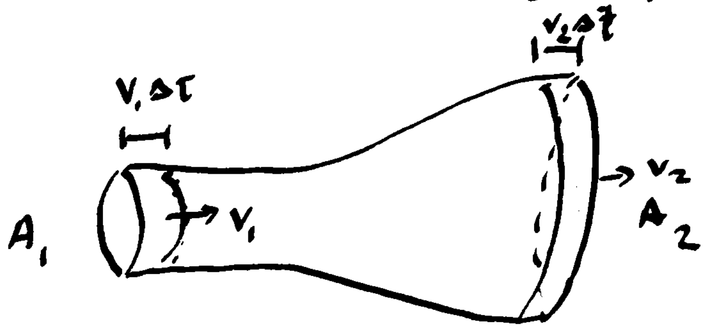
Hydrodynamics or Fluid Dynamics

Consider an ideal fluid

(NO internal friction → viscosity
and is NOT compressible)



A pipe

fluid is incompressible so ρ is constant

Amount of fluid to flow in at ①
= Amount of fluid to flow out at ②

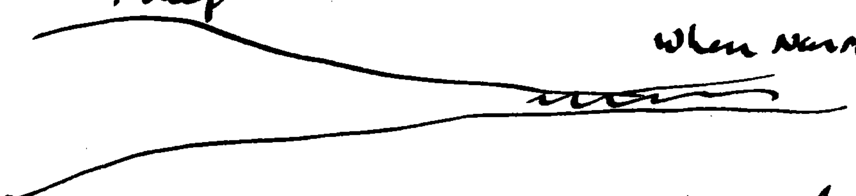
$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$\boxed{A_1 v_1 = A_2 v_2}$$ equation of Continuity

River

when broad + deep ... water flows slowly

when narrow or shallow water flows fast!



height variable can be confusing

Example

check this

Blood flow

radius Aorta = 1.0 cm

$|\vec{\text{Velocity}}|$ of blood thru Aorta is 30 cm/s

Capillary (Typical) $r = 4\times10^{-4}$ cm

$V_{\text{blood}} \sim 5\times10^{-4}$ m/s

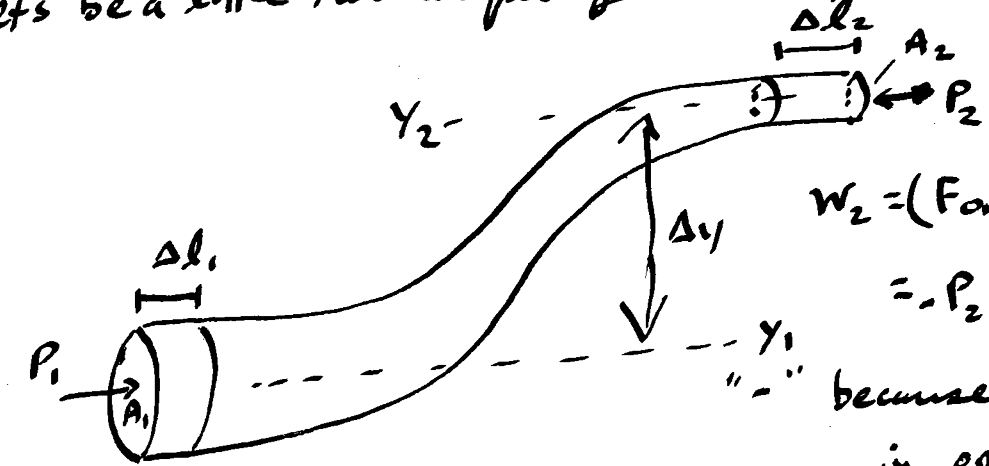
Estimate how many capillaries there are in body

$$A_1 V_1 = A_2 V_2$$

$$\left(\pi(.01)^2_{m^2}\right)(.3\ m/s) = N\,\pi\,(.04\times10^{-4}m)^2\,(5\times10^{-4}\,m/s)$$

$$N = 4\times10^{9} = 4 \text{ billion!}$$

let's be a little more careful about grav. P.E. + work, etc



$$W_2 = (F_{\text{on }A_2})\,\Delta l_2$$

$$= -P_2 A_2 \Delta l_2$$

"$-$" because $\vec{F}$ and $\vec{\Delta x}$ in opposite directions

$$W_1 = (F_{\text{on }A_1})\,\Delta l_1$$

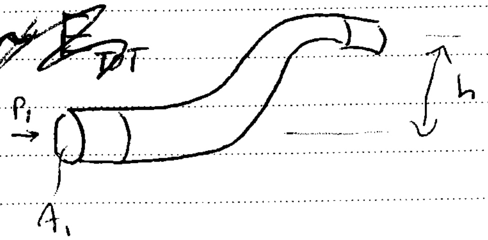
$$W_1 = P_1 A_1 \Delta l_1$$

W_1, W_2 = work done on fluid by pressure

Also have work done on fluid by gravity

$$W_3 = -mg\,\Delta y = -mg\,(y_2 - y_1)$$

$$W + \frac{1}{2}mv^2 + mgh \cancel{E_{TOT}} = 0$$



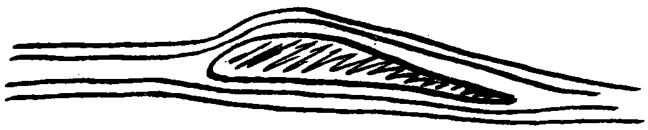
$$\frac{F \cdot d}{V} + \frac{1}{2}\frac{m}{V}v^2 + \frac{m}{V}gh$$

$$\frac{F \cdot d}{A \cdot d} + \frac{1}{2}\rho v^2 + \rho g h$$

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{CONSTANT}}$$

Bernoulli's equation

Why do you curl ?



Air foil on wing of airplane

Air on top moves faster than air on bottom

$$y_{top} \approx y_{bottom}$$

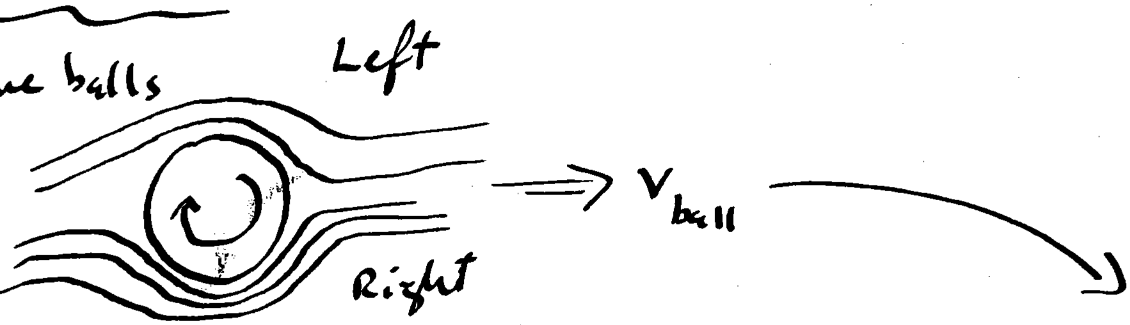
$$P_b + \frac{1}{2}\rho v_b^2 = P_{Top} + \frac{1}{2}\rho v_{Top}^2$$

$$V_{Top} > V_{bottom} \quad \therefore \quad P_{Top} < P_{BOTTOM}$$

This difference in pressure provides "Lift" (an upward force) !!

This is one component of why airplanes can fly !

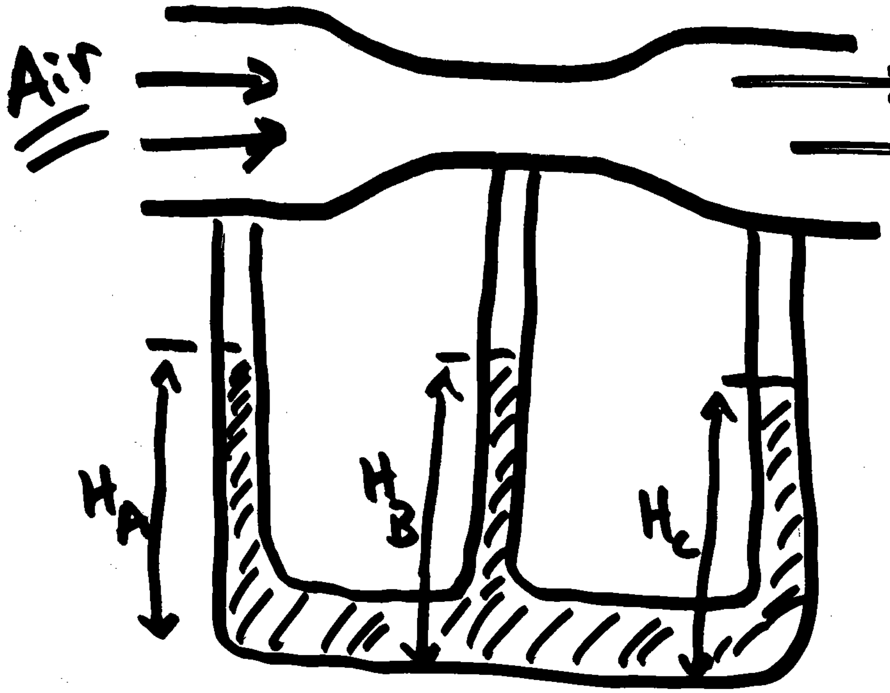
Curve balls



Rotating ball : on ~~bottom~~ Right friction between ball surface and air causes increase in Velocity of air passing by

on ~~top~~ left friction slows velocity of air passing by

$$\therefore \quad P_{\cancel{top}\ \text{Left}} > P_{\cancel{bottom}\ \text{right}} \ \ldots \Rightarrow \text{ball curves } \cancel{\text{left}}\ \text{right} !$$



Initially, NO Air Moves Thru Top tube

$H_A = H_B = H_C$

what happens when fast Air stream injected into top tube?

① $H_A > H_B > H_C$

② $H_A < H_B < H_C$

③ $H_A < H_B > H_C$, $H_A = H_C$

④ $H_A > H_B < H_C$, $H_A = H_C$

Bernoulli tells us p least where v greatest

$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{const}$$

(Area)

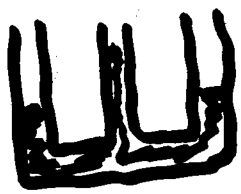
$$V A = \text{const}$$

Area A smallest above "B" tube

$\Rightarrow$ V greatest

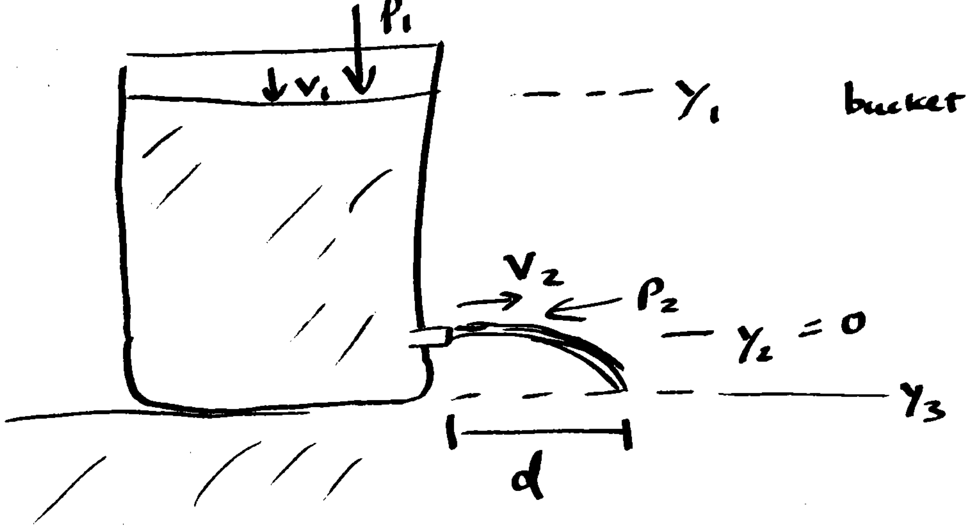
$\Rightarrow$ P least

$\Rightarrow$ liquid highest



$H_A = H_C$ due to symmetry of Tube

Bernoulli's eqn example



$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$P_1 \sim P_2 = P_{ATM}$$

$$h = y_1 - y_2$$

$$v_1 \approx 0$$

$$\therefore \cancel{\rho} g h = \tfrac{1}{2}\cancel{\rho} v_2^2$$

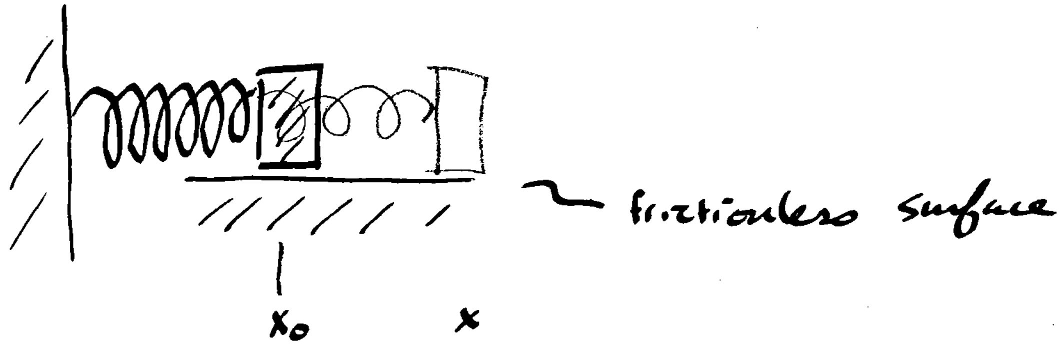
$$V_2 = \sqrt{2gh}$$

Standard Projectile Problem to find d

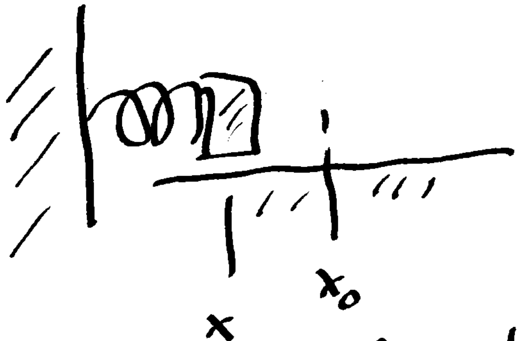
$$y_3 = y_2 + \underset{\overset{\parallel}{0}}{v_{0y}} t + \tfrac{1}{2} \overset{-9.8}{a} t^2 \Rightarrow t$$

$d = v_2 t$ subst. in

Simple Harmonic Motion (SHM)

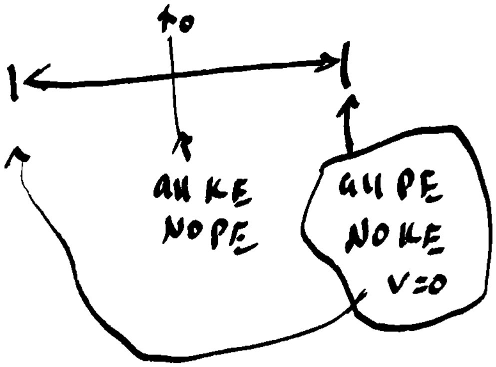


$$\leftarrow \vec{F} = -k(x - x_0)$$



$$\rightarrow \vec{F} = -k(x - x_0)$$

goes back + forth



Energy flow

$KE \rightarrow PE \rightarrow KE$

$\frac{1}{2}mv^2 \quad \frac{1}{2}k(x-x_0)^2$

$F = -kx \qquad x_0 = 0$

$ma = -kx$

$m\frac{d^2x}{dt^2} = -kx$

2ND order ordinary differential equation

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

differential equation
equation of Motion for a Simple Harmonic Oscillator

Solve this for $X(t)$ → tells where spring is as a fn of time

This is what you need usually

Let $x = A\cos(\omega t + \phi)$

Amplitude (A), Frequency (ω), initial phase (ϕ) — all just constants for now

general soln ... constants set by "initial" and "boundary" conditions specific to problem

$$\frac{dx}{dt} = -A\omega \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi)$$

Substitute into differential eqn

$$-A\omega^2\cos(\omega t + \phi) + \frac{k}{M}A\cos(\omega t + \phi) = 0$$

True if $\omega^2 = k/M$ or $\omega = \pm\sqrt{k/M}$

Let $x = A\sin(\omega t + \phi)$

$$\frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

substitute into differential eqn —

$$-A\omega^2\sin(\omega t + \phi) + \frac{k}{M}A\sin(\omega t + \phi) = 0$$

True if $\omega^2 = k/M$!

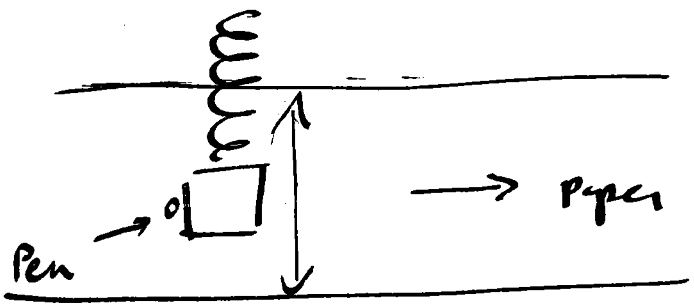
So our little differential eqn has solns

either $x(t) = A\cos(\omega t + \phi)$ oR $x(t) = A\sin(\omega t + \phi)$

Does it make sense?

Harmonic Functions

⇒ why we call it Simple Harmonic Motion!



let mass on spring oscillate on frictionless horizontal plane

View from above