$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = I \vec{\alpha}$$

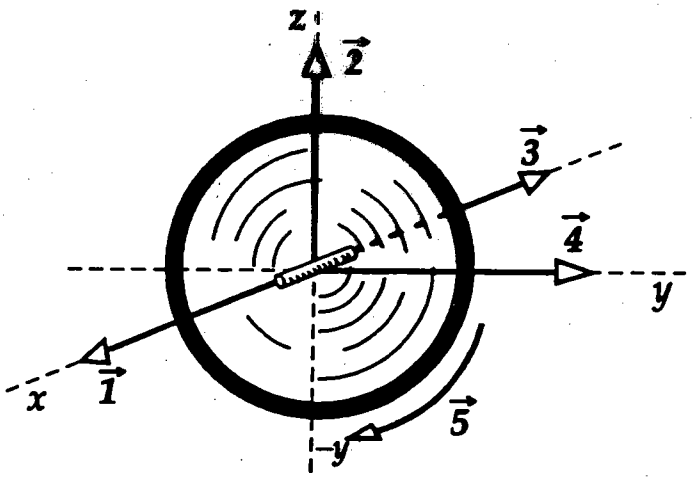
$$\vec{L} = \vec{r} \times \vec{p}$$

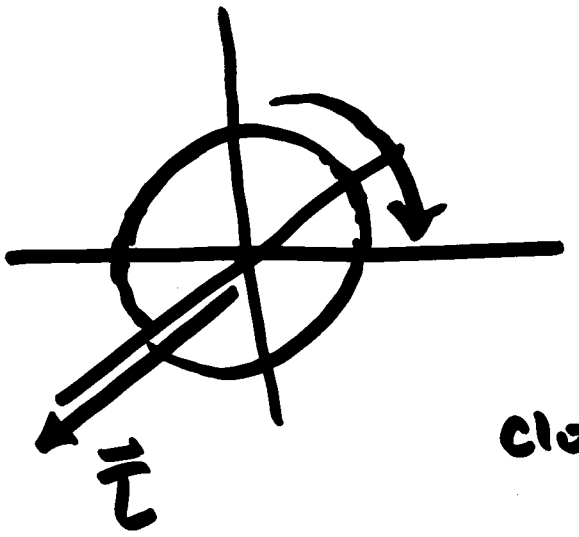
$$\vec{L} = I \vec{\omega}$$

In absence of External forces $\vec{L}$ is Conserved.

A wheel is rotating clockwise on a fixed axis perpendicular to the page (z). A torque that causes the wheel to slow down is best represented by the vector

a. $\vec{1}$
b. $\vec{2}$
c. $\vec{3}$
d. $\vec{4}$
e. $\vec{5}$





Clockwise Rotation

$\Rightarrow \vec{\omega}$ into paper

WANT Torque That causes wheel to slow down.

Slow down $\vec{\omega}$ smaller $\Rightarrow \vec{\alpha}$ out of paper

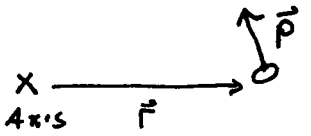
$\vec{\tau} = I\vec{\alpha}$ Torque in direction of $\vec{\alpha}$

$\therefore$ $\vec{\tau}$ out of paper

$\Rightarrow$ choice (1)

MPHZ

Angular Momentum



$$\vec{L} = \vec{r} \times \vec{p}$$

discrete body

$$\vec{L} = I\vec{\omega}$$

extended rigid body

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau} \quad \text{(just like } \frac{d\vec{p}}{dt} = \vec{F} \text{ in linear case)}$$

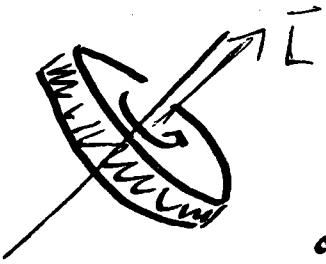
In the absence of an $\vec{\tau}_{NET}$ $\quad \frac{d\vec{L}}{dt} = 0$

Also, Angular Momentum is Conserved just as in Linear Momentum.

$$\Sigma \vec{L}_{init} = \Sigma \vec{L}_{final}$$

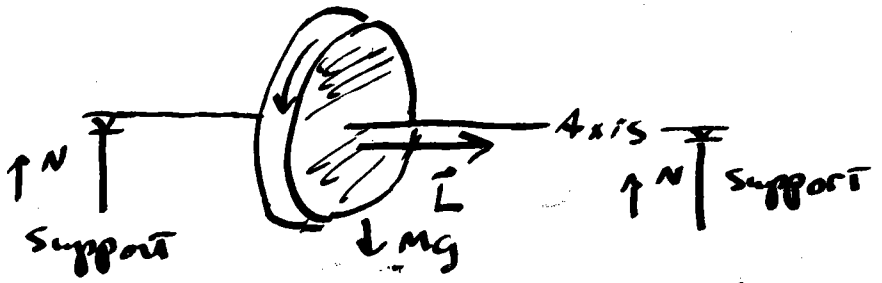
Precession and Gyroscopes

recall $\frac{d\vec{L}}{dt} = \vec{\tau}$



heavy disk spinning on an axis

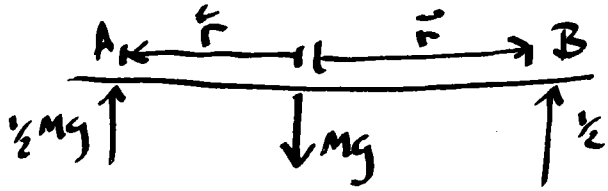
Linear Analog is $\frac{d\vec{p}}{dt} = \vec{F}$



heavy wheel spinning on horizontal axis

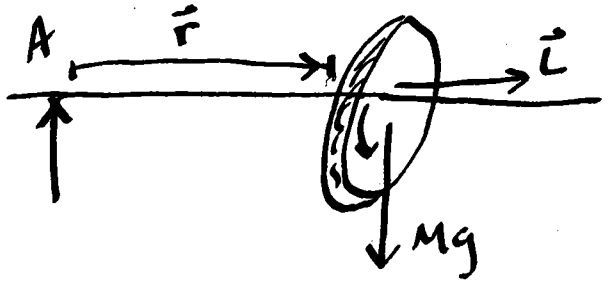
~~TAKE away one support~~

System is STABLE



+ Wheel Spins

Now Take away one support (remove support B)



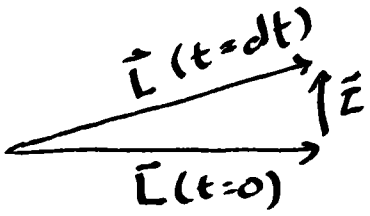
Now there is a Torque about point A

$$\vec{\tau}_A = \vec{r} \times \vec{F} = \vec{r} \times m\vec{g} = rmg \text{ into page}$$

Now recall

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

view from above



⇒ Axis + spinning disk supported at one end and free to rotate about support does NOT fall ⇒ IT rotates in the horizontal Plane !!

Is this wireless or what!?

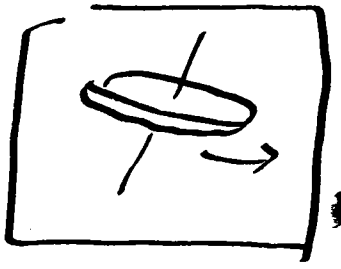
This is known as precession

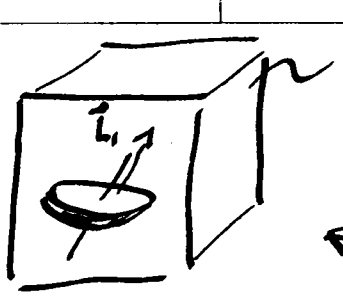
Have you ever noticed a Top spinning?

It Precesses

Imagine a system w/ a disk free to spin ~~in 3d~~ along an axis in 3d

i.e. Axis is free to rotate

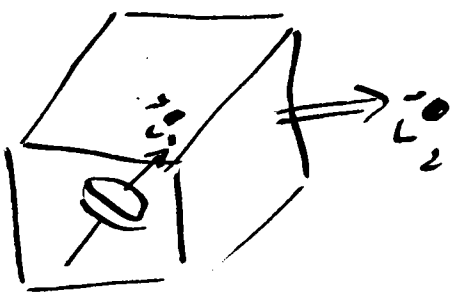




suppose box is rotated

what happens ?

TOTAL $\vec{L}$ is conserved



If $\vec{L}_2 \neq 0$

$\vec{L}_1$ must ~~change~~ change

to

cancel out $\vec{L}_2$ in vector sum !

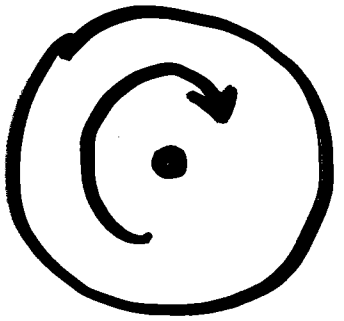
There is a change in the orientation of the spinning disk in response to the rotation of the box (about Any Axis)

Gyroscopes are a priceless tool for Navigation

$$mgr = \frac{dL}{dt} = |L| \frac{d\phi}{dt} = I\omega\Omega$$

$$\Omega = \frac{mgr}{I\omega}$$

Wheel ROTATES
As Shown



You pull down on the
end of Axle nearest you.
That end of the axle Moves

1. up
2. down
3. to the right
4. to the left
5. directly inward