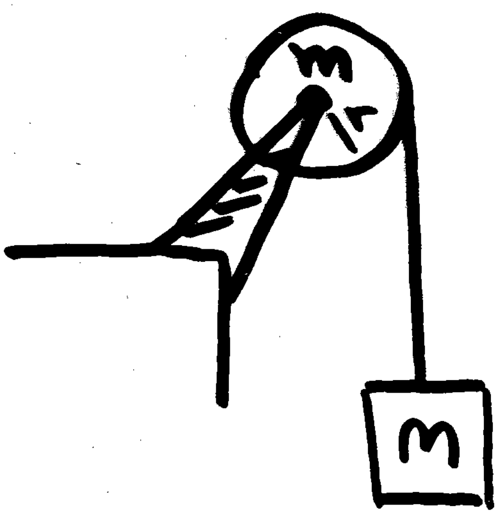


$I_{cylinder} = \frac{1}{2} m r^2$

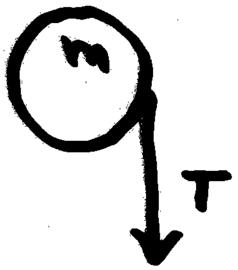
Find correct expression for a of M

① g

② $\left(\frac{M-m}{M+m}\right)g$

③ $\left(\frac{1}{2}mr^2 + M\right)g$

④ $\left(\frac{2M}{2M+m}\right)g$



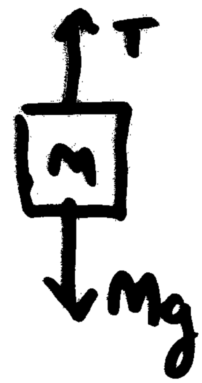
$$\tau = I\alpha$$

$$Tr = I\alpha$$

$$I = \tfrac{1}{2} m r^2 \qquad a = r\alpha$$

$$Tr = \frac{1}{2} m r^2 \frac{a}{r}$$

$$T = \frac{1}{2} m a$$



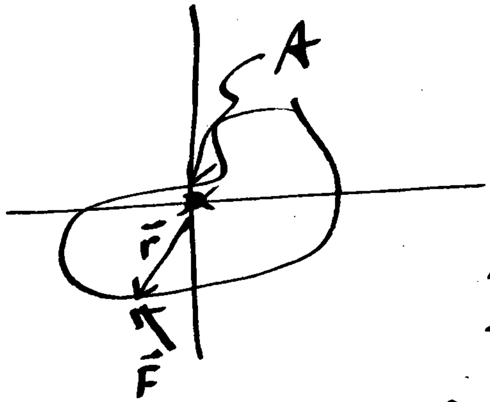
$$Mg - T = Ma$$

$$Mg - \frac{1}{2} m a = Ma$$

$$2Mg = 2Ma + ma$$

$$\boxed{a = \frac{2Mg}{2M + m}}$$

rigid body able to rotate about an Axis A



A force $\vec{F}$ acts on The body at a position $\vec{r}$ relative to the Axis of rotation.

The Torque of $\vec{F}$ about A is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

in example above $\vec{\tau}$ would be into paper

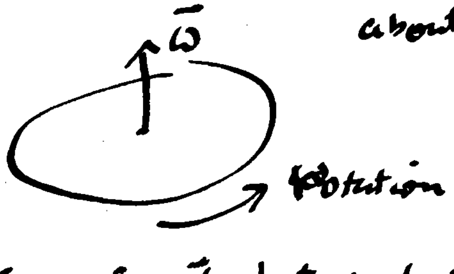
it would act to produce a clockwise Angular Acceleration

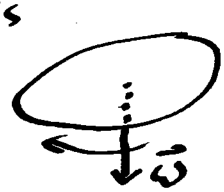
$\vec{\alpha}$, $\vec{\omega}$ are vectors, too! What are their directions?

determine direction w/ the second of our right hand rules

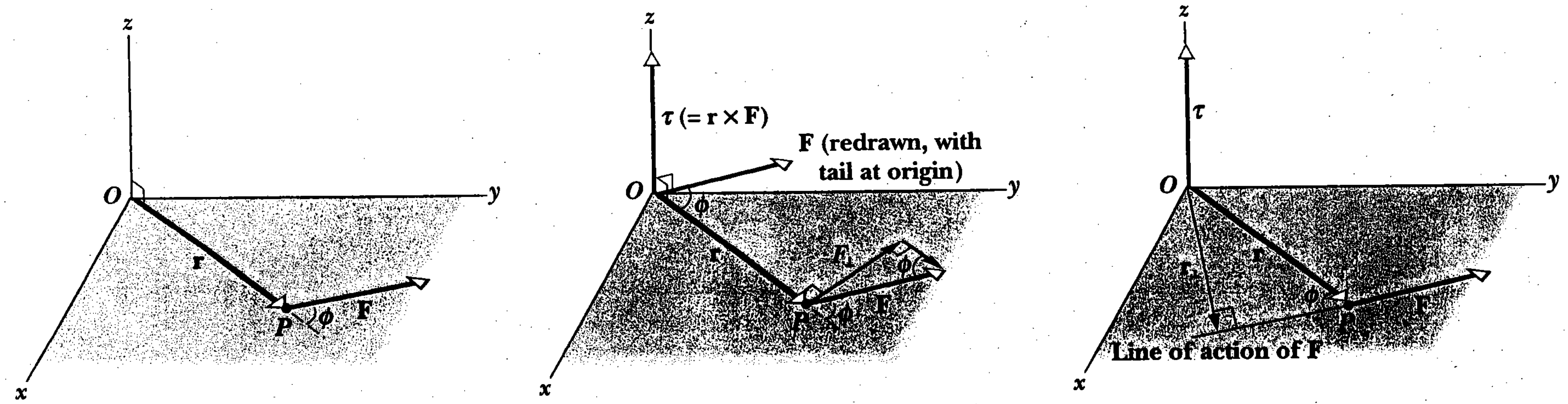
curl fingers of Right hand in direction of rotation ... Thumb will point in direction of ω

about Axis

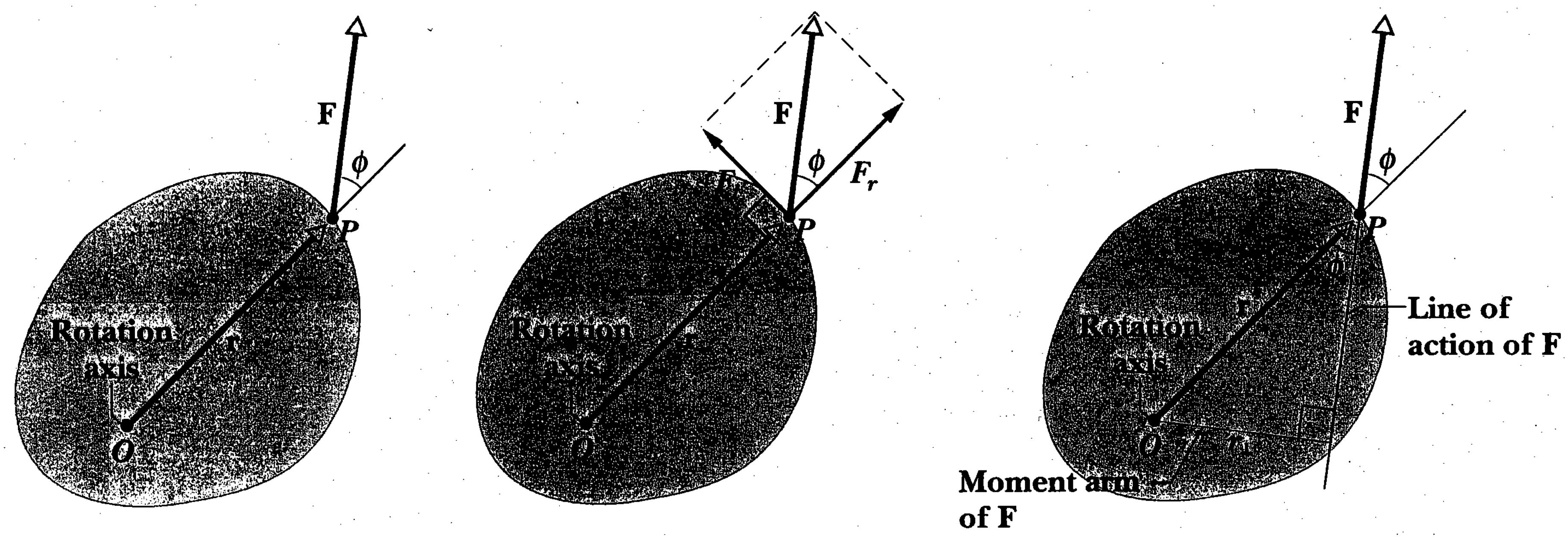




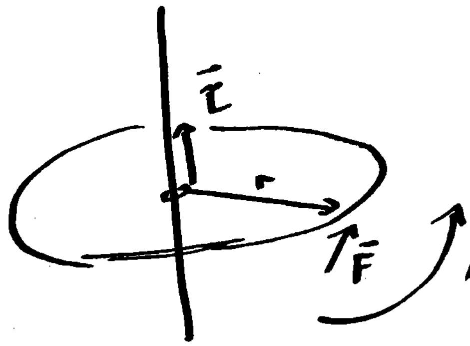
Same for $\vec{\alpha}$ but curl fingers in the direction of α increasing ω about axis. Thumb will point in direction of $\vec{\alpha}$


z
y
x
O
r
P
ϕ
F
τ (= r × F)
F (redrawn, with tail at origin)
$F_\perp$
τ
Line of action of F

(a) (b) (c)


F
ϕ
P
Rotation axis
r
O
F_r
Line of action of F
Moment arm of F

(a) (b) (c)

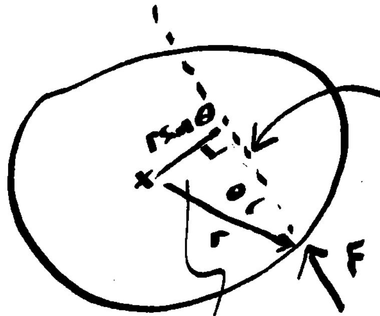


Torque is a vector up

usually drawn at axis of rotation

Motion

Counter clockwise



line of Action

or

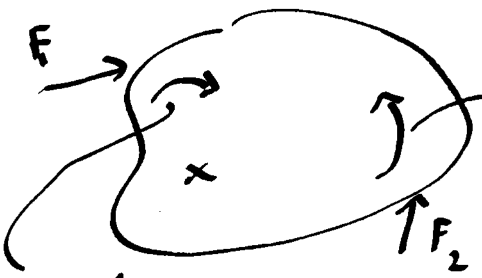
$\vec{r} \times \vec{F} =$

$|\vec{F}||\vec{r}|\sin\theta$

lever Arm

$\vec{r} \times \vec{F} = |\vec{r}||\vec{F}| \sin\theta$

Torque



sense of rotation is counterclockwise

due to force 2

(+) Torque about Axis

sense of rotation caused by F_1 is Clockwise

(−) torque about Axis

Unit of torque is N·M

Right Hand Rules ... Practice

what direction is $\vec{\omega}$?

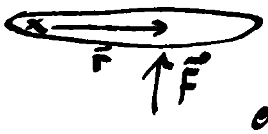
$\vec{\omega}$ out of paper

Suppose $\vec{\alpha}$ is increasing $|\vec{\omega}|$ ... what direction is $\vec{\alpha}$ ...

out of paper

Suppose $\vec{\alpha}$ is reducing $|\vec{\omega}|$ ... what direction is $\vec{\alpha}$

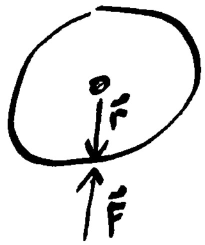
into paper



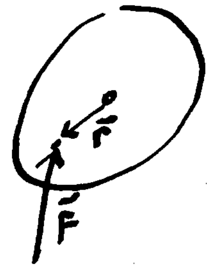
$\vec{\tau}$ is in what direction?

$\vec{\tau} = \vec{r} \times \vec{F}$

out of paper



0 Torque



into paper

Get good at these Right hand Rules !!

- use to find $\vec{\omega}$, $\vec{\alpha}$ directions
- use to find $\vec{C} = \vec{A} \times \vec{B}$ direction
- use to find $\vec{\tau} = \vec{r} \times \vec{F}$ direction

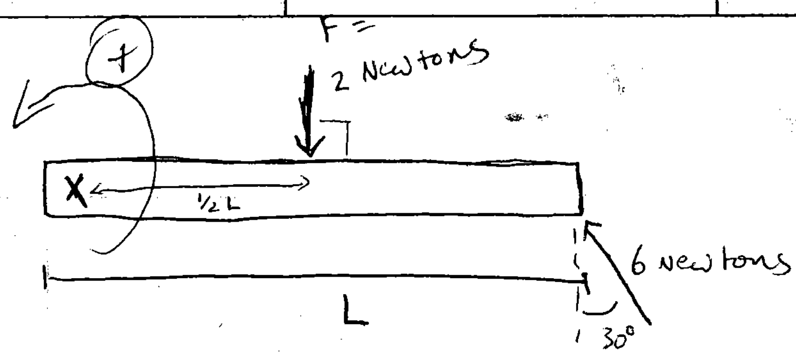
OK So pro on rotational Motion + linear Motion

Do demo on precession of bicycle wheel

Grinding wheel

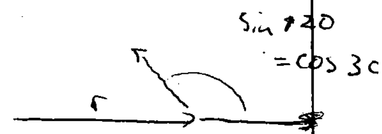
What is The direction of $\vec{\alpha}$?

1. up
2. down
3. Right
4. Left
5. in
6. out
7. in direction of $\vec{F}$



A rod of length, L, is free to rotate about ~~a point~~ an axis at one end
The forces shown act on the rod at a given instant.
The mass of the rod is 3 kg, L = 2 M
Find $\vec{\alpha}$ of rod at this instant.

$\Sigma \vec{\tau} = I \vec{\alpha}$



$I = \frac{1}{3} ML^2$ from Table

$$\Sigma \vec{\tau} = -(2)(\tfrac{1}{2}L) + L[6 \cos 30]$$

(clockwise)

Component of F ⊥ to r

$$\Sigma \vec{\tau} = -2 + (2)(5.2) = 8.4 \ N \cdot M$$

$$\therefore \quad 8.4 (N \cdot m) = \underbrace{\tfrac{1}{3} ML^2}_{\frac{1}{3}(3)(2)} (kg\, m^2)\, \alpha$$

$kg\, M/s^2$

$kg \frac{M^2}{s^2} = kg\, M^2 \qquad \alpha \sim \frac{1}{s^2}$ ✓

$\frac{8.4}{2} = \alpha = 4.2 \ rad/s^2$ counterclockwise as you look

$$\boxed{\vec{\alpha} = 4.3 \ rad/s^2 \text{ out of paper}}$$

We will do more w/ torque when we get to Static equilibrium.

Angular momentum

What do we mean by Momentum?

$$\frac{dp}{dt} = F \qquad dp = F\,dt$$

To change momentum you must exert a force for a time.

if Momentum is larger ... must exert a larger force to stop an object in same time interval

what is Angular ~~----~~ analogue of Momentum?

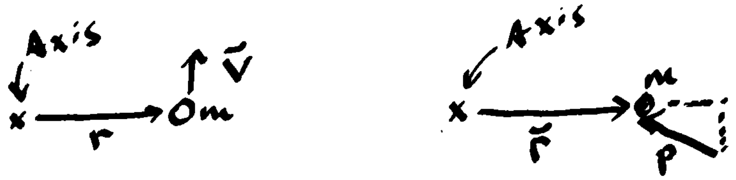
$$P = mv \qquad v = r\omega$$

Angular Momentum $\sim I\omega$

$$\sim mr^2\omega \sim r\,mr\omega \sim r\,mv \sim rp$$

Angular Momentum is a vector $\equiv \vec{L}$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = I\vec{\omega}$$ same direction as $\vec{\omega}$



Again cross product projects out the component of $\vec{P}$ $\perp$ to $\vec{r}$

JUST as in linear Momentum

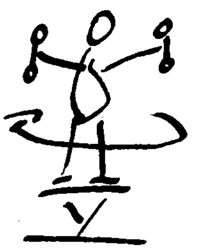
Angular Momentum is Conserved!

$$\sum \vec{L}_{initial} = \sum \vec{L}_{Final}$$

Let's do a couple of example problems to get the hang of some of this New Stuff —

Example

~~An ice skater with outstretched hands spins with angular velocity $\omega = 0.60$ rev/s~~



A student stands on a turntable with dumbbells held in outstretched hands. Initially the student is given an angular velocity of 0.5 revolutions/second.

The student then pulls the weights in close to his/her body. What is the students Angular velocity in the new configuration?

Given info

M of each dumbbell = 5 kg

Length of arm (to center of chest) ~ 0.6 m

$I_{body} \sim 0.40\ kg \cdot m^2$

Masses held close ... dist from body center = 0.2 m

Use Momentum Conservation:

$$L_{initial} = L_{final} \rightsquigarrow \sum I\omega\ \text{initial} = \sum I\omega\ \text{final}$$

$$(I_{body} + I_{mass1} + I_{mass2})\,\omega_{initial} = (I_{body} + I_{mass1} + I_{mass2})\,\omega_{final}$$

$$\left[0.4\ kg\,m^2 + (5\ kg)(0.6\ m)^2\, 2\right] 0.5\ \tfrac{rev}{s}\ 2\pi\ \tfrac{rad}{rev}$$

$$= \left[0.4\ kg\,m^2 + (5\ kg)(0.2\ m)^2\, 2\right] \omega_{final}$$

$$12.56\ \tfrac{rad}{s}\ kg\,m^2 = 0.8\ kg\,m^2\ \omega_{final} \qquad \omega_{final} = 15.7\ rad/s = 2.4\ rev/s$$

P113
S. Manly

NAME Soln key Slm

University of Rochester
Fall 1999

Problem 10 (10 pts) – zero/half/full credit:

The matter in stars is in equilibrium between the gravitational force pulling in radially and the "radiation pressure" of energy released by thermonuclear reactions pushing out. When they run out of nuclear fuel in the center (core), the radiation pressure is reduced and they collapse.

Under gravitational collapse, the radius of a spinning spherical star of uniform density shrinks by a factor of 2, with the resulting increased density remaining uniform throughout as the star shrinks. What will be the ratio of the final angular speed ω_2 to the initial angular speed ω_1? Select the correct answer below. You must show your supporting work to get credit.

$\omega 2/\omega 1$ = (a) 2 (b) 0.5 (c) 4 (d) 0.25 (e) 1.0

Angular Momentum is Conserved

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$I \text{ for Solid sphere} = \frac{2}{5} MR^2$$

$$\frac{2}{5} M R_i^2 \omega_i = \frac{2}{5} M R_f^2 \omega_f$$

$$R_i^2 \omega_i = \left(\frac{1}{2} R_i\right)^2 \omega_f$$

$$\frac{\omega_f}{\omega_i} = 4$$