Physics 114 – April 13, 2010

- Exam 3 – Thursday during normal class time
  Both sides 8.5x11 inch paper
  w/ formulas, notes
- Q+A session Today 4:30 pm B+L 106

light is a wave

- Theory well understood
  Maxwell → wave equations → derive laws of optics
  Interference, Diffraction, Refraction } → wave characteristics
- Numerous experiments show light behaves exactly like other wave phenomena such as sound, waves on strings, etc.
- Only strangeness is that light is a wave that can travel in a vacuum apparently.

Max Planck

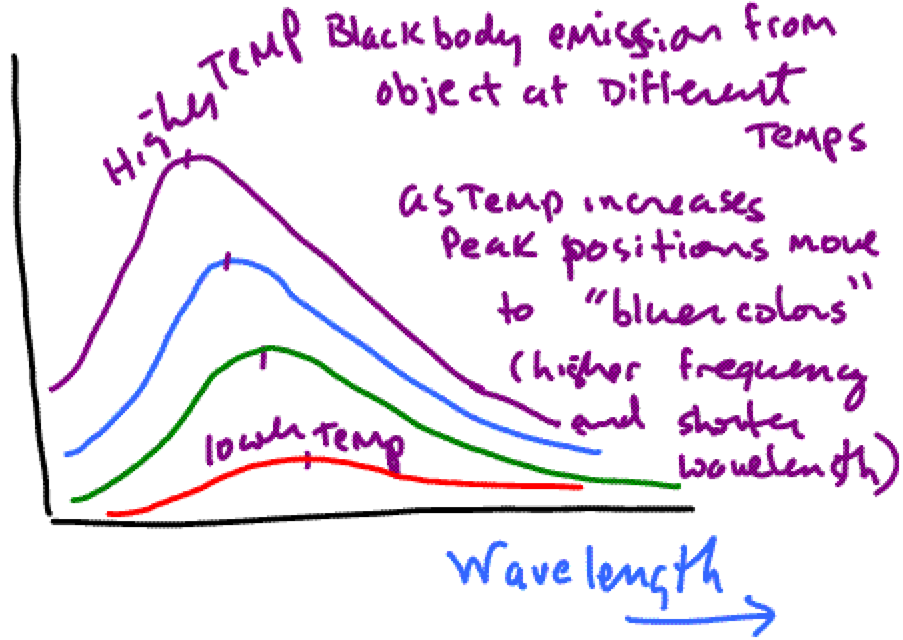
(1858 - 1947)

German national

Awarded 1918 Nobel Prize in physics
for analysis of blackbody radiation
which contributed to rise of
quantum Mechanics

http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Planck.html

"Blackbody"
radiation
(emitted by object)



Planck succeeded in describing
Black body radiation

light exists in packets of energy
Discrete, Particle-like

"photons"
Gilbert Lewis
~1926

Planck's constant

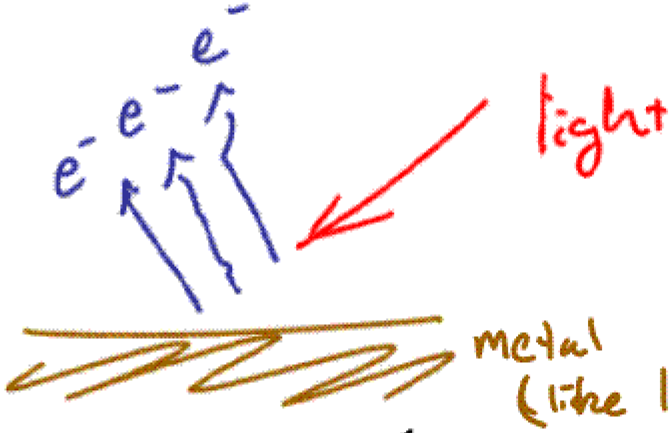
$$E = h\nu$$

Energy

frequency

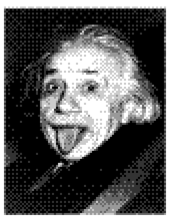
Planck's Theory worked perfectly ... but physicists thought it was a fortuitous Accident ... after all, light is a wave.

Another phenomenon that was NOT understood



Photoelectric effect

Study electric current depends on Intensity + color of light

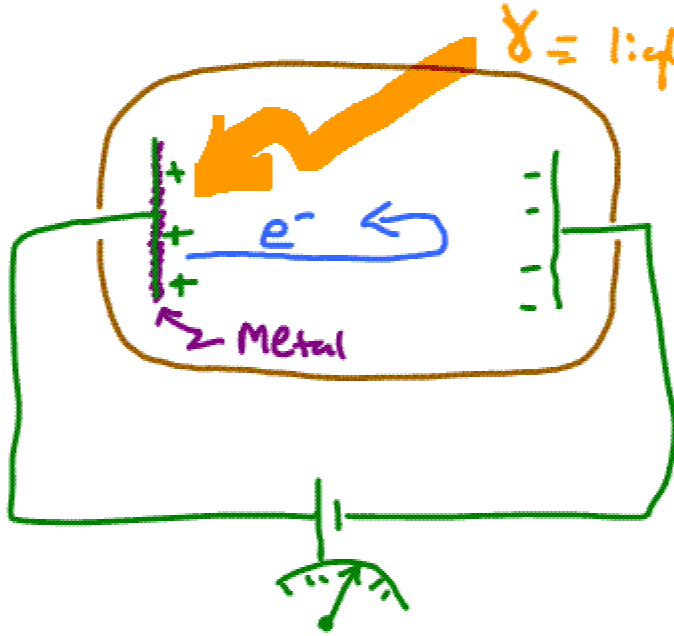


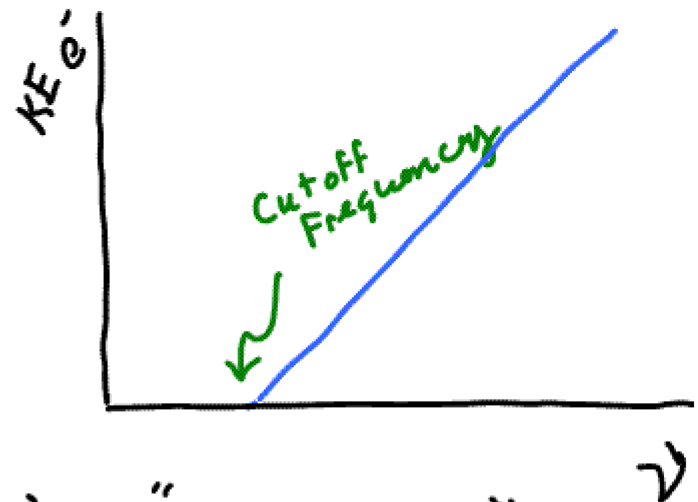
Einstein in 1905

$\underline{\underline{E = h\nu}}$

- Einstein Able to explain photoelectric effect Assuming light to come in little packets with energy $E = h\nu$
- Light as wave theory could NOT account for data
- Suddenly world has to face fact that light is both a wave and a particle ... Sounds crazy ... but that's what nature tells us.

Photoelectric effect – Einstein 1905



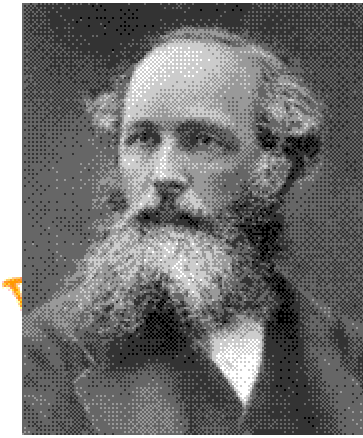


get current
increase potential
until current stops
→ PE = KE

$$KE_{e^-} = h\nu - \text{"work function"}$$

Constant the same as needed by Planck!!

e^- binding energy in metal



Maxwell's equations
interference
diffraction
Refraction
dispersion
⋮

light is a wave!

Planck

Blackbody Radiation

light comes in little packets with energy

$E = h\nu$

Einstein

Photoelectric effect

1905

light is a particle!

Louis Victor Pierre Raymond
7th duc de Broglie

mid-1920's

de Broglie hypothesized matter can have wave characteristics with a wavelength given by

So-called de Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

From special relativity ← Form of "$E=mc^2$" when particle moving

$$E = \sqrt{p^2c^2 + m^2c^4}$$

→ For photon, $m=0$

$E = pc$ or $p = \frac{E}{c}$

From Blackbody + Photoelectric

$$E = h\nu \longrightarrow E = \frac{hc}{\lambda} \rightarrow \frac{h}{\lambda} = \frac{E}{c}$$

$v = \lambda\nu$

$\lambda = \frac{h}{p}$ for photon

deBroglie: Perhaps Also true for Particle with Mass

$\lambda = \frac{h}{p}$

baseball at 92 mph

Mass 142 g

41 m/s

P of baseball = mv

$p = 5.7 \text{ kg m/s}$

$$\lambda_{\substack{92\text{ mph} \\ \text{baseball}}} = \frac{\overbrace{6.6\times10^{-34}}^{h} \text{ J}\cdot\text{s}}{5.7 \text{ kg m/s}}$$

$\lambda = 1.\times10^{-34}$ m

incredibly small wavelength — not noticeable to us.

1 eV electron

energy of 1.6×10^{-19} J

$= \frac{1}{2} m v^2 \longrightarrow v = 596 \, m/s$

$p = (9 \times 10^{-31} \text{ kg})(596) = 5 \times 10^{-28} \text{ kg m/s}$

$$\lambda_{e^-} = \frac{6.6 \times 10^{-34}}{5 \times 10^{-28}} = 1.3 \times 10^{-6} \text{ m}$$

$\sim 1 \, \mu m$

Not so incredibly small

careful experiments can see wave effects like diffraction and interference of electrons

electron Microscope

can use electron waves to image very small objects because electron wavelength is smaller than wavelength of visible light

$\lambda = h/p$

higher Energy or $p \rightarrow$ smaller λ of electron

Niels Bohr

(1885 - 1962) (Denmark)

1922 Nobel Prize in Physics

~1912, 1913 (before deBroglie)

Atomic (planetary) model with fixed orbits

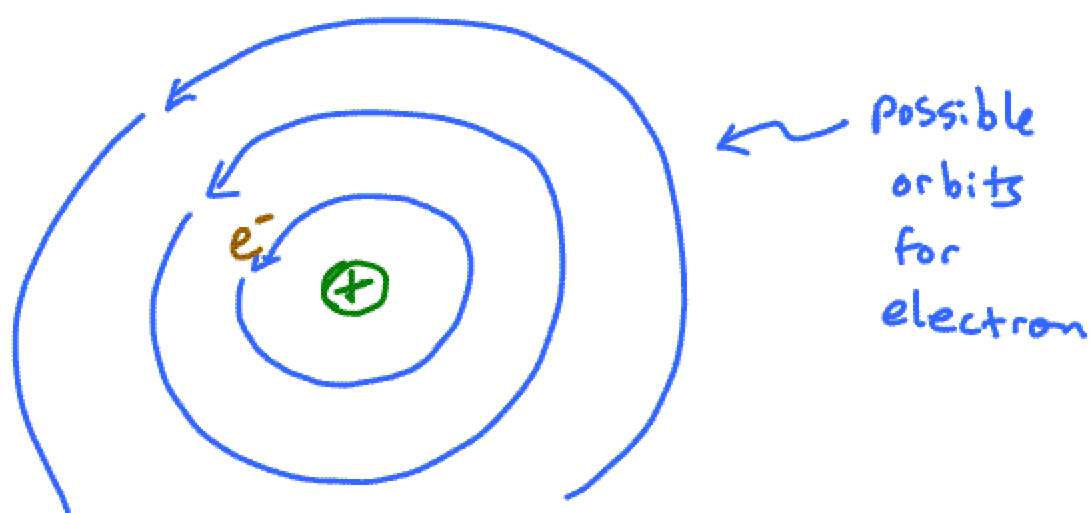
nicely motivated by de Broglie's matterwaves in 1924

Bohr model of the atom

- Positive Nucleus
- electrons orbit in circles, Proton too heavy to Move
- only particular "discrete" orbits allowed

known as quantization

- electric (coulomb) force holds electron on Circle as it orbits

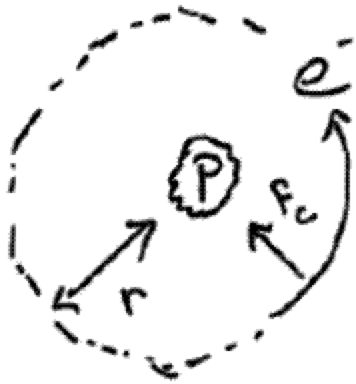


Important pre-quantum mechanics Model of Atom

Bohr Atom   1913

impt for giving us intuition about Atoms

Assume e^- orbits proton in a circular orbit



$$F_{cent} = \frac{m_e v^2}{r}$$

$\frac{kZe^2}{r^2}$ if single e^- Atom of Atomic # Z

$$\underbrace{\frac{k|e|^2}{r^2}}_{\text{Coulomb attraction between p and } e^-} = \underbrace{\frac{m_e v^2}{r}}_{\text{Centripetal force}}$$

Equation I

only works for certain λ that satisfy

Since e^- is a wave

imagine e^- as a Circular STANDing wave

$$2\pi r = n\lambda \qquad n = 1, 2, 3 \ldots$$

↓ from DeBroglie

$$2\pi r = n\frac{h}{p}$$

This is where Things become discrete or quantized

$$\hbar = \frac{h}{2\pi} \equiv \text{called h-bar}$$

Analogy with drinking STRAWS

So, $pr = n\hbar$

Angular Momentum

$$mvr = n\hbar \quad \text{same as} \quad L = n\hbar$$

So can also say Bohr quantized Angular Momentum

Eqn 2

$$v = \frac{n\hbar}{mr}$$

Substitute into Equation 1 Above

$$\frac{k|e|^2}{r^2} = \frac{mv^2}{r} \longrightarrow \frac{ke^2}{r^2} = \frac{mn^2\hbar^2}{r\,m^2r^2}$$ solve for r

$$r_n = \frac{n^2\hbar^2}{ke^2m} \qquad n = 1, 2, 3 \cdots$$

says that electron only exists at discrete radii

If done for single e^- atom of atomic # Z in initial eqns

$$\frac{ke^2}{r^2} \longrightarrow \frac{kZee}{r^2}$$

$$\boxed{r_n = \frac{n^2\hbar^2}{kZe^2m}}$$

good for single e^- atom w/ At. # Z

r_1 = Ground state orbital radius

$$\boxed{r_n = \frac{n^2}{Z}r_1}$$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{n\hbar}{mr}\right)^2 = \frac{n^2\hbar^2}{2mr^2}$$

from Eqn 2

Substitute in for r

$$K.E. = \frac{n^2\hbar^2}{2m}\,\frac{k^2Z^2e^4m^2}{n^4\hbar^4} = \frac{mk^2Z^2e^4}{2n^2\hbar^2}$$

$$E_{TOTAL} = KE + PE$$

What is P.E.? recall Potential = $\frac{\text{work}}{\text{chg}}$

$$P.E. = -k\frac{Ze^2}{r}$$

"–" because ATTRACTIVE

Sub in for r

$$P.E. = -\frac{kZe^2\,kZe^2m}{n^2\hbar^2} = -\frac{k^2Z^2e^4m}{n^2\hbar^2}$$

$$E_{TOTAL} = KE + PE = \frac{1}{2}\frac{mk^2Z^2e^4}{n^2\hbar^2} - \frac{mk^2Z^2e^4}{n^2\hbar^2}$$

$$E_{n\,(TOTAL)} = -\frac{mk^2Z^2e^4}{2n^2\hbar^2}$$

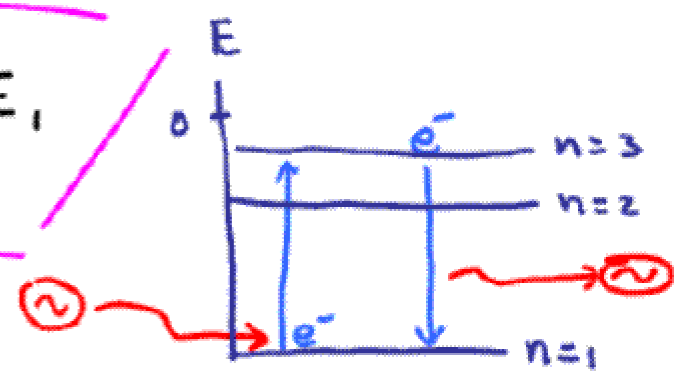
What is meant by the negative sign here?

for $Z=1$, $n=1$ $\longrightarrow$ Most tightly bound Energy level for H

$$E_1 = -13.6\,\text{eV}$$

$$E_n = -\frac{Z^2}{n^2}E_1$$

Discrete Spectra!!



photons emitted or Absorbed as e^- shifts levels $E_\gamma = h\nu$

Discrete Atom Emission + Absorption lines come from e^- transitions between discrete Energy levels

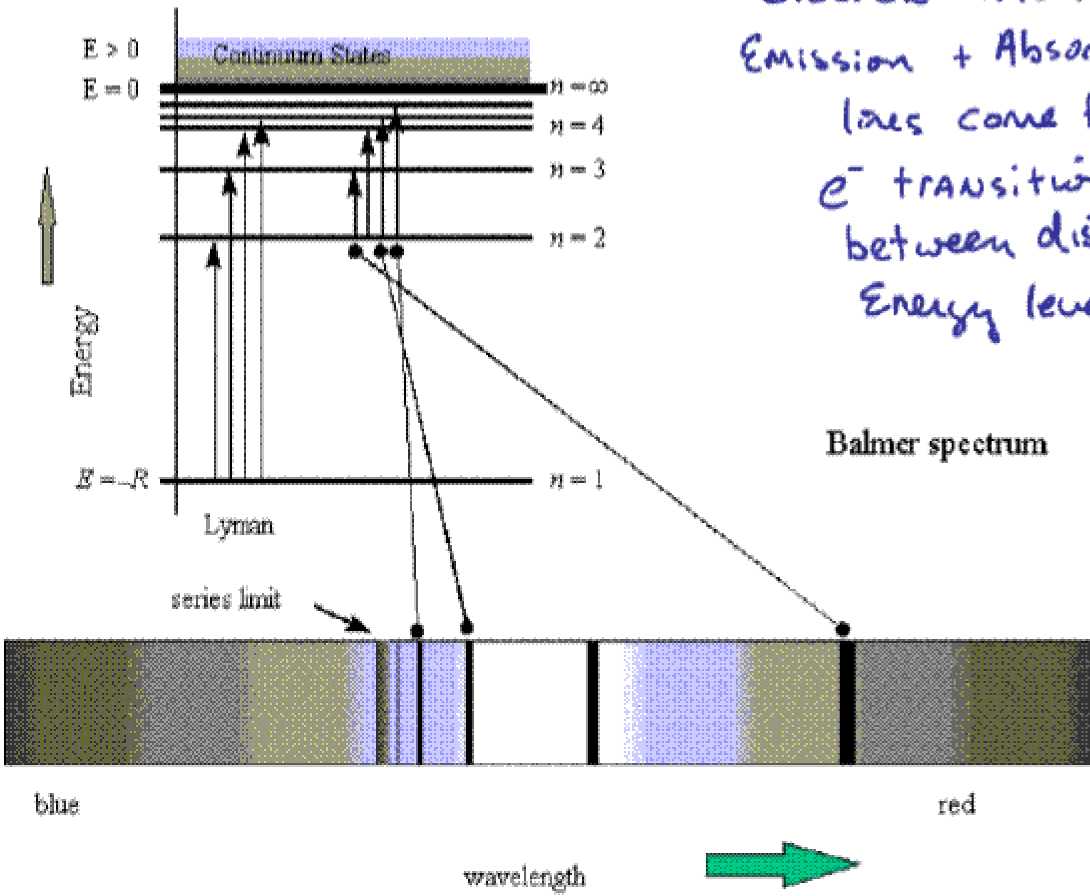
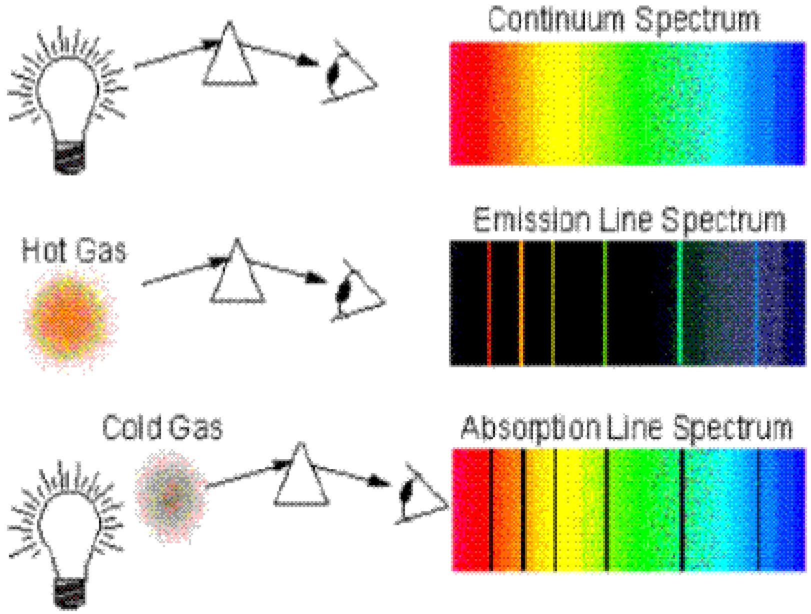
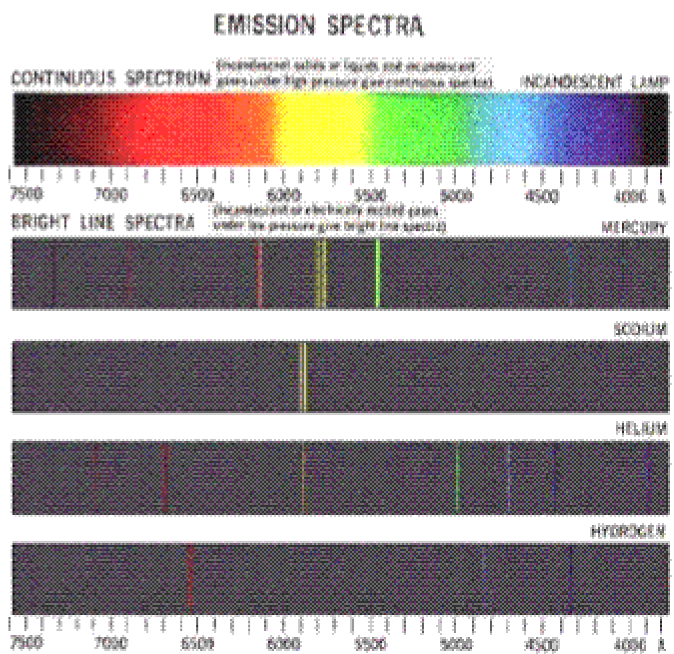


Figure from http://www.uclan.ac.uk/facs/science/physastr/x99/PAM98/UCert/Ch06/6_6ato~1.htm

Emission vs. Absorption



Different Atoms
→ different discrete frequency pattern



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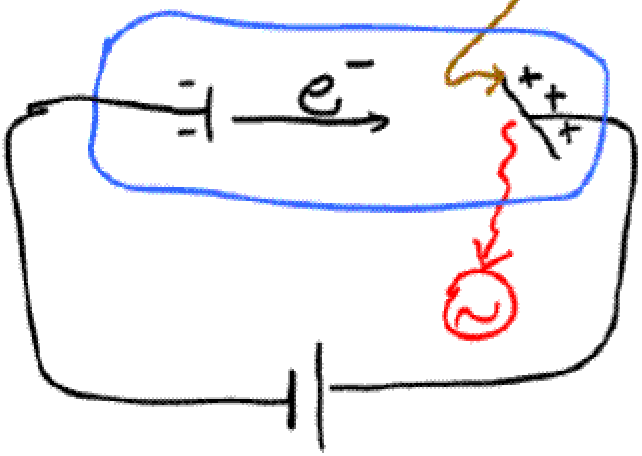
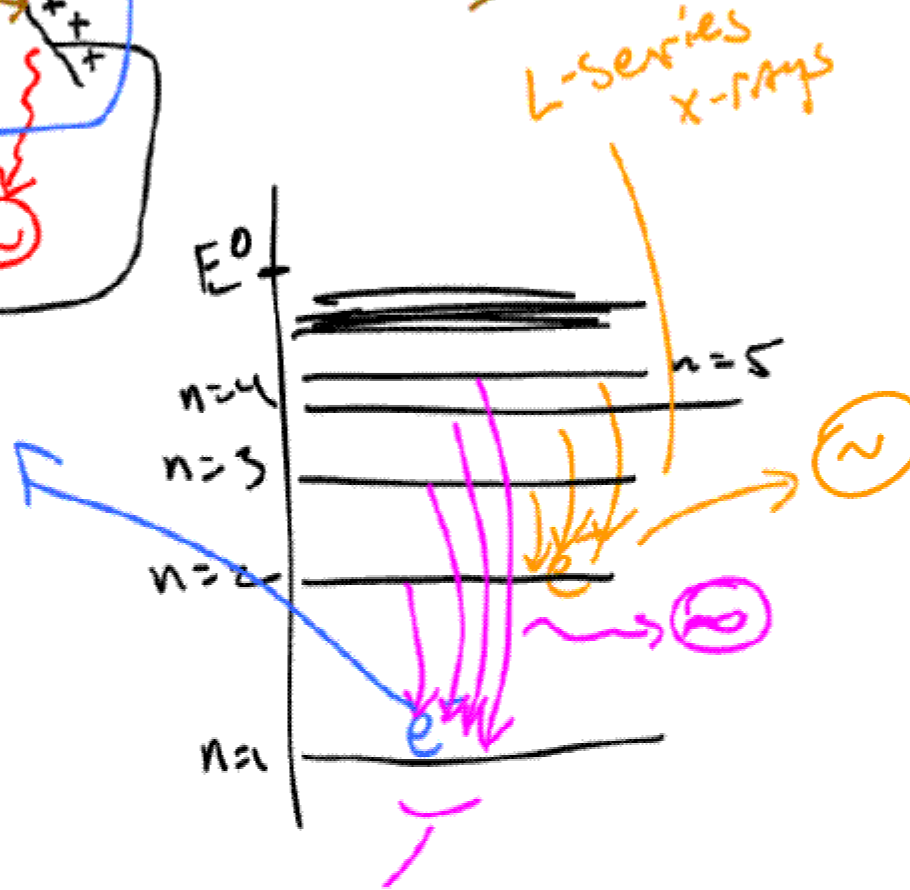
Atomic Fingerprinting

X-rays

Heavy metal (High Z) → means inner electrons bound tightly!





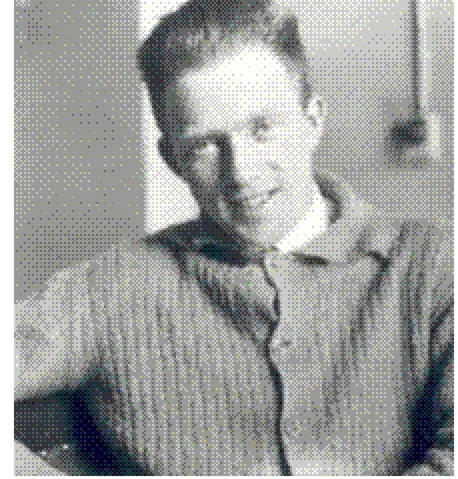
X-rays

100 ~ 100,000 eV

Remove inner electron. When e^- from higher n cascades down to fill open place in inner shell, high energy γ is emitted

Werner Karl Heisenberg

(1901 - 1976)



Nobel Prize in physics - 1932
for "the creation of quantum Mechanics"
(Max Born, Pascual Jordan - co-workers)

Erwin Rudolf Josef Alexander Schrödinger
(1887 - 1961) Austria

1933 Nobel Prize in physics

1926 - Paper on wave Mechanics of Matter
Annalen der Physik

"For discovery of new and productive forms of atomic theory"

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U\psi(x) = E\psi(x)}$$

Schrödinger's EQUATION → 1d

1-d

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U\,\psi(x) = E\,\psi(x)$$

Potential energy function

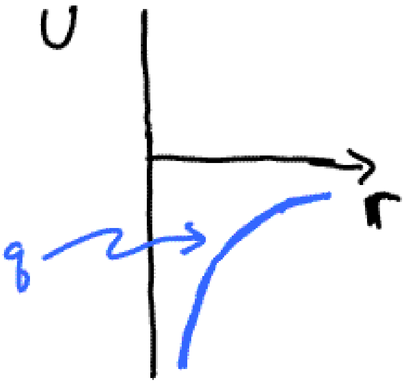
Pop in diffent U's

Solve for E and ψ

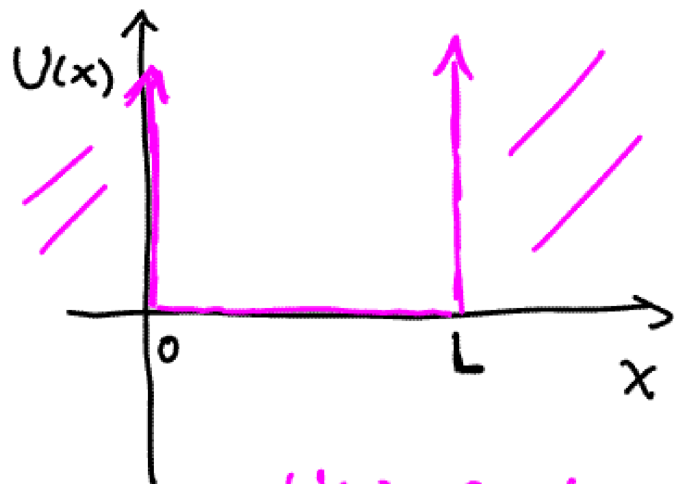
Example

$Q\cdot \quad \cdot q$

$$U = \frac{kQq}{r}$$



infinite Square Well potential



$$\frac{d^2\psi(x)}{dx^2} = -\boxed{\frac{2ME}{\hbar^2}}\,\psi(x)$$

$$\equiv k^2$$

$U(x) = 0$ for $0 < x < L$

$U(x) = \infty$ for $x < 0,\ L < x$

$$\frac{d^2\psi(x)}{dx^2} = -k^2\,\psi(x)$$

$F = -kx \rightarrow d^2x/dt^2 = -k/m\,x$

SHM-ish

$$\psi(x) = A\sin kx + B\cos kx$$ general solution

Boundary conditions

$$\psi(x) = A\sin kx$$ using $\psi(x)=0$ at $x=0$

$$\psi(L)=0 \rightarrow A\sin kL = 0$$

quantization

quantum number

$$k_n = \frac{n\pi}{L} \quad n = 1, 2, \ldots$$

Sub in

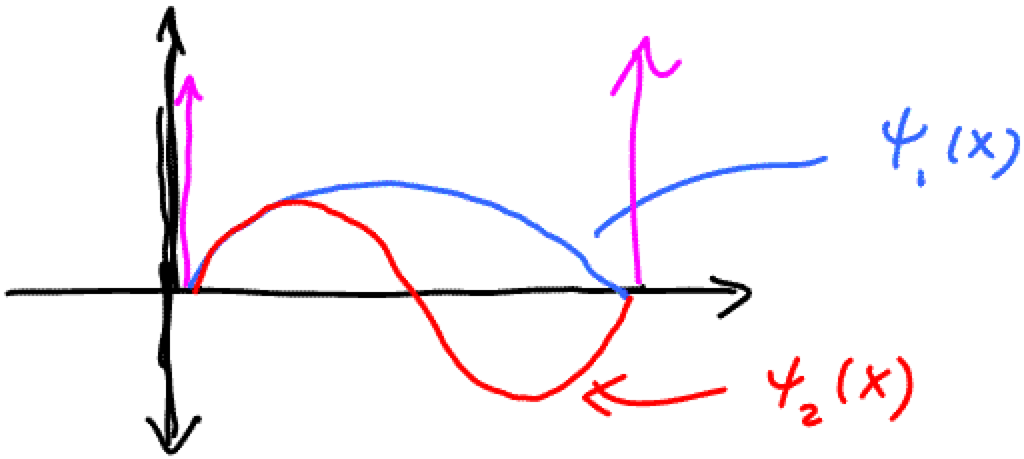
Solve for E_n

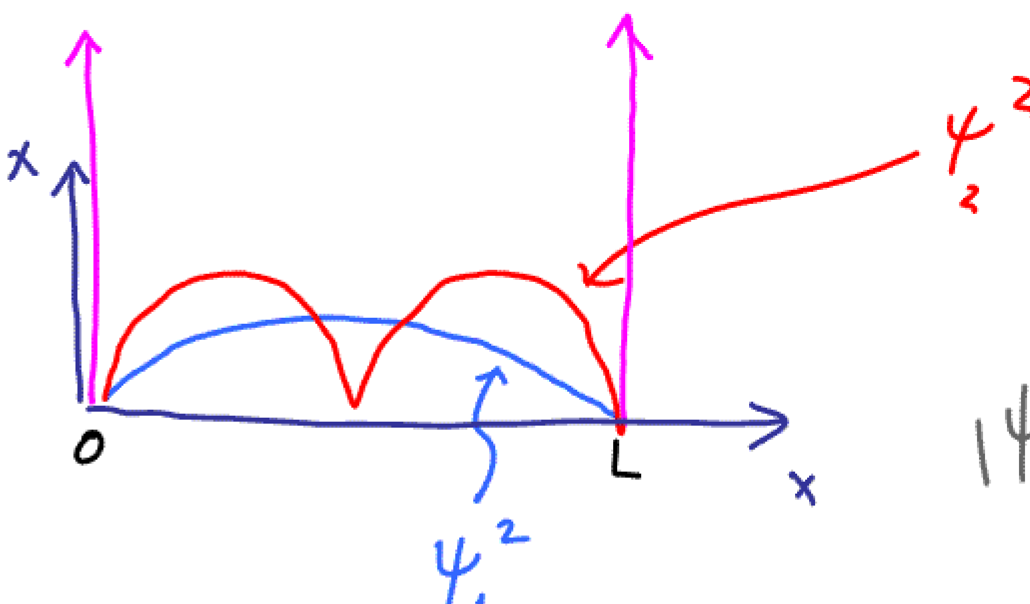
$$E_n = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2}{L^2}\frac{\hbar^2}{2m} \quad n = 1, 2, \ldots$$

$$E_1 = \frac{\pi^2 h^2}{L^2 2m(2\pi)^2} = \frac{h^2}{8mL^2} \qquad E_2 = 2^2\left(\frac{h^2}{8mL^2}\right)$$

$$\psi_n(x) = A \sin\left(\frac{n\pi}{L} x\right) \qquad n = 1, 2 \ldots$$

Think of Standing Waves on String





$|\psi|^2 \rightsquigarrow$ Probability