Physics 114 – February 16, 2010

Slides meant to accompany Posted Mp3 Audio file

Happy Valentine's Day

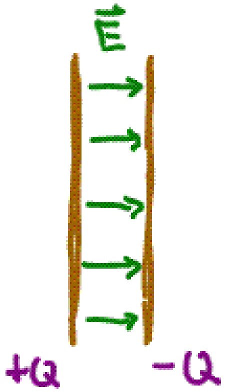
Last Time —

$$V \propto Q$$

Define Capacitance as constant of Proportionality

$$Q = CV$$

$\equiv$ Capacitance



Parallel Plate configuration

ΔV or V

Units = $\dfrac{\text{Coul}}{\text{NM/coul}} = \dfrac{\text{Coul}^2}{\text{NM}}$

$\equiv$ Farad

Capacitance Depends only on geometry

For example: For // plate geometry we found

$$C = \frac{\epsilon_0 A}{d}$$

area of plates

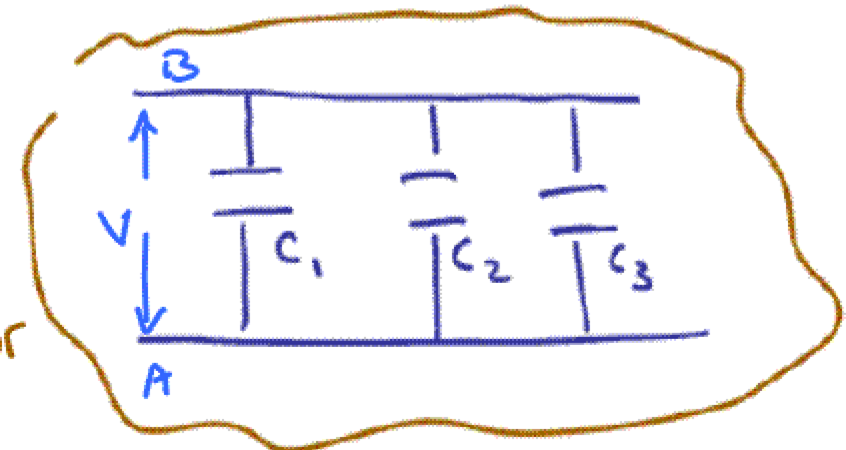
Geometry only

distance bet plates

SM shorthand for Parallel

For Capacitors in parallel

Same Voltage Across Each capacitor



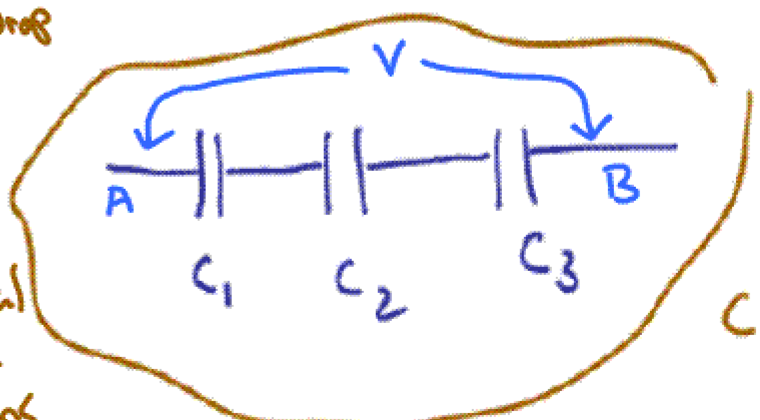
C_{TOT}

$$C_{TOT} = \sum_i C_i$$

Useful

For Capacitors in Series

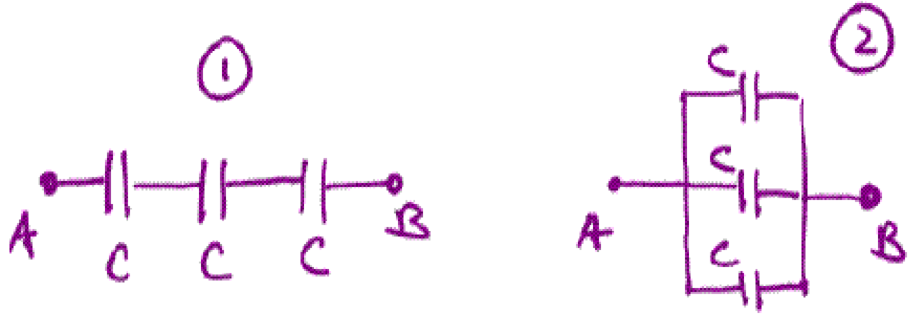
Voltage Drop Across All is sum of individual Voltage Drops



C_{TOT}

$$\frac{1}{C_{TOT}} = \sum_i \frac{1}{C_i}$$

3 identical capacitors are connected in two different configurations



A voltage is applied between pts $\underbrace{A + B}_{V_{AB}}$

Some time passes

how does the amount of charge stored in the LH config. compare to that on the right?

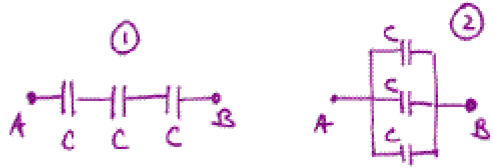
Ⓐ $Q_1 > Q_2$ Ⓑ $Q_1 < Q_2$ Ⓒ $Q_1 = Q_2$

Take a minute ... what do you think?

5

Go back + think
about the problem
I posed if you
didn't ...

3 identical capacitors are connected in two different configurations



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Ⓐ $Q_1 > Q_2$ Ⓑ $Q_1 < Q_2$ Ⓒ $Q_1 = Q_2$

$$Q_1 = C_{1_{TOT}} V_{AB}$$

$$Q_2 = C_{2_{TOT}} V_{AB}$$

$$C_{2TOT} = C + C + C = 3C$$

$$\frac{1}{C_{1TOT}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

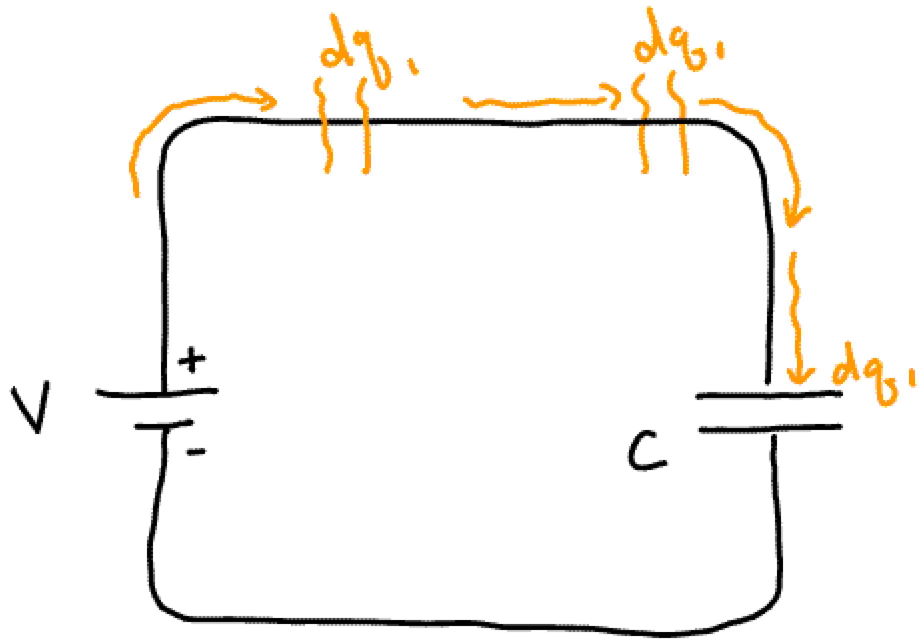
$$C_{1TOT} < C_{2TOT}$$

$$C_{1TOT} = \frac{C}{3}$$

So $Q_1 < Q_2$

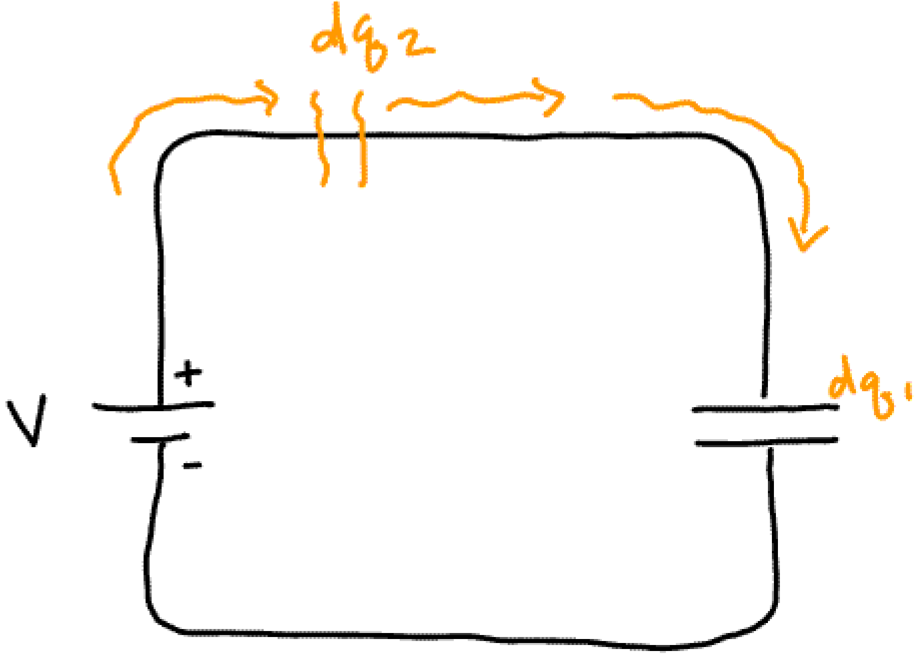
The BiG question...

How much work (or energy) does it take to charge a capacitor?



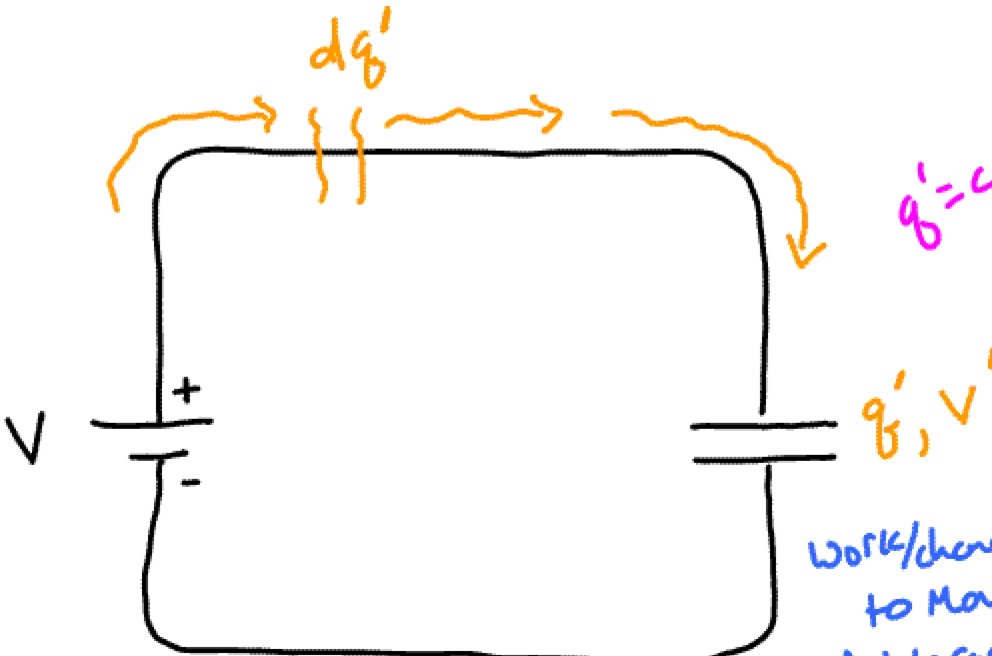
at $t=0$ connect capacitor across an emf

emf does No work to move 1st dq to capacitor at time $0+dt$



Work must be done by emf to move dq_2 to capacitor against the repulsion of dq_1

at some general time t during the charging of the capacitor



$q' = CV'$

For simplicity, lets define the potential at "+" plate of emf to be 0

work/charge to move dq' to capacitor $\angle$ $V' = dW/dq'$

$$dW = V'dq'$$

$$dw = v' dq'$$

$$dw = \frac{q'}{c} dq'$$

note c is a constant for given capacitor (geometry only)

$$c = q/V = q'/V'$$

$$\underline{\underline{W}} = \int dw = \int_0^Q \frac{1}{c} q' dq' = \frac{1}{2c} Q^2$$

Total work it takes to charge the capacitor to its final charge Q, where $Q = CV$

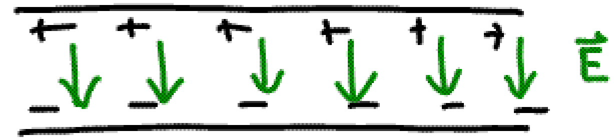
$$\boxed{\text{Energy required to charge a capacitor} = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV}$$

Also work required. Sometimes symbol is U

$$U = \frac{1}{2} CV^2$$

Charged capacitor

↳ have created an electric field in given volume

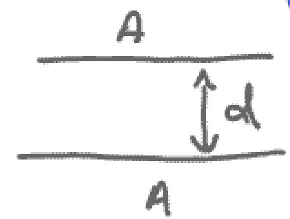


where does the work/energy go that was used to charge the capacitor? → in the electric field

$$\text{Energy density of Electric field} \equiv u \equiv \frac{U}{\text{Volume between} \parallel \text{plates}}$$

$$u = \frac{U}{\text{Vol. bet plates}} = \frac{U}{A\,d}$$

$\frac{1}{2}CV^2$



Also recall $C_{/\!/\,\text{plate cap}} = \frac{\epsilon_0 A}{d}$

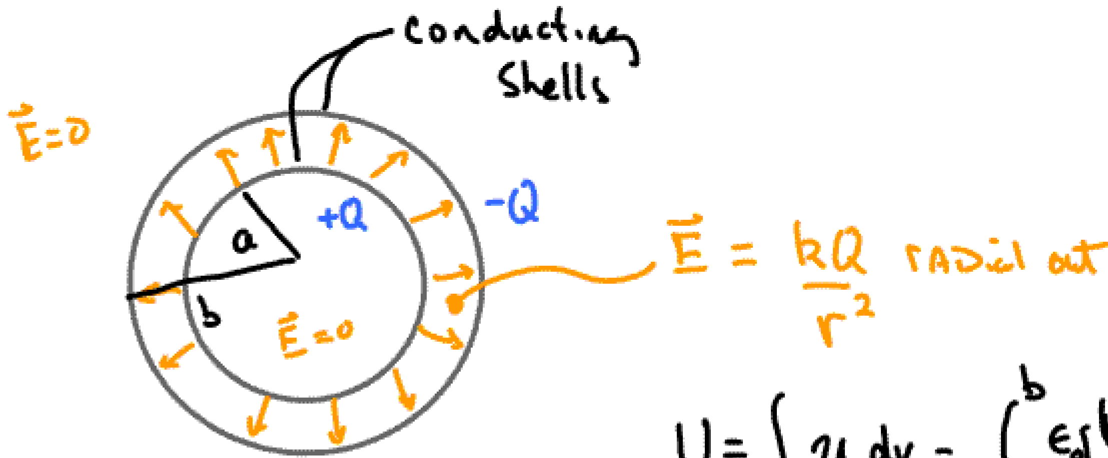
$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\frac{\epsilon_0 A}{d}V^2}{Ad} = \frac{\epsilon_0}{2}\frac{V^2}{d^2} = \frac{\epsilon_0}{2}|\vec{E}|^2 = \frac{\epsilon_0}{2}E^2$$

makes use of $E = V/d$ for // plate system

Derived using specific geometry but result is general

$$\boxed{u = \frac{\epsilon_0}{2}E^2}$$

Awesome! Is it not ?!!



Total Energy Stored in this configuration

$$U = \int_{vol} u\, dv = \int_a^b \frac{\epsilon_0}{2}\left[\frac{kQ}{r^2}\right]^2 4\pi r^2\, dr$$

$$U = \frac{\epsilon_0}{2} k^2 Q^2 4\pi \int_a^b r^{-2}\, dr$$

$$U = \frac{\epsilon_0}{2} k^2 Q^2 4\pi \left[-r^{-1}\right]_a^b$$

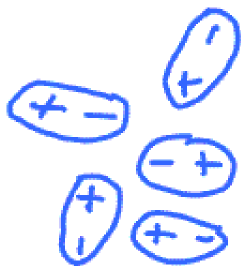
$$U = \frac{\epsilon_0}{2} k^2 Q^2 4\pi \left(\frac{1}{a} - \frac{1}{b}\right)$$

use $k = \frac{1}{4\pi\epsilon_0}$

$$U = \frac{kQ^2}{2}\left(\frac{1}{a} - \frac{1}{b}\right)$$

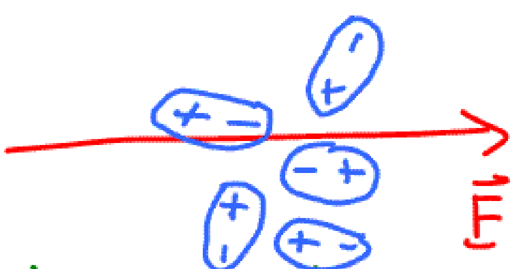
Electric Fields in Materials ≡ dielectrics
(nonconductors, insulators)

Polar materials



inherent dipoles

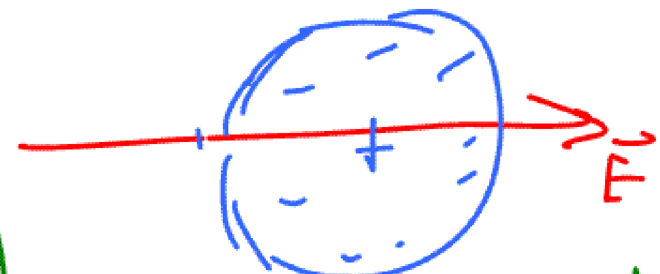
Random orientations



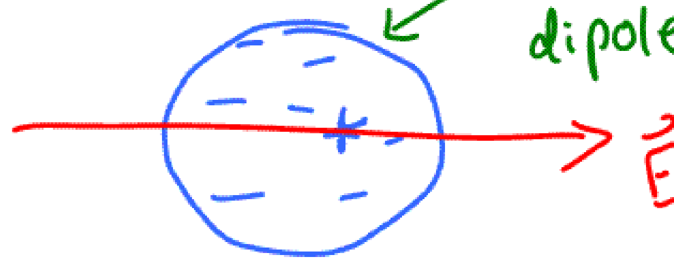
Net torque → align particles along $\vec{E}$

nonpolar materials

indiv. Atoms/Molecules
No net dipole

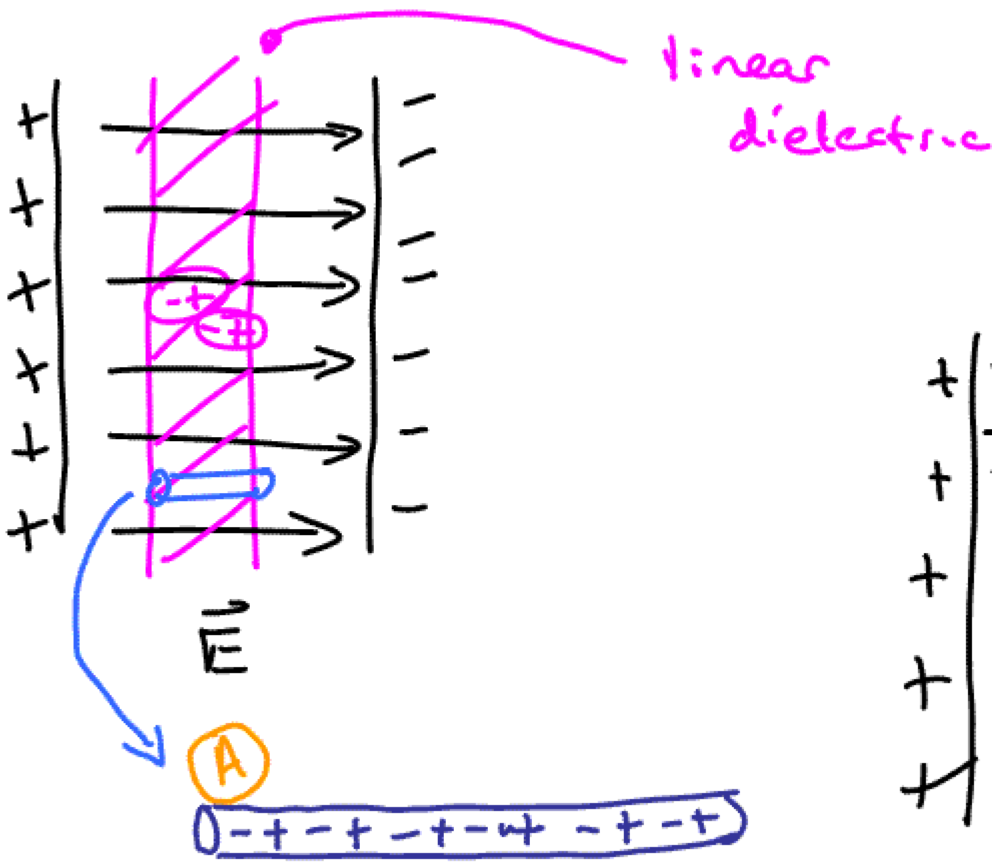


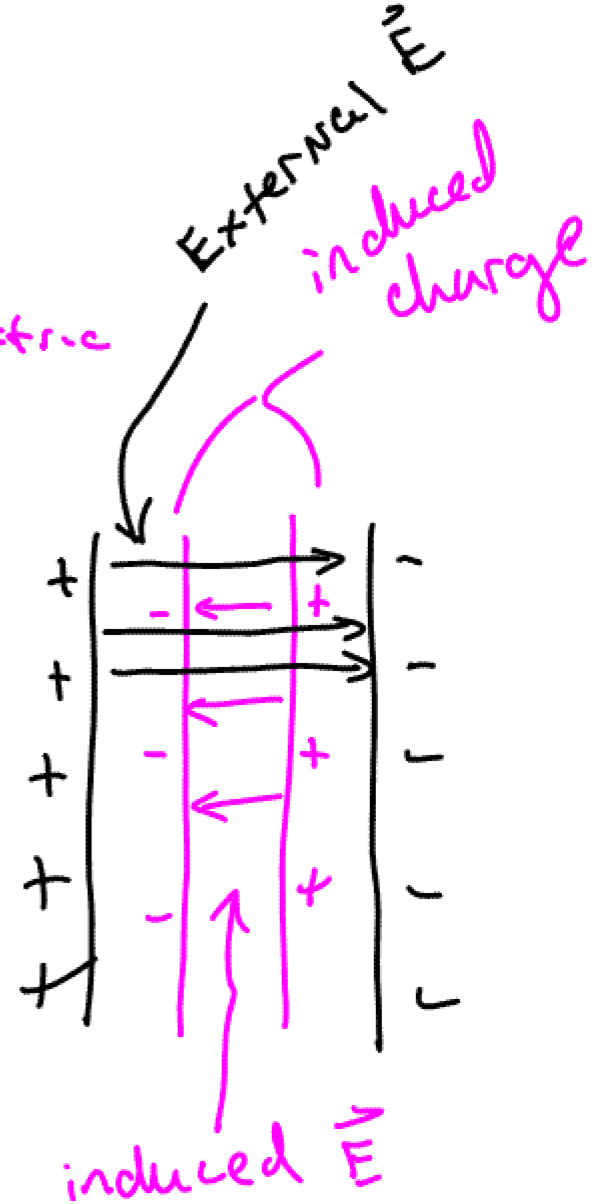
induced dipole



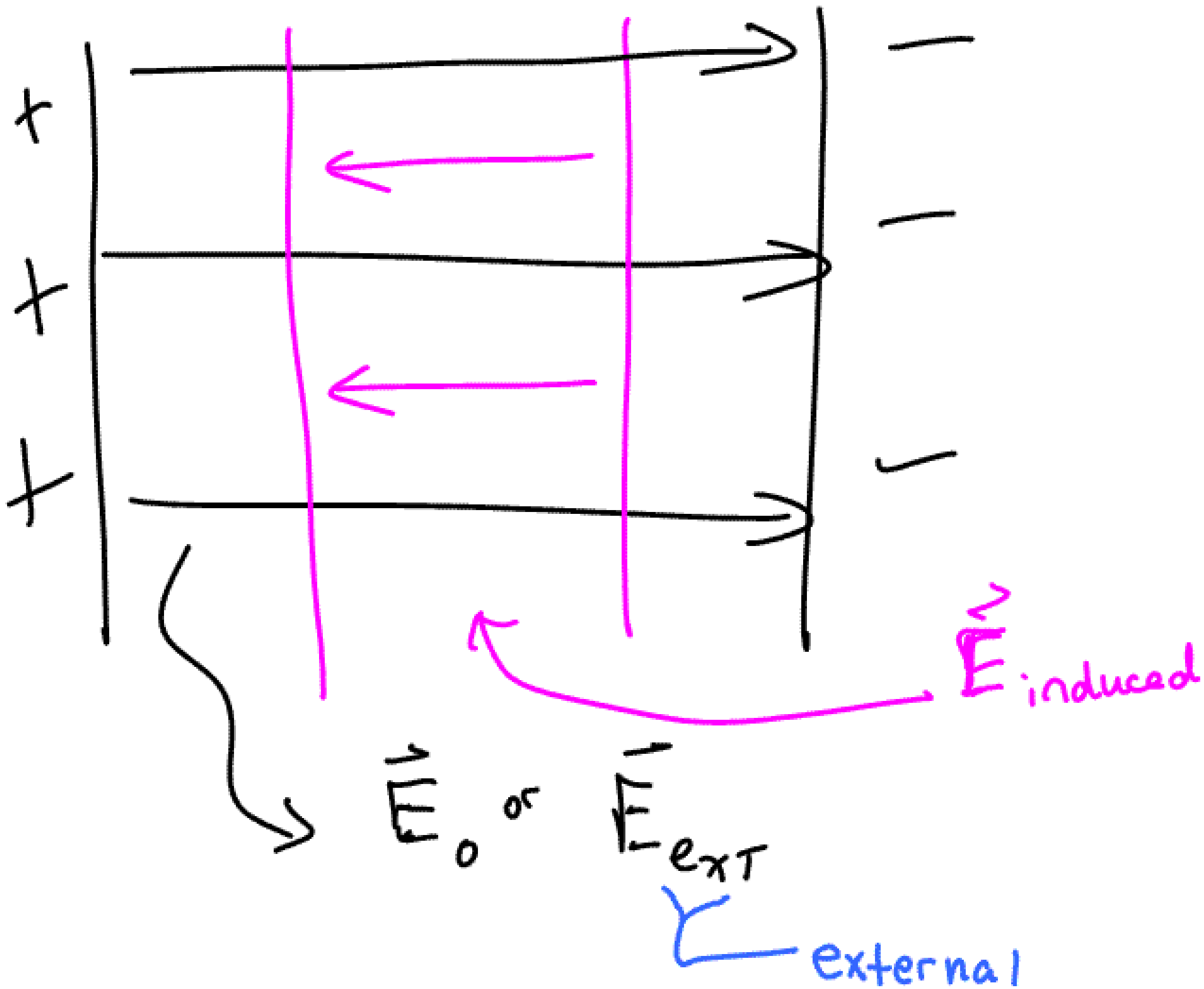
\# Dipoles Aligned $\propto |\vec{E}|$

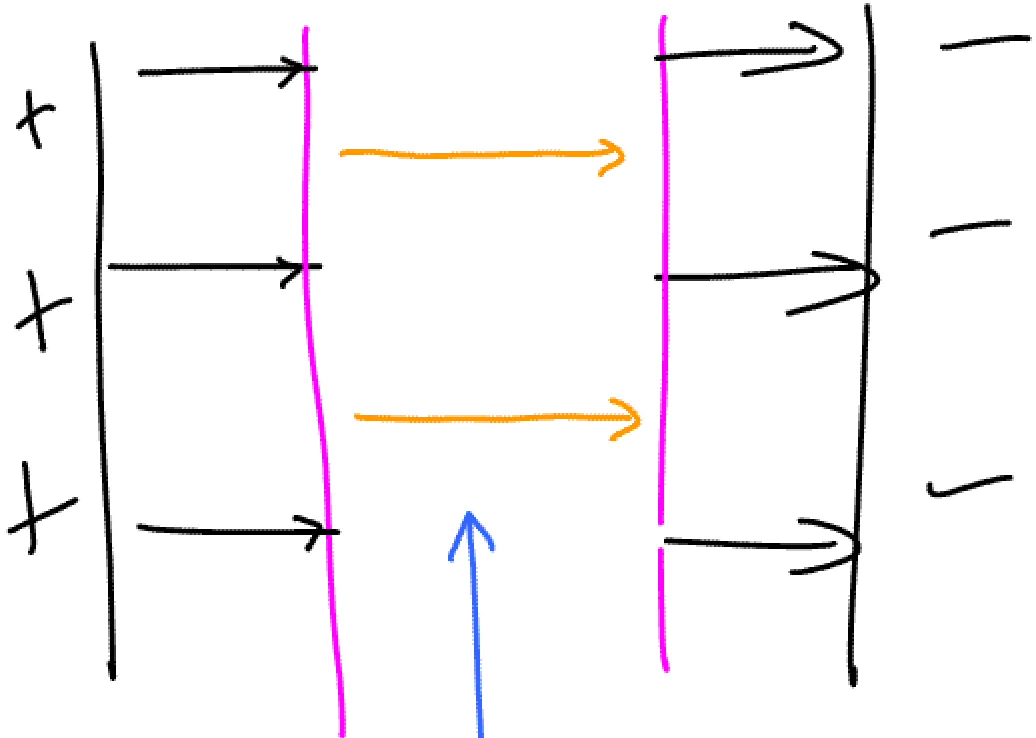
"Linear dielectrics"





$\vec{E}_{induced} \propto \vec{E}_{external}$ For linear dielectrics





linear dielectric

$$\vec{E} = \frac{\vec{E}_0}{K}$$

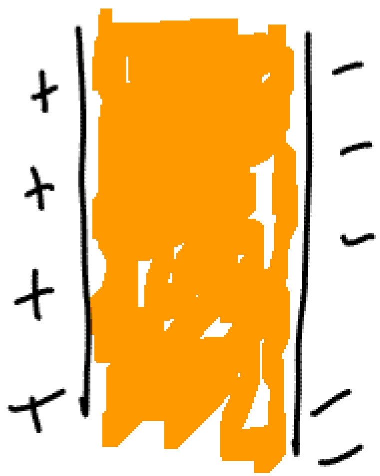
Net $\vec{E}$ in material

$$\vec{E} = \vec{E}_{\text{external}} + \vec{E}_{\text{induced}}$$

Dielectric constant $K > 1$

$K > 1$

water	$K = 80.4$
Air	1.00054
Vacuum	1
Oil	4.5



A $\vec{E}_0 = \dfrac{\sigma}{\epsilon_0}$

B $\vec{E} = \dfrac{\sigma}{K\epsilon_0} = \dfrac{\sigma}{\epsilon}$

permittivity of free space

$K\epsilon_0 \longrightarrow \epsilon \equiv$ permittivity