Physics 114 – January 26, 2010

- Problem Sets
  - inbox vs. outbox
  - Posted Scores
- Labs
- Workshops

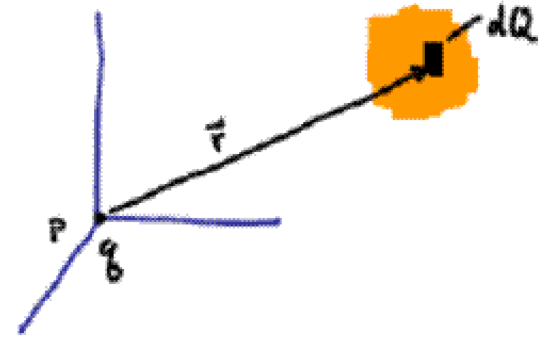
Start this week

- LAS study group

last Time

$\vec{E}_{\text{at } p}$ due to system of discrete charges $= \sum_{i=1}^{N} \frac{kQ_i}{r_i^2}\hat{r}_i$

$\vec{E}_{\text{at } p}$ due to continuous charge distribution

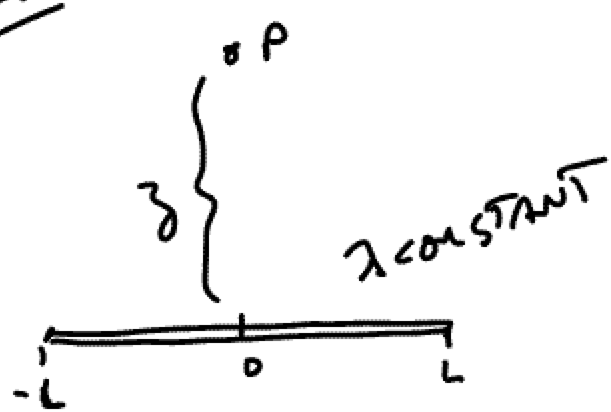


$$= \int_{\text{vol}} \frac{k\,dQ}{r^2}\,\hat{r} = \int_{\text{volume}} \frac{k\,\rho(\vec{r})\,dV}{r^2}\,\hat{r}$$

All of These things might change as you integrate over the volume

- might be hard.
- might not be so bad.

Example



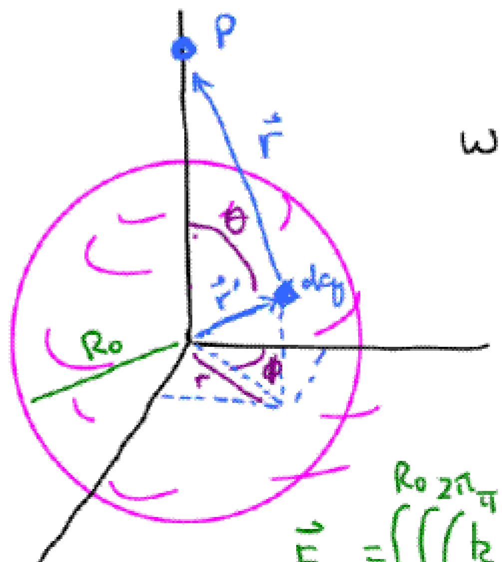
See Teal Demos for line charge + loop integration

Another "simple" example

Charge Q is dist evenly in sphere

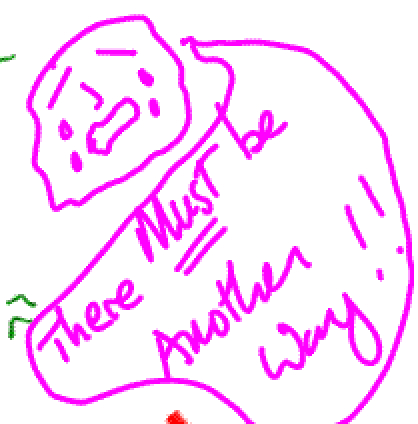
$$\rho(\vec{r}\,') = \frac{Q}{\frac{4}{3}\pi R_o^3} = \text{const}$$

What is $\vec{E}_p$?



$$\vec{E}_p = \int_{\text{chg dist}} \frac{k \rho \, dv \, \hat{\vec{r}}}{|\vec{r}|^2}$$

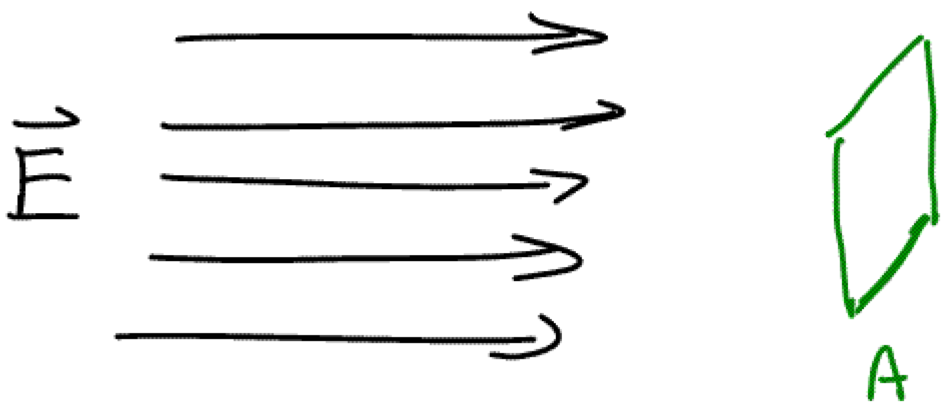
$$\vec{E}_p = \int_0^{R_o}\int_0^{2\pi}\int_0^{\pi} \frac{k \rho r'^2 \sin\theta \, d\theta \, d\phi \, dr}{r^2} \, \hat{r}$$

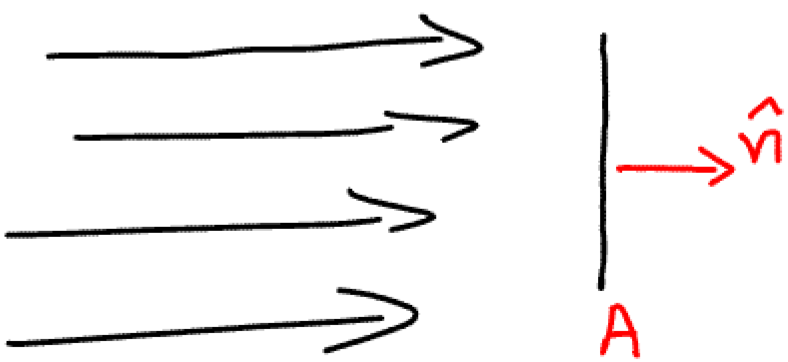


Holy Crap!

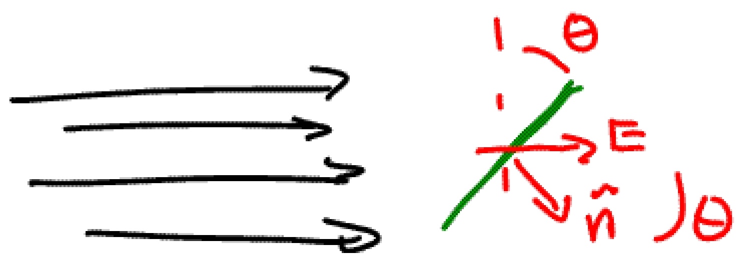
Electric flux

$$\equiv \phi = |\vec{E}| A$$





$$\phi = \vec{E} \cdot \hat{n} \, A$$



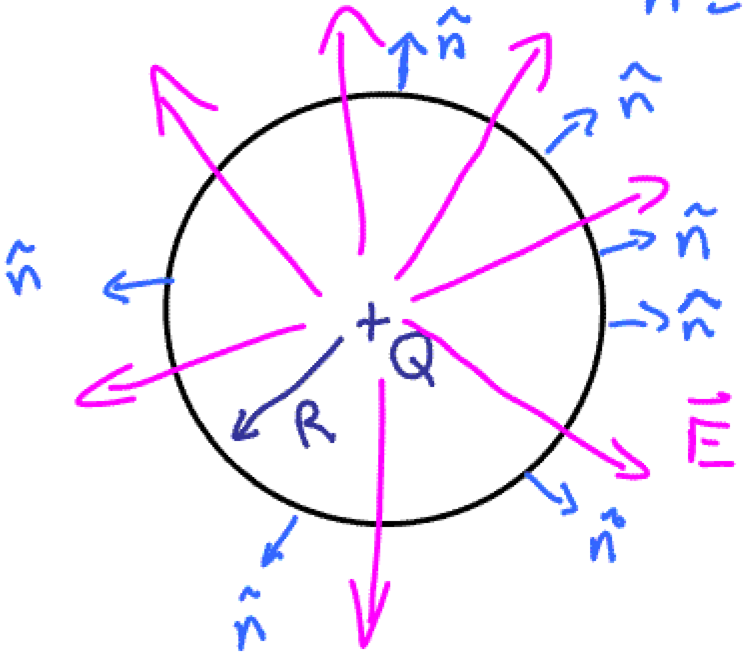
$$\phi = \vec{E} \cdot \hat{n} \, A$$

$$= |\vec{E}| \cos\theta \, A$$

$$\Phi_{surface} = \int_{surface} \vec{E}\cdot\hat{n}\, dA = \oint_{s} \vec{E}\cdot\hat{n}\, dA$$

$\hat{n} \equiv$ unit vector normal to surface

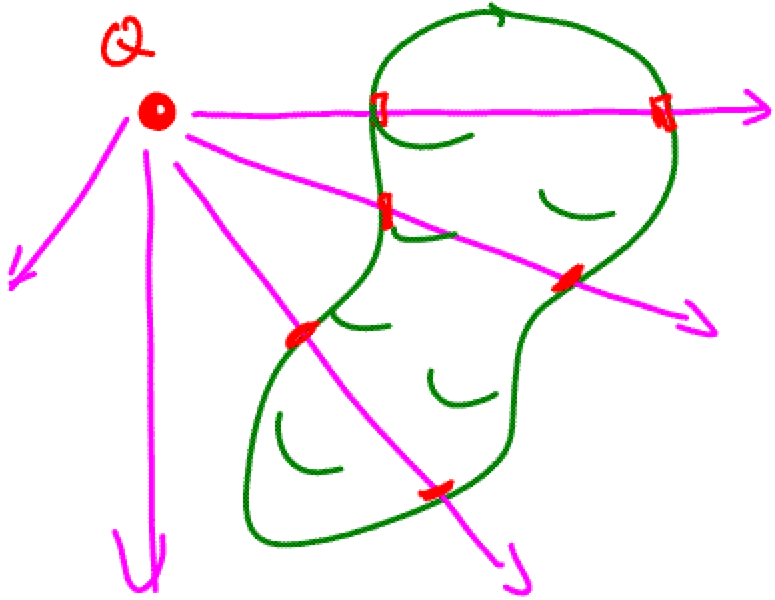
Spherical surface surrounding point charge

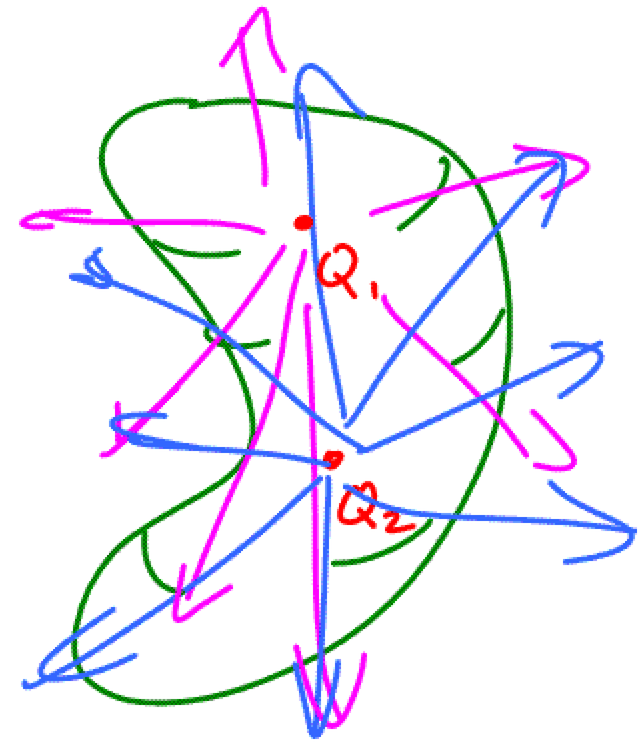


radial symmetry

$$\Phi = \int_{\text{surface}} \vec{E} \cdot \hat{n}\, dA = \int_{\text{surf}} |\vec{E}|\, dA = |\vec{E}| \overbrace{\int_{\text{surf}} dA}^{4\pi R^2}$$

$$\Phi = \frac{kQ}{R^2} 4\pi R^2 = \frac{1}{4\pi \epsilon_0} \frac{Q\, 4\pi R^2}{R^2} = \frac{Q}{\epsilon_0}$$





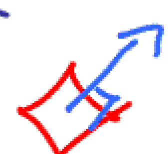
$\Phi = 0$

without proof

$$\Phi = \int_{surf} \vec{E}\cdot\hat{n}\, dA = \frac{Q_{enclosed}}{\epsilon_0}$$

$$\int \vec{E}\cdot\hat{n}\, dA \equiv \int \vec{E}\cdot\vec{dA}$$

or

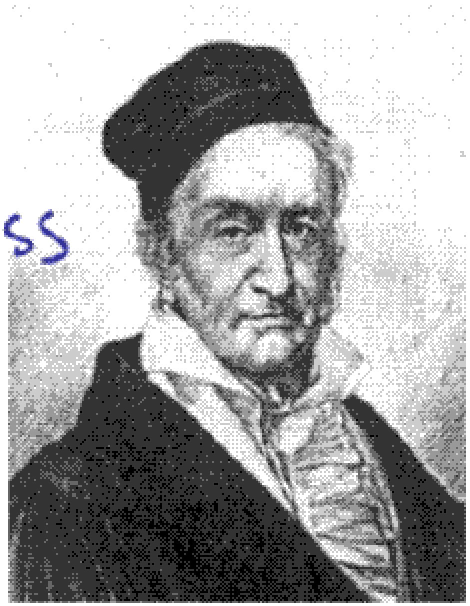


$\vec{dA} \equiv \hat{n}\, dA$

$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}}$$

Gauss' Law

Important

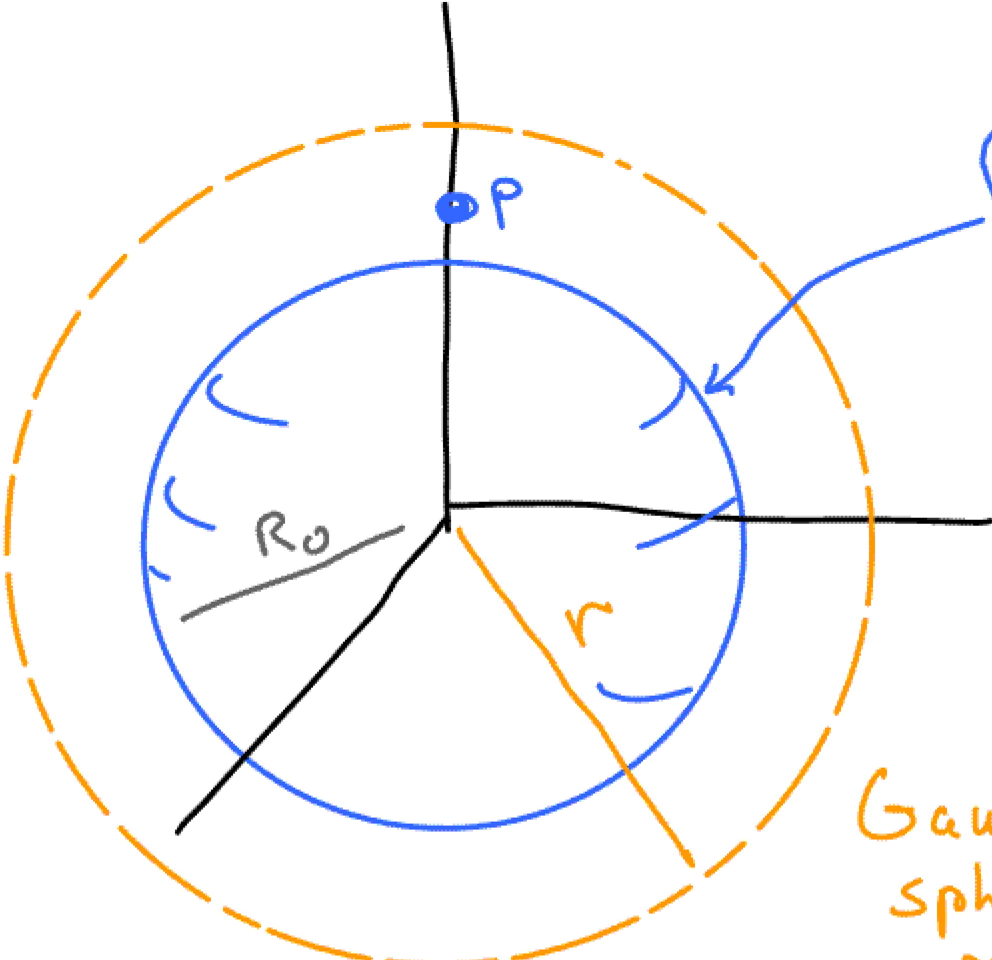


Johann Carl Friedrich Gauss
(1777-1855, Germany)

Very Smart
Plus

A mad cool sense of fashion!

When a philosopher says something that is true then it is trivial. When he says something that is not trivial then it is false



Spherical dist of charge

$\rho \equiv$ CONSTANT

What is $\vec{E}_p$?

What is $\vec{E}$ everywhere?

Gaussian Surface
spherical
radius $r > R_o$

evalute Gauss' law for chosen "gaussian surface" at $r > R_o$

$$\oint \vec{E} \cdot \hat{n} \, dA = \frac{Q_{encl}}{\epsilon_o}$$

Spherical / radial symmetry $\vec{E}$ is radial

$$\Phi = \int_{surf} \vec{E} \cdot \hat{n}\, dA = \int_{surf} |\vec{E}|\, dA = |\vec{E}| \underbrace{\int_{surf} dA}_{4\pi r^2}$$

left hand side

$$\Phi = |\vec{E}|\, 4\pi r^2$$

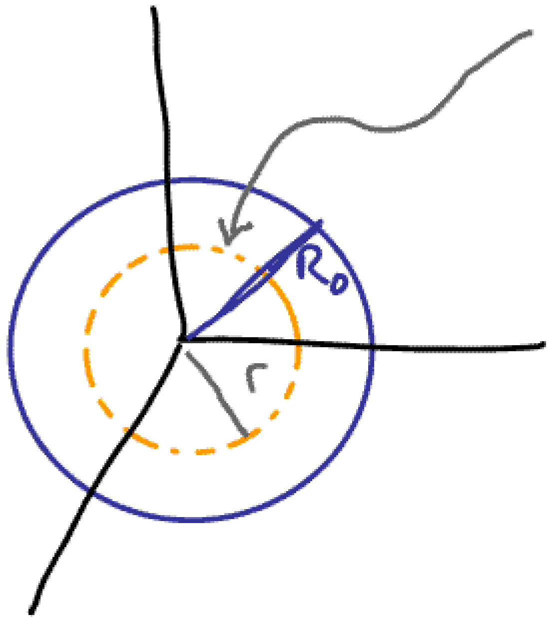
Right hand side

$$\frac{Q_{encl}}{\epsilon_0} = \frac{\rho}{\epsilon_0} \underbrace{\frac{4}{3}\pi R_0^3}_{\text{Vol of sph of radius } R_0} = \frac{Q_{TOT}}{\epsilon_0}$$

$$|\vec{E}|\, 4\pi r^2 = \frac{\rho}{\epsilon_0} \frac{4}{3}\pi R_0^3 \qquad |\vec{E}|_{r>R_0} = \frac{\rho R_0^3}{\epsilon_0 3 r^2}$$

$$\rightarrow |\vec{E}| = \frac{Q_{TOT}}{4\pi \epsilon_0 r^2}$$

Evaluate $|E|$ for $r < R_0$



Choose gaussian surface with $r < R_0$

$$\int_{\text{surf}} \vec{E} \cdot \hat{n} \, dA = \frac{Q_{encl}}{\epsilon_0}$$

$$\int |\vec{E}| \, dA = \underset{(\text{at } r)}{|\vec{E}|} \, 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

$$= \int_{\substack{\text{gauss} \\ \text{surf}}} \rho \, dv \; \frac{1}{\epsilon_0}$$

$$= \rho \int_{\text{sphere}} dv \; \frac{1}{\epsilon_0}$$

$$|\vec{E}| \, 4\pi r^2 = \rho \, \frac{4}{3}\pi r^3$$

$$|\vec{E}|_{r<R_0} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\rho r}{3} \frac{1}{\epsilon_0}$$