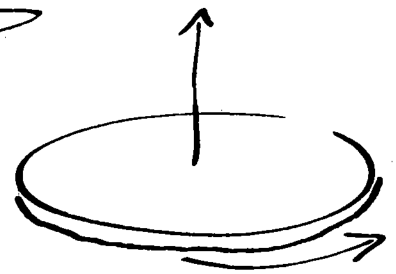
All you know about solving 1-d linear const-Acceleration problems $\longrightarrow$ carries over to

Constant Angular Acceleration problems

Example



A disk initially rotating at 120 Rad/s slows down w/ constant angular acceleration of 4.0 rad/s^2

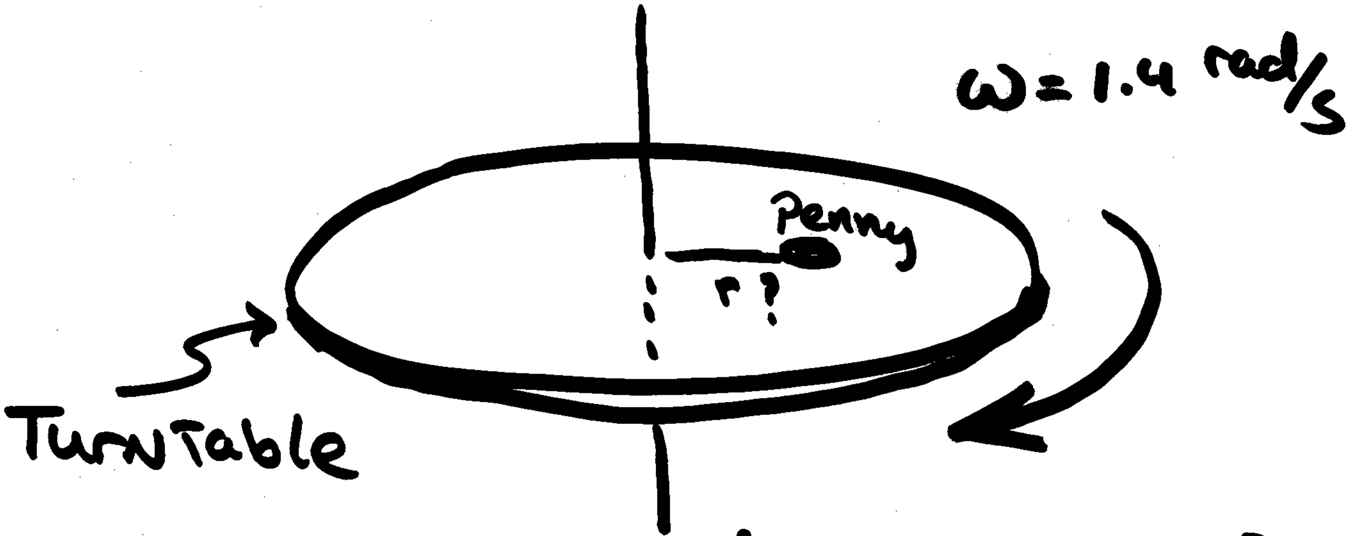
$\Rightarrow$ How much time passes before the disk stops rotating?

$$\omega = \omega_0 + \alpha t$$
$$0 = 120 - 4.0\,t$$
$$t = 30 \text{ seconds}$$

So far relating linear to angular variables works well

Linear $\quad F = Ma$

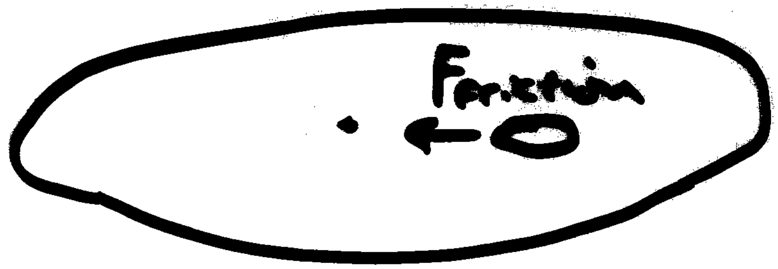
Angular? $\quad ? = ?\,\alpha$

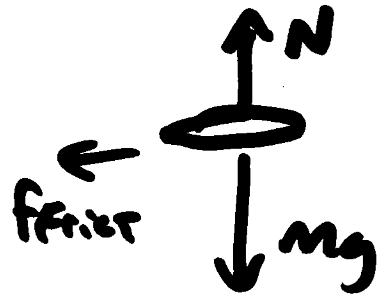


$\mu_{s\ \text{penny-surface}} = 0.20$

What is Maximum distance penny can be placed from center of Turntable Without Sliding?

1. 0.50 m
2. 1.0 m
3. 1.4 m
4. 2.0 m
5. 4.4 m





$$\Sigma F_y = 0 \Rightarrow N = mg$$

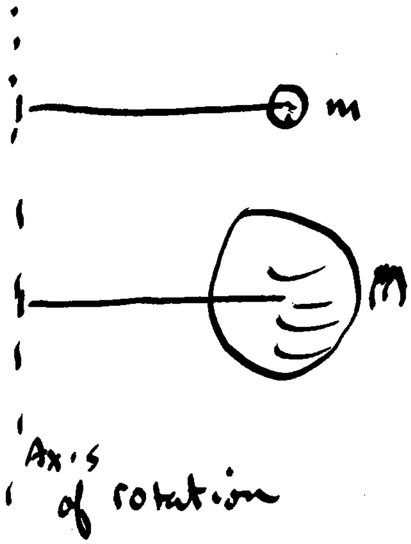
$$\Sigma F_{radial} = \frac{mv^2}{r}$$

$$N\mu_s = mg\,\mu_s = \frac{mv^2}{r}$$

$$v = r\omega$$

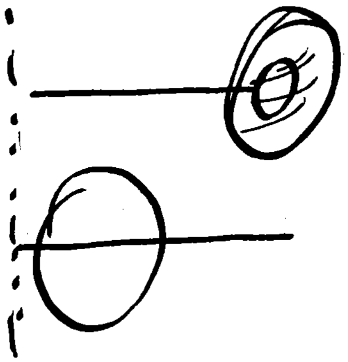
$$g\mu_s = \frac{r^2\omega^2}{r}$$

$$r = \frac{g\mu_s}{\omega^2} = 1.0\ m$$



Which is harder to rotate?

requires more force



Which of these?

~~Also door example~~

$F = Ma$ linear

$F = ?\,\alpha$ Angular

obviously depends on M ... magnitude of body

And Mass distribution

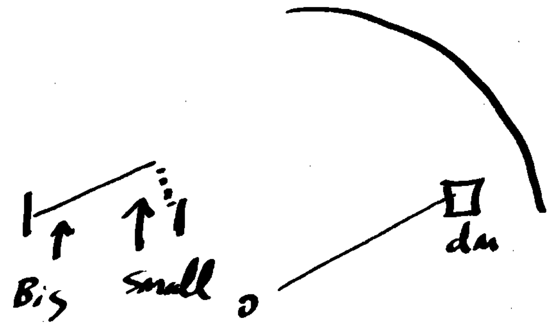
Do demos

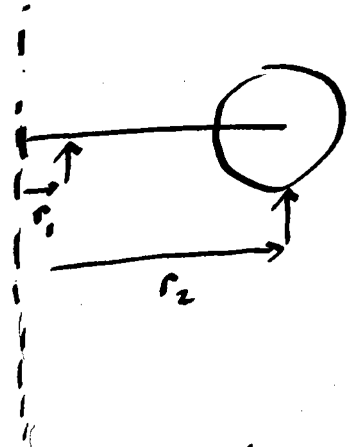
3 from Thang

Door demo

$F = f(m,r)\,\alpha$

Do door Demo
2 students





consider Application of force
does it matter where you
Apply it

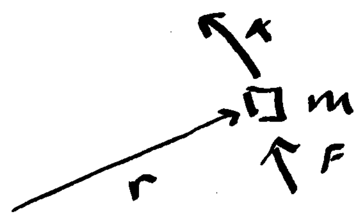
So ... α depends on Mass, dist of Mass, force
And point of Application of force!

$F = ma$

$s = r\theta$
$v = r\omega$
$a = r\alpha$

ignore vector character for Now

$F = mr\alpha$



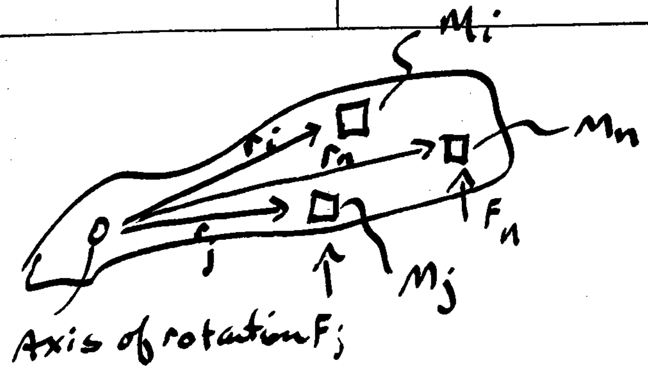
$rF = rmr\alpha$

$rF = Mr^2 \alpha$

Angular Acceleration

Angular Force
Known as Torque
τ

Angular Mass
Known as Moment of Inertia of Mass M
I



$$\left(\sum r_i F_i\right) = \left(\sum m_i r_i^2\right)\alpha$$

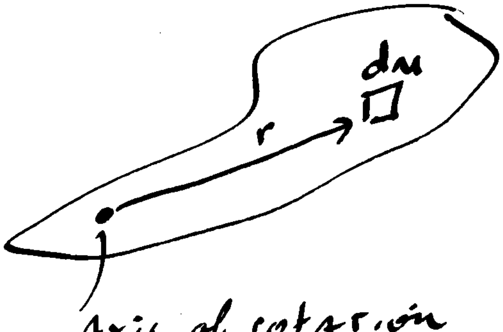
Net "angular" force

net "angular" mass

$$\sum \tau_i = \sum \underbrace{m_i r_i^2}_{I_i} \alpha$$

$$\tau_{NET} = \sum m_i r_i^2 \alpha = I\alpha \qquad \boxed{\tau_{NET} = I\alpha}$$

go to continuous limit



~~$d\tau = r^2 dm\, \alpha$~~

$dI = r^2 dm$ — dI for differential element

$I = \int_V r^2 dm$ — Total I from integral over volume

Vector vs. Scalar?

Does I have a direction?

$\Rightarrow$ Scalar ... just like regular mass

Soln's to Prob set #7 posted (2 days?)

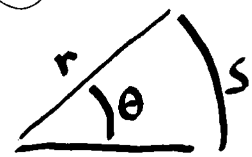
Prob set 8 is posted

Worry about Procrastination

Rotational Motion NOT easy to draw

Motivation Slide → Think Circular

Think



$s = r\theta$

$v = r\omega$

$a = r\alpha$

→ const a eqns

⇓

const α eqns

$F = ma$

$F = m r \alpha$

$rF = r m r \alpha$

$$\underbrace{rF}_{\tau} = \underbrace{mr^2}_{I} \alpha$$

Angular force — Torque

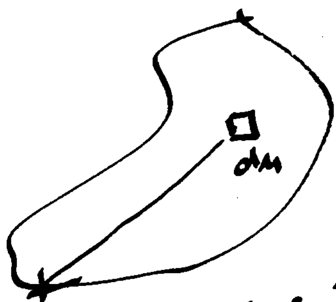
Angular Mass — Moment of Inertia

$$\boxed{\tau_{NET} = I\alpha}$$

$$I = \int_{\text{volume}} r^2\, dm$$

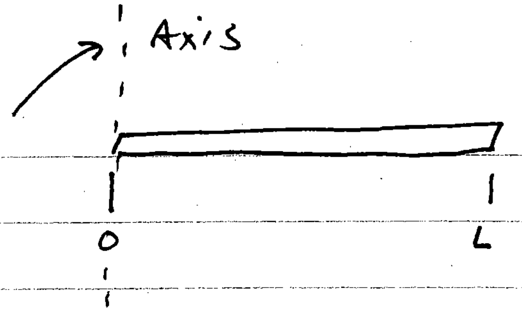
↑

depends on Mass distribution



See Table p.2~~78~~ 252

What is I thru



$\lambda = \lambda_0 = \frac{M}{L}$

$$I = \int r^2 \, dM \qquad r = x \qquad dM = \lambda_0 \, dx$$

$$I = \int_0^L x^2 \lambda_0 \, dx = \lambda_0 \frac{x^3}{3}\Big|_0^L = \frac{\lambda_0 L^3}{3}$$

what is $\lambda_0 = \frac{M}{L} = \frac{ML^2}{3}$

bicycle Wheel

Hollow Sphere

Solid Sphere

Each rotate about central axis

m is the same, R is the same

Roll w/out slipping down a ramp

Which gets to bottom 1st, which last?

1. Hollow Sphere 1st, wheel last
2. Wheel 1st, solid sphere last
3. Solid sphere 1st, wheel last
4. Solid Sphere 1st, Hollow sphere last

$$\vec{\tau} = I\vec{\alpha} \Rightarrow |\vec{\tau}| = I|\vec{\alpha}|$$

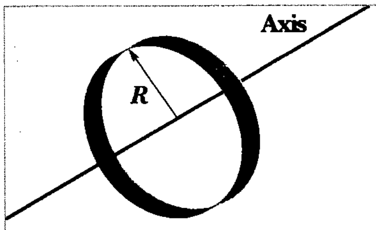
- α largest, get to bottom fastest
- α largest when I smallest because τ is the same
- Mass distributed at largest R for wheel
- Mass distributed w/ large contribution from small R → solid sphere

$$I_{wheel} = MR^2$$

$$I_{\substack{spherical\\shell}} = \frac{2}{3}MR^2$$

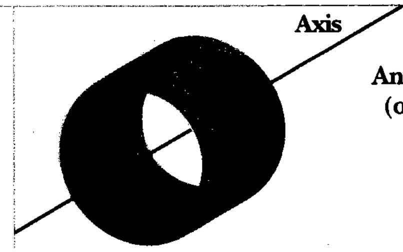
$$I_{\substack{solid\\sphere}} = \frac{2}{5}MR^2$$

place at BOTTOM Solid Sphere 1ST shell 2ND wheel 3RD



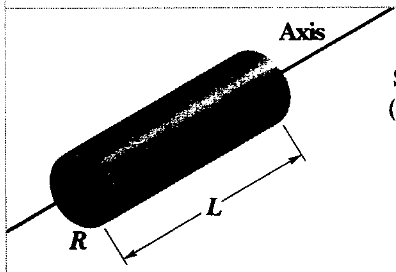
Hoop about central axis

$I = MR^2$ (a)



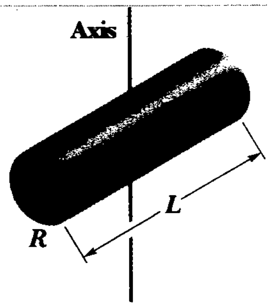
Annular cylinder (or ring) about central axis

$I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)



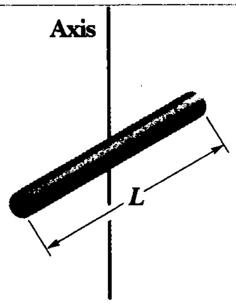
Solid cylinder (or disk) about central axis

$I = \frac{1}{2}MR^2$ (c)



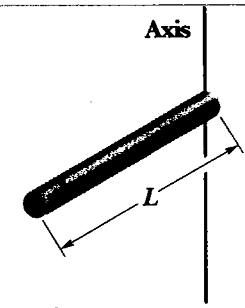
Solid cylinder (or disk) about central diameter

$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)



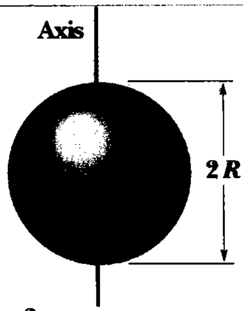
Thin rod about axis through center perpendicular to length

$I = \frac{1}{12}ML^2$ (e)



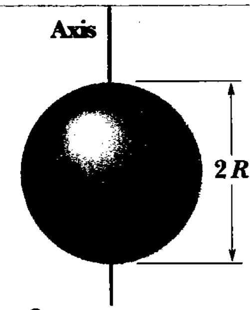
Thin rod about axis through one end perpendicular to length

$I = \frac{1}{3}ML^2$ (f)



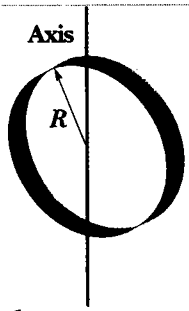
Solid sphere about any diameter

$I = \frac{2}{5}MR^2$ (g)



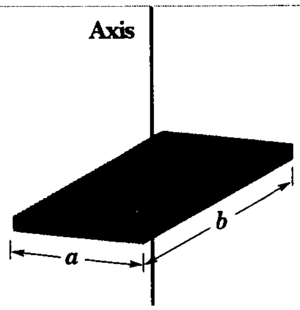
Thin spherical shell about any diameter

$I = \frac{2}{3}MR^2$ (h)



Hoop about any diameter

$I = \frac{1}{2}MR^2$ (i)



Slab about perpendicular axis through center

$I = \frac{1}{12}M(a^2 + b^2)$ (j)

Does $\vec{\tau}$ have a direction ?

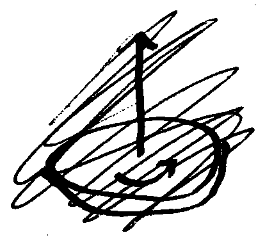
Does α have a direction ?

$$\boxed{\vec{\tau} = I\vec{\alpha}}$$

How do we define these vectors

Specify
- Axis of rotation
- Magnitude of rotation (force / accel)
- direction of rotation (force / accel.)

simultaneously



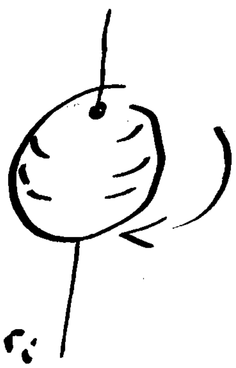
~~Right hand rule~~

~~fingers along direction of rotation~~

~~Thumb points along direction~~

Think about it ... we'll come back to it

let's continue to think in terms of scalars



rotating ball $\vec{V} = 0$

Is there kinetic energy associated with ~~rotation~~ this system?

small m_i at r_i

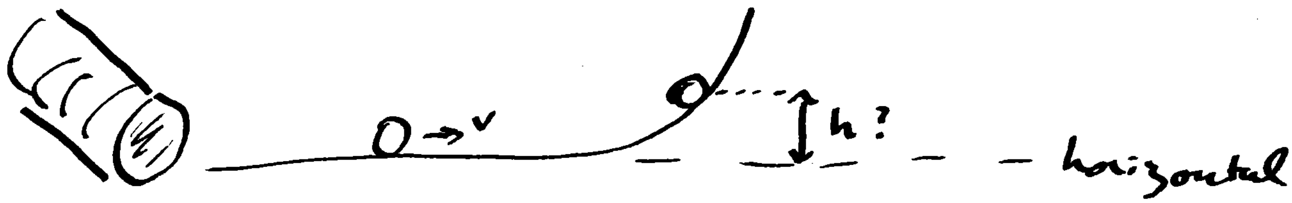
$$KE_i = \frac{1}{2} m_i v_i^2 \qquad v = r\omega$$

$$K.E_i = \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2} I_i \omega^2$$

$$\sum KE_i = KE = \frac{1}{2}\omega^2 \sum m_i r_i^2 = \frac{1}{2} I \omega^2$$

$$\boxed{\text{rotational } KE = \tfrac{1}{2} I \omega^2}$$

Example Problem



Solid homogeneous cylinder

mass $M = (50 \text{ kg})$

radius $R = (15 \text{ cm})$

rolles w/ linear velocity V toward ramp $= (6 \text{ m/s})$

How far up ramp does cylinder climb?

Use Energy conservation

$E_{TOT} = KE_{init} + PE_{init}$

~~$KE_{final} - KE_{initial} = \Delta PE$~~

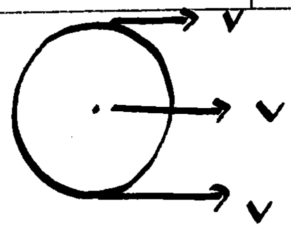
$E_{TOT} = KE_{fin} + PE_{final}$

$$0 = \underbrace{KE_{Final} - KE_{init}}_{\Delta KE} + \underbrace{PE_{Final} - PE_{init}}_{\Delta PE}$$

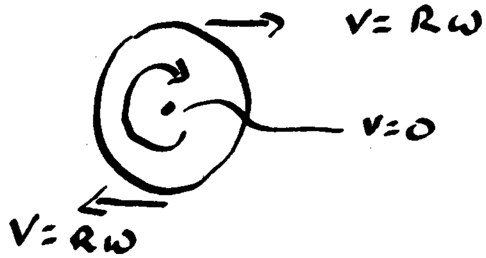
$-\Delta KE = \Delta PE$

$K.E_{init} - KE_{final} = \cancel{PE_{End}}\ \Delta PE = mgh$

? 0

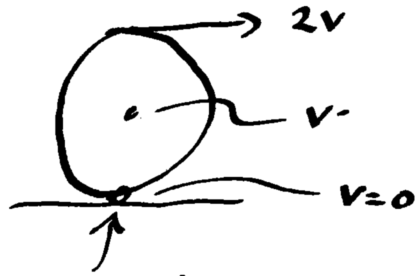


Translation only



Rotation only

combine the two



actual Axis of rotation
at a given moment!

Think of Motion as

1) ~~[illegible]~~ rotational Motion about an axis

2) Translational Motion of the ~~Mass~~ Axis + Associated Mass

Typically ... axis passes thru center of Mass
So you can think of part two as linear translation of C. M.

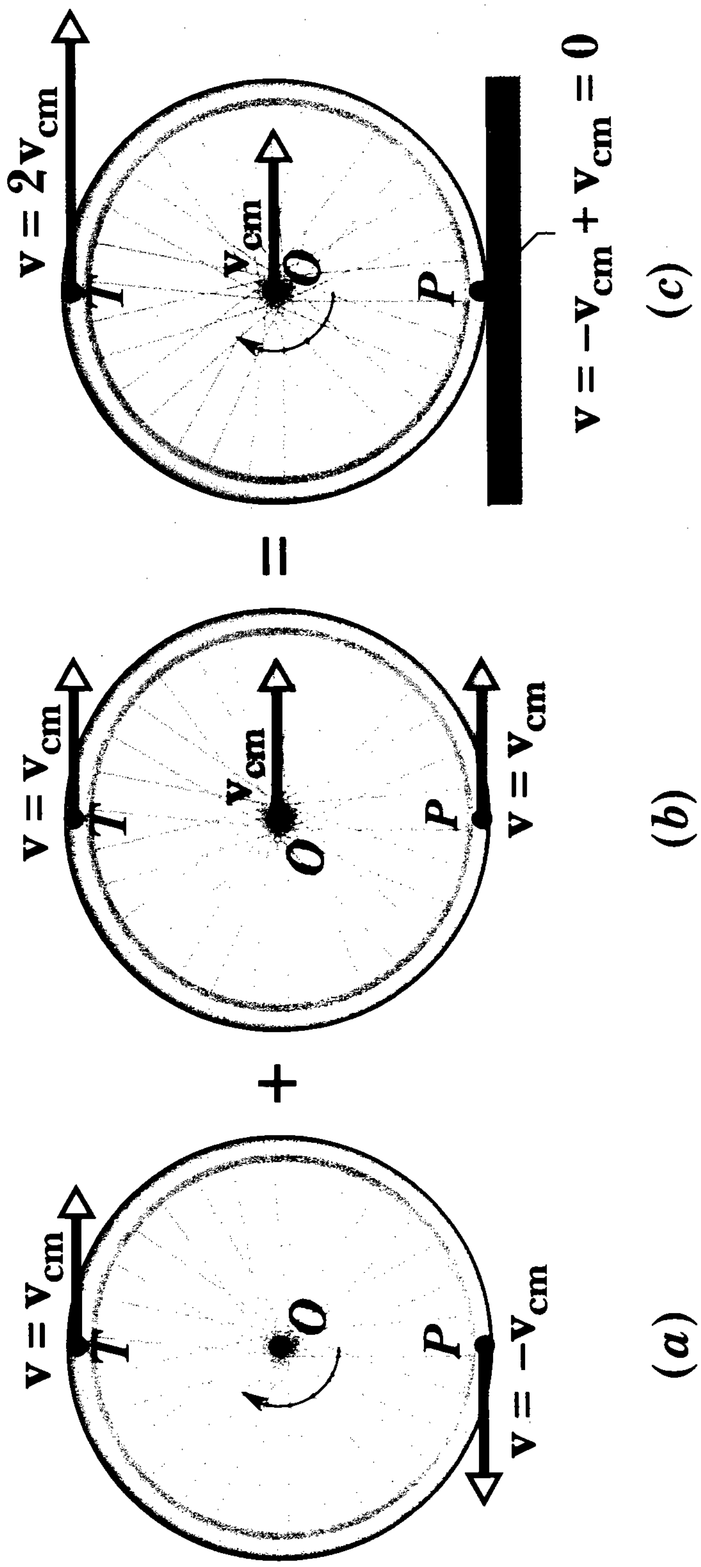
So

$$KE. = \frac{1}{2} I \omega^2 + \frac{1}{2} M V^2 = Mgh$$

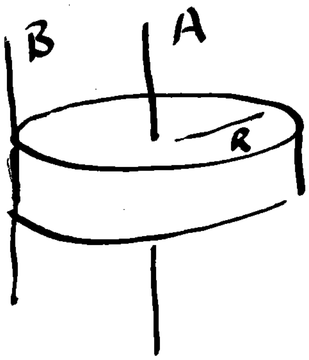
$V = R\omega$ so, know ω

What is I?

Must ~~[illegible]~~ evaluate or look at table $I = \frac{1}{2} MR^2$
in this case



v = v_cm
T
O
P
v = −v_cm
(a)
+
v = v_cm
v_cm
T
O
P
v = v_cm
(b)
=
v = 2v_cm
v_cm
T
O
P
v = −v_cm + v_cm = 0
(c)



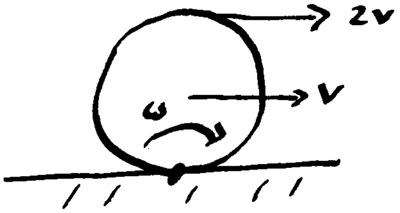
$I_A = \frac{1}{2} MR^2$

$I_B = \frac{1}{2} MR^2 + MR^2$ from Parallel Axis Thm

$= \frac{3}{2} MR^2$

2 WAYS TO look at the Problem:

1)



As Body rotating about Axis at bottom where cylinder Meets the ground.

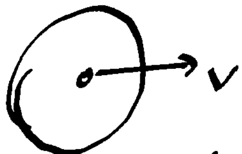
In This case, view as a rotating body abt Axis B

$KE = \frac{1}{2} I \omega^2$

$KE = \frac{1}{2} \left(\frac{3}{2} MR^2\right) \frac{v^2}{R^2}$

$KE = \frac{3}{4} mv^2$

2)

I 

Moving CM

+

II 

rotation About center of Mass

$KE_I = \frac{1}{2} mv^2$

$KE_{II} = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} MR^2\right) \frac{v^2}{R^2}$

$= \frac{1}{4} Mv^2$

$KE_{TOTAL} = \frac{3}{4} Mv^2$

$$\frac{1}{2}\,\frac{1}{2}MR^2\,\frac{v^2}{R^2} + \frac{1}{2}Mv^2 = Mgh$$

demos
MS-1
MS-2

$$\frac{1}{4}Mv^2 + \frac{1}{2}Mv^2 = Mgh$$

$$\frac{3}{4}Mv^2 = Mgh$$

$$\boxed{\frac{3}{4}\frac{v^2}{g} = h}$$

$$\frac{m^2/s^2}{m/s^2} = m \quad \text{units} \checkmark$$

Tell story about David's Pinewood Derby car

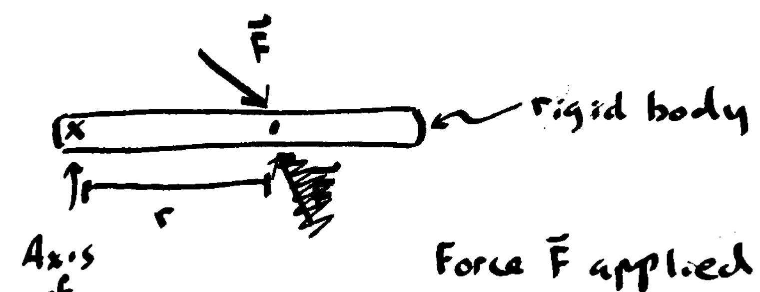
If sliding (non rotating)

$$\frac{1}{2}Mv^2 = Mgh$$

$$h = \frac{1}{2}\frac{v^2}{g}$$

Goes higher if moving at V and rotating because system starts w/ More KE.

Rotational Motion and Vectors



Force $\vec{F}$ applied to rigid body at a distance r from axis of rotation