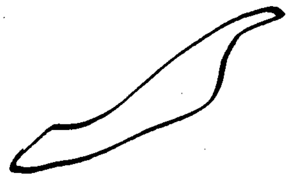
Static Equilibrium



A rigid body is in static equilibrium if $\sum \vec{F} = \vec{F}_{net} = 0$

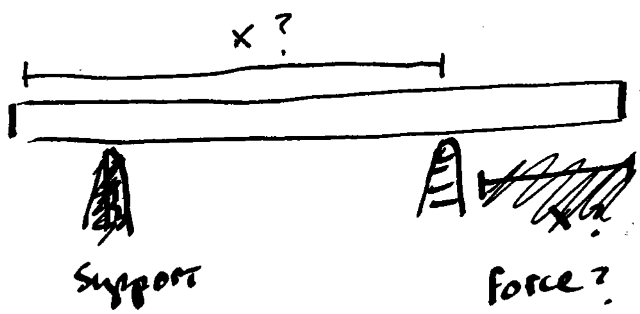
No linear Acceleration

And $\sum \vec{\tau} = \vec{\tau}_{net} = 0$

No ROTATION

Both eqns are true at all points & all possible Axes of rotation

Example



uniform Beam

$M = 12$ kg

$L = 10$ m

$W = 117.6$ N

2 m

This one pushes up w/ $\left(\frac{1}{3}\right)(12\text{ kg})\,9.8\text{ m/s}^2 = 39.2$ N

System is in Static equilibrium.
Where is the 2ND support located?
How much weight does it support?

Recall:

We can treat the mass of an extended body as being concentrated at a single point called the center-of-mass.

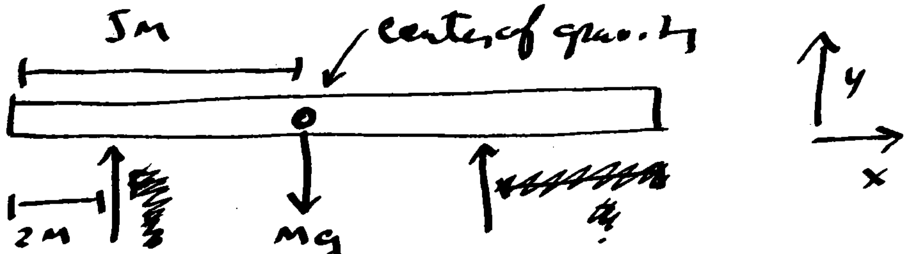
i.e., an external force acting on the body ~~is the same~~ will have the same effect as that force acting on the center of mass.

So ... when thinking about the effect of gravity in static equilibrium problems, it can be treated as if it acts on a single mass located at the center of mass of the extended body (usually obvious due to symmetry)

$\Rightarrow$ called the center-of-gravity.

<u>SO</u> back to our problem -

Can consider the mass of the plank to be concentrated at a ~~length~~ length $= \frac{1}{2}L$ (center of the plank)



$F_1 = \frac{1}{3}(9.8)(12\ kg) = 39.2\,N$

$F_2 = ?$ located where?

<u>Linear</u> equilibrium

$\Sigma F_x = 0 = ma_x = 0$ does not give us any information

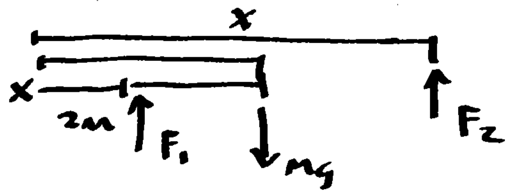
$\Sigma F_y = 0 = F_1 + F_2 - Mg$

<u>Rotational Equilibrium</u>

Pick an axis for rotation + be consistent

Where? ... Anywhere ... Make it somewhere easy for calculation

Let's sum Torques ~~about~~ around left end



$$\sum \tau = 0 = (2)F_1 - Mg(5) + x F_2$$

2 eqns

$$F_1 + F_2 - Mg = 0$$

$$2F_1 + xF_2 - Mg(5) = 0$$

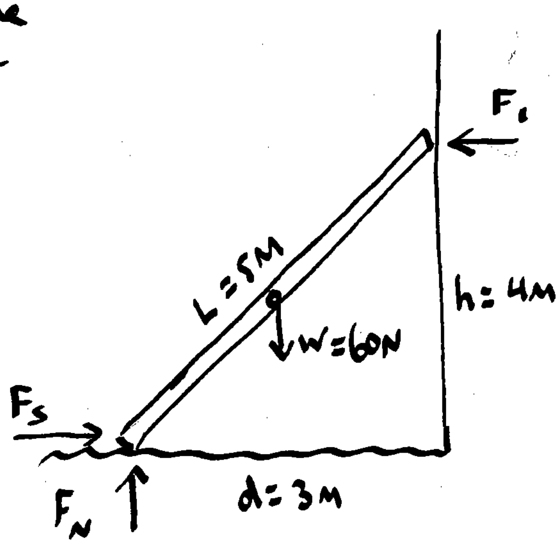
$$39.2\ N + F_2 - 117.6\ N = 0$$

$$F_2 = 78.4\ N$$

$$2(39.2\ N) + x(78.4\ N) - (117.6)5 = 0$$

$$x = 6.5\ m$$

Example



Ladder up against a frictionless ~~floor~~ wall

floor has friction

What is minimum coeffic. of static friction such that ladder will NOT slip?

Solve for μ_s such that ladder is in static equilibrium

$\Rightarrow$ this will be the minimum μ_s !

$$\Sigma F_x = m a_x = \underset{\text{static}}{0} = F_s - F_1 \quad \Rightarrow \quad F_s = F_1$$

$$\Sigma F_y = m a_y = \underset{\text{equilibrium}}{0} = F_N - W \quad \Rightarrow \quad F_N = W$$

$$\Sigma \tau_{\text{about base of ladder}} = F_1 h - W\frac{d}{2} = I\omega = 0 \quad \text{static equilibrium}$$

Could choose any axis $\quad F_1 h = W d/2$

Choose about base of ladder because it means 2 forces $\rightarrow$ give zero torque $\rightarrow$ gives the simplest calculation

3 eqns

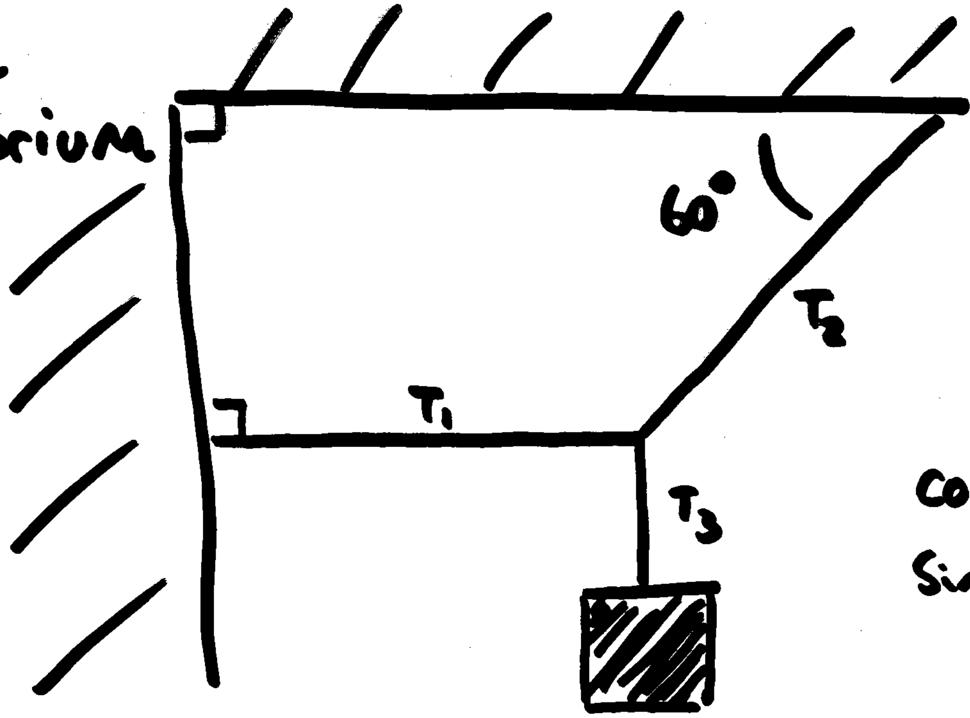
$$F_s = F_1 \quad \rightarrow \quad \mu_s F_N = F_1$$

$$F_N = W \quad \rightarrow \quad F_N = Mg = 60\,N$$

$$F_1 h = W d/2 \quad \rightarrow \quad \mu_s F_N h = W d/2$$

$$\mu_s = \frac{60 \frac{3}{2}}{4 \cdot 60} = \frac{3}{8}$$

Object is in STATIC Equilibrium

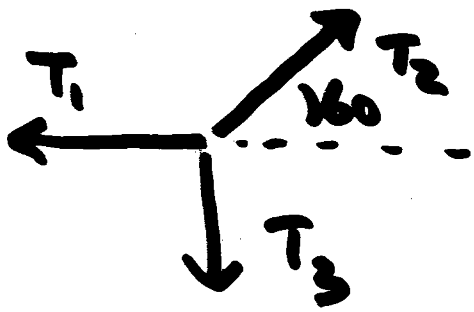


$\cos 60 = 0.5$

$\sin 60 = 0.87$

Which STATEMENT is True?

1. $T_1 > T_2$ and $T_2 > T_3$
2. $T_2 > T_1$ and $T_1 > T_3$
3. $T_2 > T_3$ and $T_3 > T_1$
4. $T_3 > T_1$ and $T_1 > T_2$
5. $T_1 > T_3$ and $T_2 > T_3$



$$\Sigma F_x = 0 = T_1 - T_2 \cos 60 \rightarrow T_1 = T_2 \overbrace{\cos 60}^{0.5}$$

Equilibrium

$$T_2 > T_1$$

$$\Sigma F_y = 0 = T_2 \sin 60 - T_3 \rightarrow T_3 = T_2 \underbrace{\sin 60}_{0.866}$$

$$T_2 > T_3$$

Eliminate T_2 to find relation between T_1 and T_3

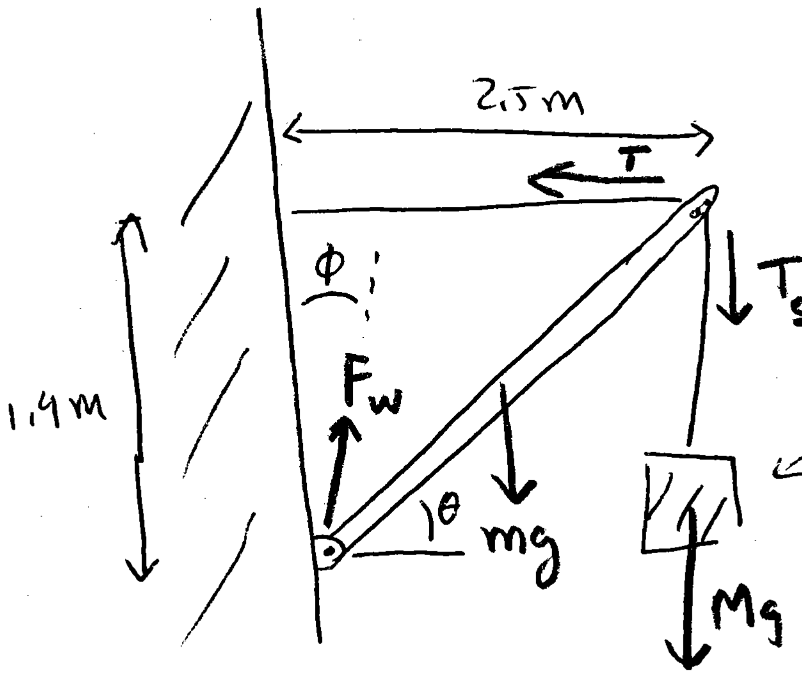
$$T_1 = \frac{T_3}{\sin 60} \cos 60 = T_3 \frac{(.5)}{(.87)}$$

$$T_3 > T_1$$

$$\boxed{T_2 > T_3 > T_1}$$

Ans. is $\boxed{3}$

beam $M = 85$ kg



Safe w/ Mass = 430 kg

Find T in uppercable!

how do I know direction of F_w ??

I $\quad \sum F_x = 0 = F_w \sin\phi - T$

II $\quad \sum F_y = 0 = F_w \cos\phi - mg - T_s$

T_s = Mg from FBD of Safe

III $\quad \sum \tau_{\text{abt WAll support}} = mg(1.25) + Mg(2.5) - T(1.9)$

3 eqns 3 unknowns T, ϕ, F_w

Solve

from $\underline{\text{III}}$ $\quad T = \dfrac{\overset{85}{m}g\,1.25 + \overset{430}{M}g\,(2.5)}{1.9} = 6093\ N$

Find ϕ

$\text{I} \qquad F_w \sin\phi = T$

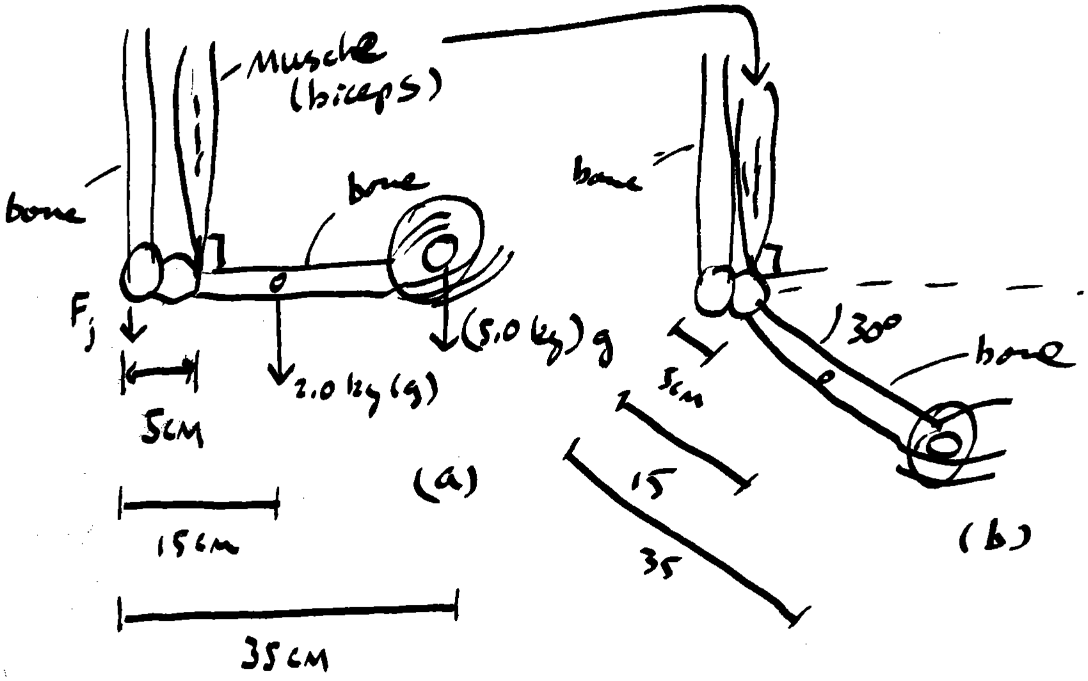
$\underline{\text{II}} \qquad F_w \cos\phi = mg + Mg$

$$\tan\phi = \frac{T}{(m+M)\,g} = \frac{6093}{(85+430)\,9.8}$$

$$\phi \simeq 50.3^\circ$$

$$F_w = \frac{T}{\sin\phi} = \frac{6093}{\sin 50.3} = 7919\ N$$

Similar example ... Weight lifter



Assume Mass of Forearm + hand = 2.0 kg and cog as shown

Find Force exerted by biceps muscle in config (a) and in config. (b)

(a) $\Sigma\tau$ about elbow ~~joint~~ joint $= 0$

$$(.05\,m)\,F_M - (.15\,m)(2.0\,kg)\,g - (.35\,m)(5.0\,kg)\,g = 0$$

$$F_M \cong 400\,N$$

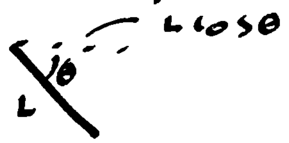
(b) $\Sigma\tau$ abt elbow joint $= 0$

lever arm of all 3 forces in torque sum

Shrinks by

$\cos\theta$

$\cos 30$ factor



$$\therefore\quad F_M = \frac{.15\cos 30(2)\,g + .35\cos 30(5.0)\,g}{.05\cos 30} = 400\,N\ !$$

$F_M \gg$ Mass of barbell ~~Insertion point of Muscle with~~

Suppose your insertion point of muscle is 6 cm from joint instead of Average 5

$$F_M = \frac{(.15)(2)(9.8) + (.35)(5.0)(9.8)}{(.06)} = 335\ N$$

So when you go to the gene splicer to design your child . . . if you want an athlete, give them Muscle insertion points a little Farther from the joint than average!

Skip Stress, Strain, elasticity

We will come back to Periodic Motion

$\Rightarrow$ Fluid dynamics (Chapt 14)