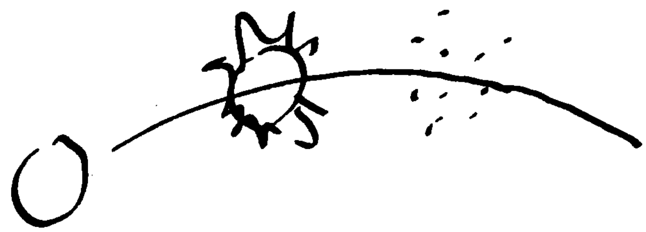
$$x_{c.m.} = \frac{\int x\, dm}{\int M} = \frac{1}{M} \int x\, dm = \frac{1}{M} \int_0^L \lambda dx$$

$$x_{c.m.} = \frac{\lambda_0}{M} \left(\frac{x^2}{2} + \frac{x^4}{4L^2}\right)\Big|_0^L = \frac{\lambda_0}{M} \left(\frac{L^2}{2} + \frac{L^2}{4}\right) = \frac{\lambda_0}{M} \frac{3}{4} L^2$$

$$x_{c.m.} = \lambda_0 \frac{3}{4} L^2 \frac{3}{L 4 \lambda_0} = \frac{9}{16} L$$

back to a discrete system of Particles

$$M\vec{V}_{c.m.} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n$$



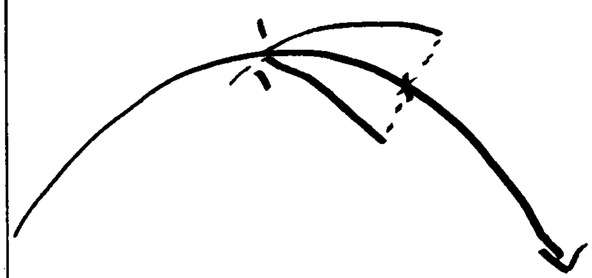
$$M\frac{d\vec{v}}{dt} = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + \cdots + m_n\frac{d\vec{v}_n}{dt}$$

$$M\vec{a}_{c.m.} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_n$$

$$\parallel$$

$$\Sigma\vec{F} = \Sigma\vec{F}_{ext} + \underbrace{\Sigma\vec{F}_{int}}_{=\,0}$$

when a body or collection of particles is acted on by external forces, the center of mass moves just as though all the masses were concentrated at that point and it were acted on by a net force equal to the sum of external forces



Exploding projectile Example

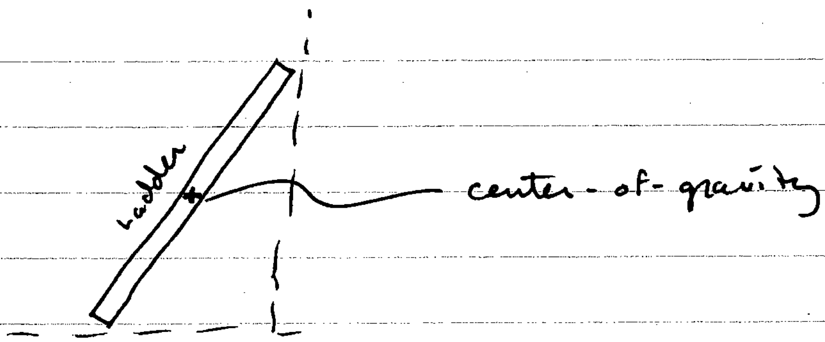
Masses on ice Example

Center-of-Mass is a concept valid under any circumstance

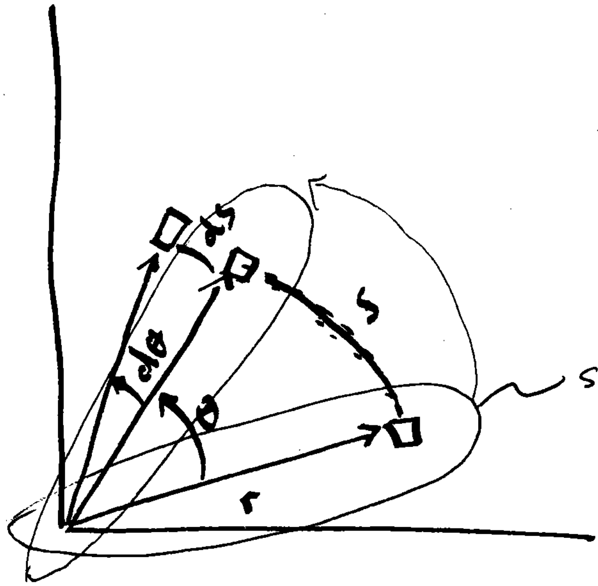
Center-of-gravity - Point on a body where one can assume all Mass is concentrated ~~for considering to~~ in considering how the force of gravity acts on a body

These points are one and the same.

Critical for rotational equilibrium



Rotational Motion



Some rigid body rotates thru Angle θ

$$S = r\theta$$

Arclength = (Radius)(θ in ~~the~~ radians)

$$ds = r\,d\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$V = r\,\omega$$

Tangential Velocity (m/s) — Angular Velocity ($radians/s$)

All you need remember

$S = r\theta$

$\left.\begin{matrix} V = r\omega \\ a_t = r\alpha \end{matrix}\right\}$ follow

$$\underbrace{\frac{d^2 s}{dt^2} = \frac{dv}{dt}}_{\text{Tangential Acceleration}} = \underbrace{r\frac{d^2\theta}{dt^2} = r\frac{d\omega}{dt}}_{\text{Angular Acceleration } \alpha}$$

$$\frac{d\theta}{dt} = \omega$$

$$d\theta = \omega\, dt$$

$$\int_{\theta_0}^{\theta} d\theta = \int_{t_0}^{t} \omega\, dt$$

$$\boxed{\theta - \theta_0 = \int_{t_0}^{t} \omega\, dt}$$

$$\frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha\, dt$$

$$\int_{\omega_0}^{\omega} d\omega = \int_{t_0}^{t} \alpha\, dt$$

$$\boxed{\omega - \omega_0 = \int_{t_0}^{t} \alpha\, dt}$$

if α is constant

$$\omega - \omega_0 = \alpha(t - t_0)$$

$$t_0 = 0 \implies \omega = \omega_0 + \alpha t$$

Does all this look familiar??

$$x, v, a, t \iff \theta, \omega, \alpha, t$$

The differential eqns that relate the variables are the same. $\therefore$ The eqns that relate them will be the same!

For a = CONSTANT

Linear variables

$$\left.\begin{aligned} v &= v_0 + at \\ x &= x_0 + v_0 t + \tfrac{1}{2} a t^2 \\ v^2 &= v_0^2 + 2a(x - x_0) \\ x - x_0 &= \tfrac{1}{2}(v + v_0)t \end{aligned}\right\} \implies$$

For α = CONSTANT

Angular variables

$$\left\{\begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \\ \theta - \theta_0 &= \tfrac{1}{2}(\omega + \omega_0)t \end{aligned}\right.$$