wave on a string $$V = \sqrt{\frac{\text{Tension}}{\mu = \text{mass/length}}}$$

sound waves

$$V = \sqrt{\frac{B}{\rho}}$$

B — Bulk modulus — defines the compressibility of medium

ρ — volume density

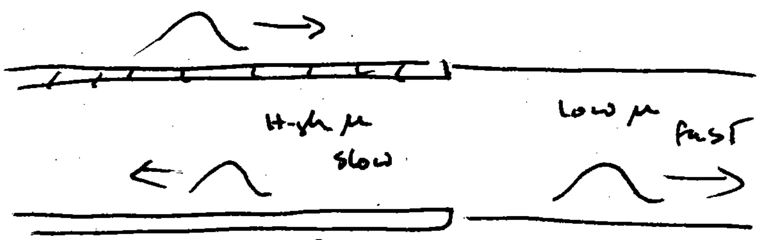
Transverse wave
Fixed end

Phase change of 180° at reflection

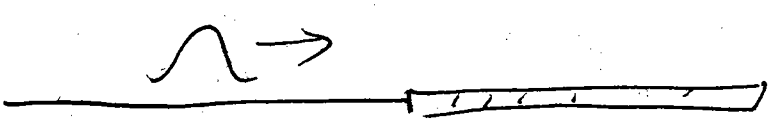
loose end

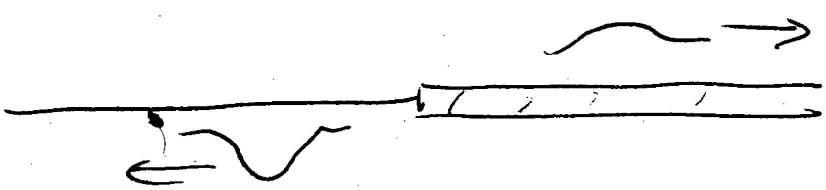
No phase change at reflection

Interface



slow to fast reflected wave has NO inversion





fast to slow reflected wave has inversion

$$y_1(x,t) = A \sin (kx - \omega t)$$

$$y_2(x,t) = A \sin(kx + \omega t + \phi)$$

Bound condition

$$\Rightarrow y_2(x,t) = A \sin(kx + \omega t + \pi)$$

Trig identity $\quad A \sin(X + \pi) = -A \sin X$

$$Y_2(x,t) = -A \sin(kx + \omega t)$$

Principle of Superposition

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= A \sin(kx - \omega t) - A \sin(kx + \omega t) \\ &= A \sin(kx - \omega t) + A \sin(-kx - \omega t) \end{aligned}$$

Trig identity

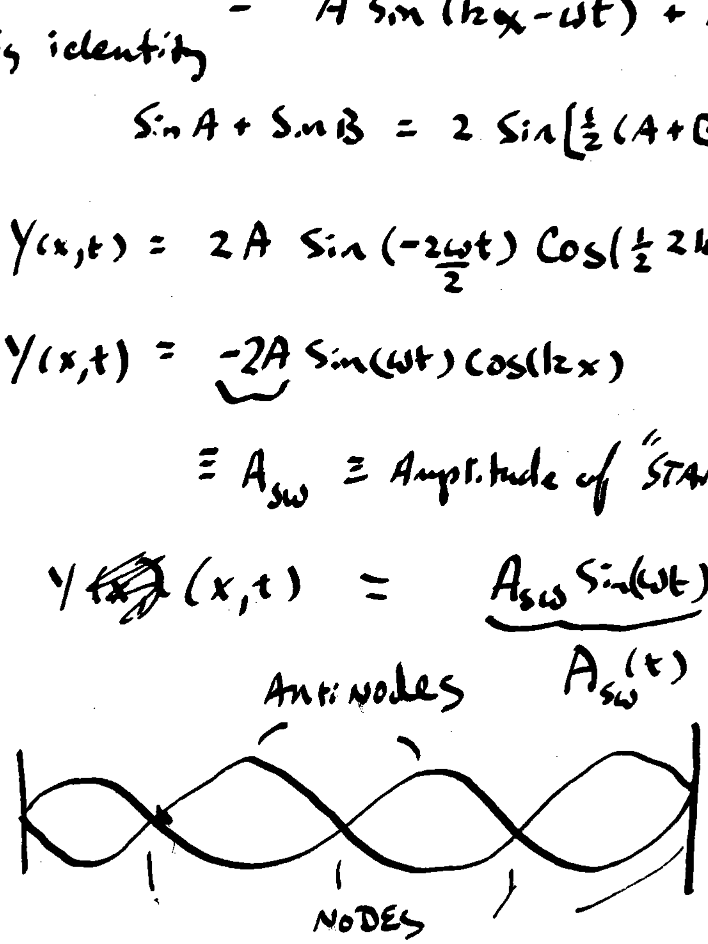
$$\sin A + \sin B = 2 \sin\left[\tfrac{1}{2}(A+B)\right] \cos\left[\tfrac{1}{2}(A-B)\right]$$

$$y(x,t) = 2A \sin\left(-\frac{2\omega t}{2}\right) \cos\left(\tfrac{1}{2} 2kx\right)$$

$$y(x,t) = \underbrace{-2A}_{} \sin(\omega t) \cos(kx)$$

$\equiv A_{sw} \equiv$ Amplitude of "STANDING Wave"

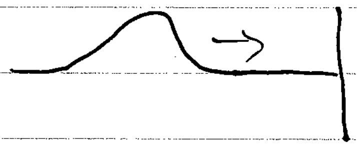
$$y(x,t) = \underbrace{A_{sw} \sin(\omega t)}_{A_{sw}(t)} \cos(kx)$$



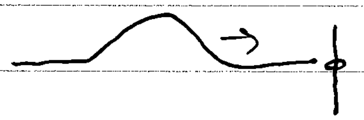
STANDING Waves

WAVES

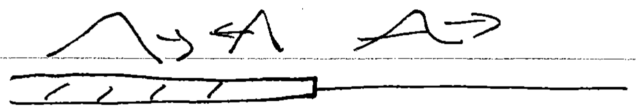
$$y(x,t) = A\sin(\overset{2\pi/\lambda}{k}x - \overset{\frac{2\pi}{T} = 2\pi\nu = 2\pi f}{\omega}t + \phi)$$

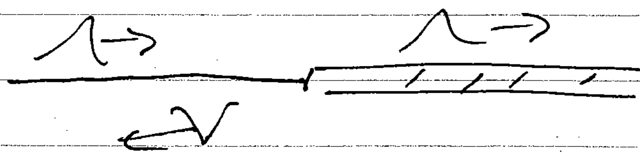


phase Δ of 180° at reflection



No phase change at reflection





Superposition – Displacement is sum of Displacement of individ. waves

reflected periodic harmonic waves on a string w/ fixed end

$$Y_1 = A\sin(kx - \omega t) \qquad Y_2 = A\sin(kx + \omega t + \pi)$$

$$Y(x,t) = \underbrace{B\sin(\omega t)}_{\text{Amplitude varies in time}}\ \underbrace{\cos(kx)}_{\lambda \text{ frozen in time along } x}$$

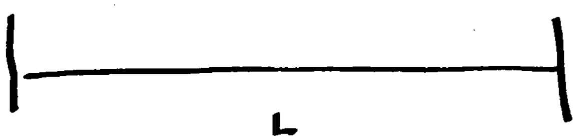
Amplitude at antinodes oscillates w/ time

at greatest it is $= 2A$

at least it is zero!

Nodes are every $\frac{1}{2}\lambda$

for a string tied on both ends ... each end must be a Node



L

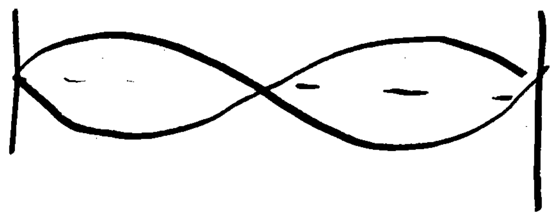
1^{ST} harmonic

These are so called Normal Modes!

fundamental frequency ... mode —

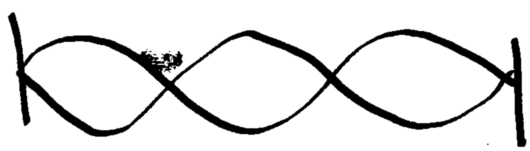
$\lambda = 2L$

overtones ↓



2^{ND} harmonic

$\lambda = L$



3^{RD} harmonic

$\lambda = \frac{2}{3}L$

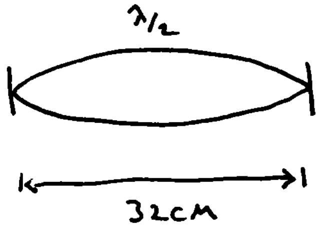
$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3 \cdots$$

wavelengths of the harmonics

ν_n = frequency of n^{th} harmonic $= \frac{v}{\lambda_n} = \frac{nv}{2L}$

Example

The G string on a violin has a fundamental frequency of 196 Hz. The length of the vibrating portion is 32 cm and has a mass of 0.68 g. What is the tension in the string?



$\lambda = 2(32\ cm) = 64\ cm$

$\nu = \frac{v}{2L}$

$(64)(196) = v$ in cm/s

$\mu = \frac{0.68\ g}{32\ cm} = $ g/cm $\quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{0}}$

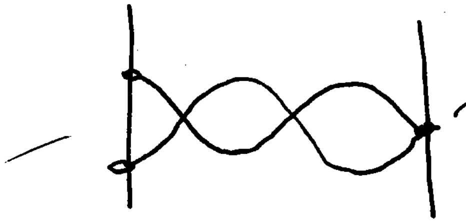
$T = v^2 \mu$

<u>Vibrating Strings</u> $\to$ violins, guitars, cellos etc...
pianos

We have "Normal Modes" for vibrating sound waves in tubes. Now, instead of string displacement we have air displacement or pressure variation.
Got to watch <u>Boundary conditions</u>!

String tied at each end $\Rightarrow$ has a node at each end
$\to$ usual, but not only case

Could have



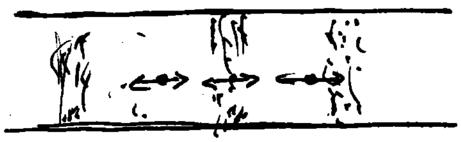
Tied end

end of ring about rod

Anti Node

because String is free to move up and down

Consider Standing Sound Waves in a tube



Tube w/ open end — Molecules ~~Air~~ can be easily moved

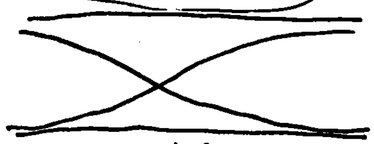
$\Rightarrow$ Displacement Anti Node

Tube w/ closed end

$\Rightarrow$ Displacement constrained

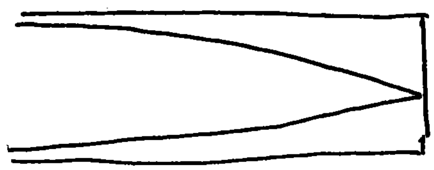
Displacement Node

Fundmental Modes



$L = \frac{1}{2}\lambda$

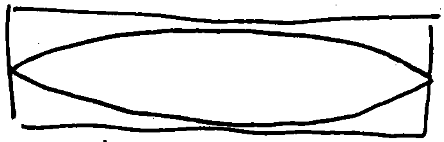
Two open ends



$L = \frac{1}{4}\lambda$

one open, one closed

graphical Measure of Longitud displacemen



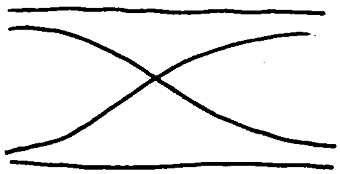
$L = \frac{1}{2}\lambda$

2 closed ends

Example What frequencies are available to an organ w/ 2 pipes of length L?
one open ended + one w/ 1 closed end

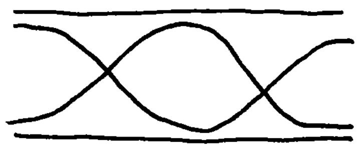
open end

fundamental



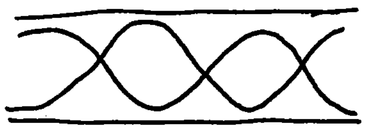
$L = \frac{1}{2}\lambda$

1st harmonic



$L = \lambda$

2ND harmonic



$L = \frac{3}{2}\lambda$

$$L = \frac{n}{2}\lambda_n$$

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3 \dots$$

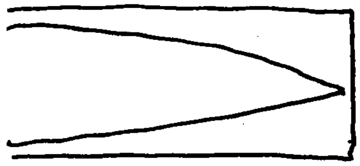
$$v = \nu_n \lambda_n$$

$$\frac{v}{\lambda_n} = \nu_n$$

$$\nu_n = \frac{vn}{2L} \qquad n = 1, 2, 3 \dots$$

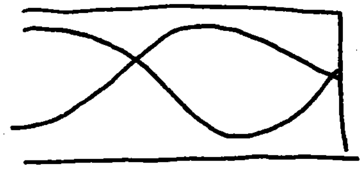
1 closed end

Fund.

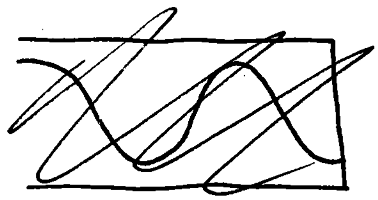


$L = \frac{1}{4}\lambda$

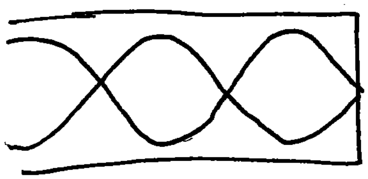
1st harmonic



$L = \frac{3}{4}\lambda$



2ND harmonic



$L = \frac{5}{4}\lambda$

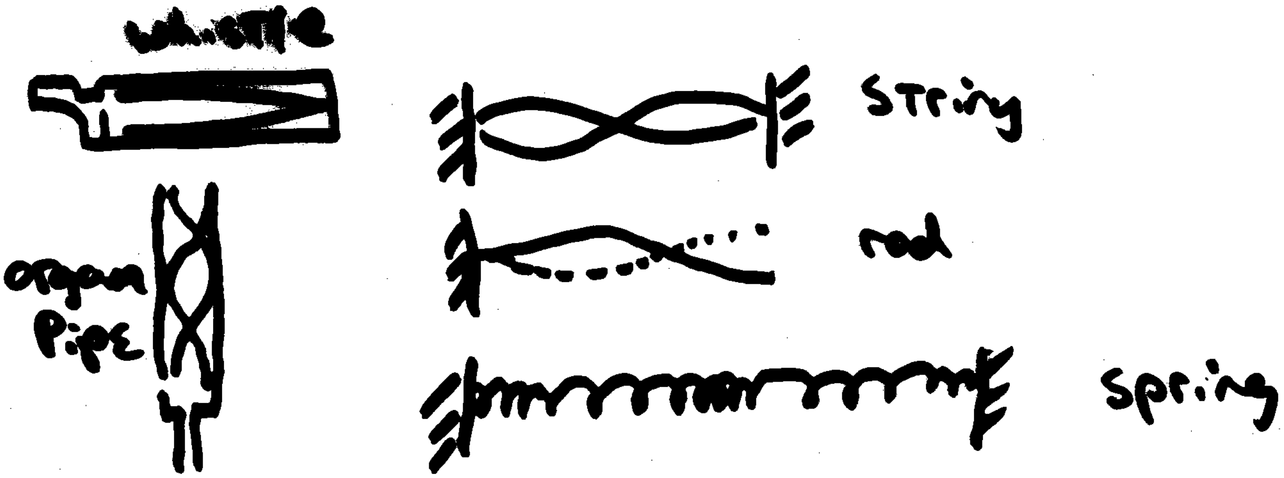
$$L = \frac{n\lambda_n}{4}$$

$$\lambda_n = \frac{4L}{n}$$

$$\frac{v}{\lambda_n} = \nu_n$$

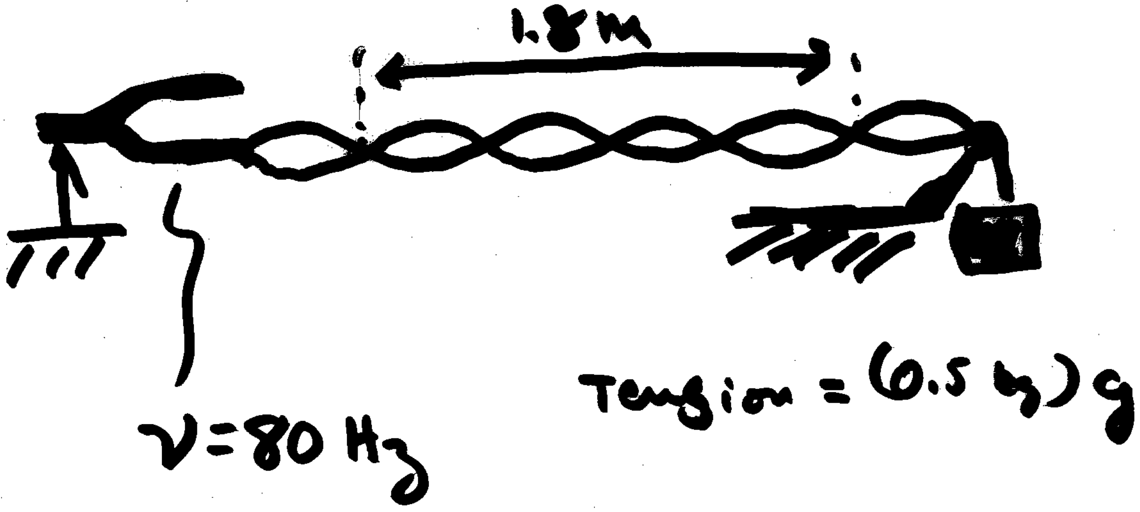
$$\nu_n = \frac{vn}{4L} \qquad n = 1, 3, 5 \dots$$

Only odd harmonics Available!



Which system vibrates with 1st harmonic?

1 Whistle
2 organ
3 string
4 rod
5 spring



The mass/length (μ) of this string is

1) 9×10^{-4} kg/m
2) 1×10^{3} kg/m
3) 4.3 kg/m
4) 3×10^{-2} kg/m
5) 6×10^{-3} kg/m
6) 3.8×10^{-2} kg/m

$$v = \sqrt{\frac{T}{\mu}}$$

$$v = \lambda \nu = (0.9\,m)(80\,Hz) = 72\ m/s$$

$$72 = \sqrt{\frac{(.5)g}{\mu}}$$

$$\mu = \frac{(.5)9.8}{72^2}$$

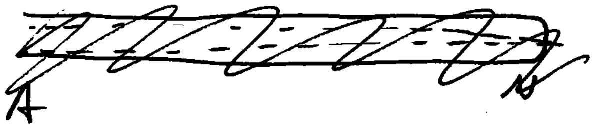
$$\mu = 9\times10^{-4}\ kg/m$$

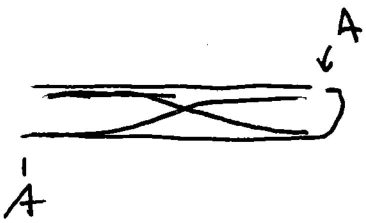
~~Example~~

Flute middle C (262 Hz)

as fund frequency

when all holes covered !





How long from mouthpiece to end.

Assume Temp = 20°c

V_{sound} Air at 20°c = 343 m/s

$\cancel{\nu = \frac{V}{4L}}$ $\cancel{L = \frac{V}{4\nu} = \frac{343}{2}}$

$\nu = \frac{V}{2L}$ $L = \frac{V}{2\nu} = \frac{343 \text{ m/s}}{2\,(262\ ^1/_s)}$ $= 0.655\text{m}$

Suppose the Temp is only 20°c

What is the fundamental frequency of the flute?

$\nu = \frac{V}{2L} = \frac{337}{2\,(.655)}$

V_{sound} at 10°c = 337 m/s

$\nu = 257$ Hz

Consider two waves passing a fixed point $(x=0)$
Let them differ slightly in frequency.
Let Amplitudes be equal.

$$x_1(t) = A\sin(k_1 x - \omega_1 t) = A\sin(\omega_1 t) \quad [\text{at } x=0]$$

Displacement due to wave 1 + wave 2

$$X_2(t) = A\sin\omega_2 t$$

$$X(t) = X_1(t) + X_2(t) = A\left[\sin(\omega_1 t) + \sin(\omega_2 t)\right]$$

Total Displacement   Principle of superposition

$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$

$$X(t) = A\,2\sin\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right]\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

$\omega_1 + \omega_2 \approx 2\bar{\omega}$ Small frequency Diff $\bar{\omega}$ = Ave freq.

$$X(t) = A\,2\underbrace{\sin(\bar\omega t)}_{\text{wave vibrates at Average frequency}}\;\underbrace{\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]}$$

Amplitude Modulated w/ Time at a frequency that depends on the difference between ω_1 and ω_2

Beat frequency!

Very sensitive way to detect small frequency differences