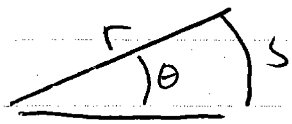


$$s = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$

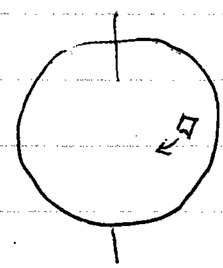
$$\omega = \omega_0 + \alpha t$$
$$\vdots$$

$$\tau = I\alpha$$

rF $\qquad$ $\sim mr^2$

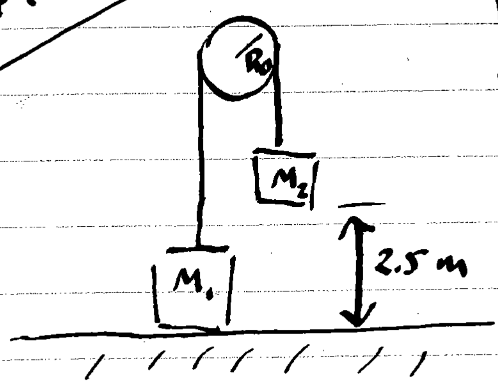
$$I = \int r^2\,dm = \int r^2 \rho\,dV$$

rotational KE



$$KE_{rot} \sim \sum \tfrac{1}{2} m_i v_i^2$$
$$= \frac{1}{2} \sum m_i r_i^2 \omega^2$$
$$KE_{rot} = \frac{1}{2} I \omega^2$$

~ 10-65 p274 Giancoli 3rd ed.



$m_1 = 35.0$ kg

$m_2 = 38.0$ kg

Pulley is uniform cylinder w/ radius $R_0 = 0.3$ m and Mass 4.8 kg

Initially m_1 is on grnd m_2 at rest 2.5 m above ground.
Assume rope is Massless and pulley axle is frictionless.
Assume rope does NOT slip.
Find Speed of m_2 just before striking the ground when system is released.

Solve using Energy conservation

$$E_{START} = E_{end}$$

$$m_2 g h = m_1 g h + \frac{1}{2} I \omega^2 + \frac{1}{2} M_2 V_2^2 + \frac{1}{2} M_1 V_1^2$$

$V_1 = V_2 = R_0 \omega$ relation between velocities

$I = \frac{1}{2} M R_0^2$ From Table

$$m_2 g h = m_1 g h + \frac{1}{2}\left(\frac{1}{2} M R_0^2\right)\frac{V^2}{R_0^2} + \frac{1}{2} M_2 V^2 + \frac{1}{2} M_1 V^2$$

$$(m_2 - m_1) g h = \frac{1}{4} M V^2 + \frac{1}{2} M_2 V^2 + \frac{1}{2} M_1 V^2$$

$$(m_2 - m_1) g h = \left(\frac{M}{4} + \frac{M_2}{2} + \frac{M_1}{2}\right) V^2$$

$$V = \pm\sqrt{\frac{(m_1 - m_2)gh}{\left(\frac{M}{4} + \frac{M_2}{2} + \frac{M_1}{2}\right)}}$$

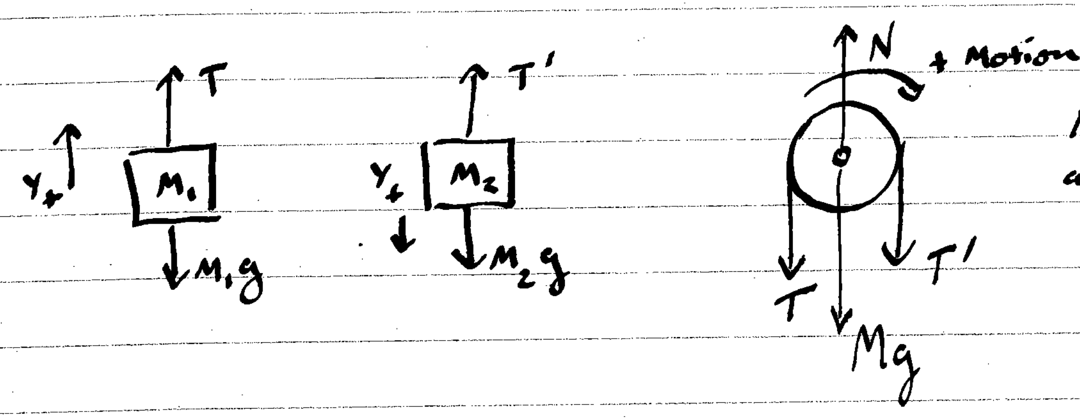
$$V = \pm\sqrt{\frac{(3)(9.8)(2.5)}{\frac{4.8\,\cancel{0.3}}{4} + \frac{38}{2} + \frac{35}{2}}} = \pm 1.4\ \mathrm{m/s}$$

$\cancel{\pm}1.4\ \mathrm{m/s}$ down

Speed

Solve using Newton's Laws

FBD^s just after M_1 in air ... no normal force



N of axle and Mg cancel

$\Sigma F_y = m_1 a_1$ $\qquad$ $\Sigma F_y = M_2 a_2$ $\qquad$ $\Sigma \tau = I\alpha$

$T - M_1 g = M_1 a_1$ $\qquad$ $m_2 g - T' = m_2 a_2$ $\qquad$ $R_0 T' - T R_0 = I\alpha$

relate Accelerations $\quad a_1 = a_2 = I\frac{a}{R_0}$

(I) $T - M_1 g = M_1 a$ $\qquad$ (II) $m_2 g - T' = m_2 a$ $\qquad$ (III) $R_0 T' - R_0 T = I\frac{a}{R_0}$

3 eqns, 3 unknowns → T, T', a $\quad$ Solve for a

once a is known use linear 1-d Const a eqns to get V

Sub I and II into III solve for a

$$R_0 m_2(g-a) - R_0 m_1(a+g) = \frac{1}{2} M R_0^2 \frac{a}{R_0}$$

$$m_2 g - m_2 a - m_1 a - m_1 g = \frac{1}{2} M \cancel{R_0}\, a$$

$$(m_2 - m_1)g = \left(\frac{1}{2} M \cancel{R_0} + m_1 + m_2\right) a$$

$$a = \frac{(m_2 - m_1)g}{\left(\frac{1}{2}M + m_1 + m_2\right)} = \frac{(3)(9.8)}{\left(\frac{\overset{4.8}{\cancel{0.5}}}{2} + 35 + 38\right)}$$

$\cancel{4.8}$

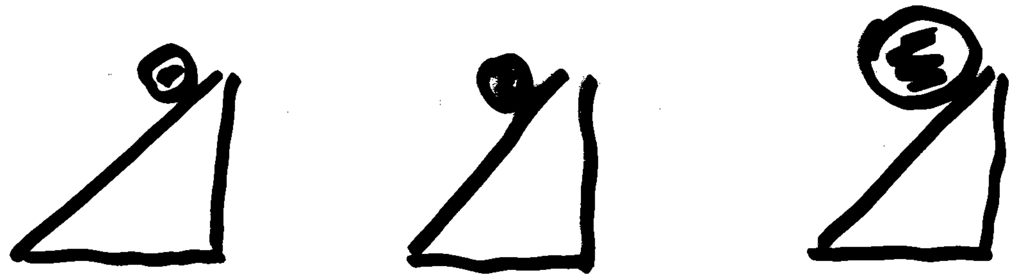
$a = 0.4\ m/s^2$ down for M_2

$$V_2^2 = V_{02}^2 + 2(a)h$$

$$V_2^2 = 2(0.4)(2.5) = 2 \qquad V_2 = 1.4\ m/s$$

Which Method of Soln is correct?

Discuss Angular "directionality" in this problem



A: Lead B: Wood C: Wood

$R_A = R_B < R_c$

$I_{sphere} = \frac{2}{5} M R^2$

Rank velocities at the bottom

1. $V_A = V_B = V_c$
2. $V_A > V_B = V_c$
3. $V_A = V_B < V_c$
4. $V_A < V_B < V_c$
5. $V_A > V_B > V_c$
6. $V_A < V_B = V_c$

Energy Conservation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$

$$mgh = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2$$

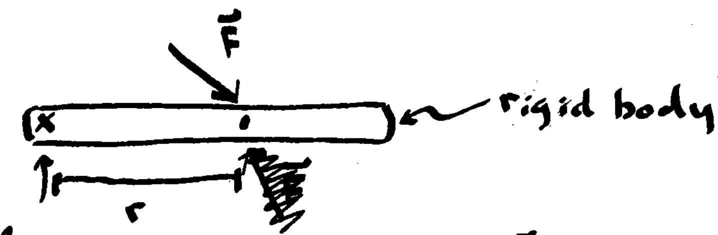
$$mgh = \frac{7}{10}Mv^2$$

$$v^2 = \frac{10}{7}gh$$

$\Rightarrow$ No R, M, or ρ dependence

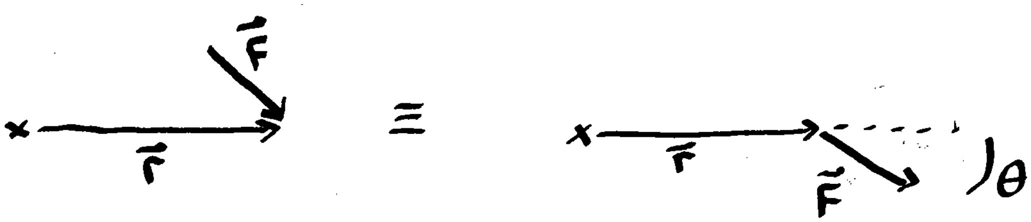
$$V_A = V_B = V_C$$

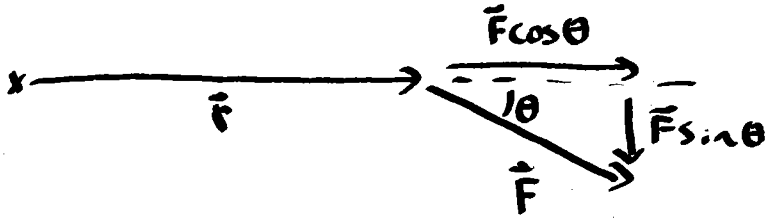
Rotational Motion and Vectors



Axis of rotation

Force $\vec{F}$ applied to rigid body at a distance r from axis of rotation





note: component of $\vec{F}$ along $\vec{r}$ cannot have an effect on the rotational motion!

... just pulls on axis more or less

component of $\vec{F}$ $\perp$ to $\vec{r}$ is the component that ~~rotates body~~ ... provides force (Torque)

Torque effective in producing rotational motion

$\hookrightarrow rF \approx (\vec{r})(\vec{F}\sin\theta)$

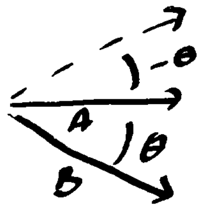
in dot product recall

$$\vec{A}\cdot\vec{B} = |\vec{A}|\,(\text{component of } \vec{B} \parallel \text{ to } \vec{A})$$

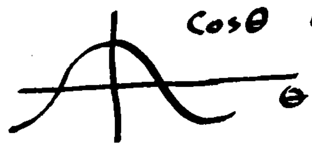
$\rightarrow$ scalar $\quad \parallel$

Another way to say $\quad \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} \qquad (\text{comp of } \vec{A} \parallel \text{ to } \vec{B})\,|\vec{B}|$

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta = |\vec{A}||\vec{B}|\cos(-\theta)$$



$|\vec{B}|\cos\theta = |\vec{B}|\cos(-\theta)$

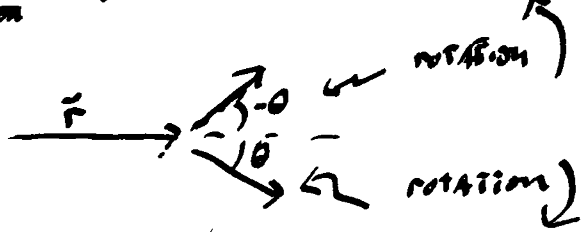


$\cos\theta$ is symmetric fn about 0

in our case this is NOT true

Torque $= |\vec{r}|\,(\text{component of } F \perp \text{ to } \vec{r})$

But now

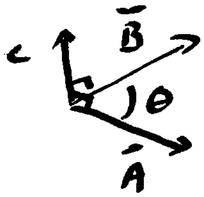


Sign of θ matters!

$\sin\theta$ is NOT a symmetric fn

Vector cross product $\Rightarrow$ (Two vectors $\rightarrow$ vector)

$\vec{A} \times \vec{B} = \vec{C} = |\vec{A}||\vec{B}| \sin\theta$ w direction given by Right hand rule



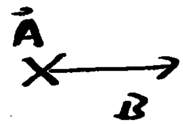
Right Hand Rule

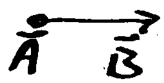
Put fingers of Right hand along vector $\vec{A}$ ... curl toward $\vec{B}$
Thumb will point along $\vec{C}$

$\vec{B} \times \vec{A} = ?$

... in opposite direction!!

Give me directions of $\vec{C}$ for practice





Tell Patti Eller story abt Right hand Screw rule

You can see this has all the parts we need for Making a Mathematical Model of Torque

$\vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin\theta$

θ is opening angle between $\vec{r}$ and $\vec{F}$

$|\vec{F}|\sin\theta$ is $\perp$ component of $\vec{F}$ (relative to $\vec{R}$)