P123 University of Rochester NAME Soln key - SEM
S. Manly Spring 2013

Final Exam (May 7, 2013)

Please read the problems carefully and answer them in the space provided. Write on the back of the page, if necessary. Show all your work. Partial credit will be given unless specified otherwise.

Problem 1 (3 pts, not necessary to show work):

In simple harmonic motion, the magnitude of the acceleration of a body is always directly proportional to its

a) Displacement
b) Velocity
c) Mass
d) Potential energy
e) Kinetic energy

Think spring

$F = ma \propto x$

Problem 2 (4 pts, show work):

A musical pitch is played at 60 dB. Another is played that sounds four times as loud. The sound intensity level of the second pitch is

a) 80 dB
b) 100 dB
c) 66 dB
d) 64 dB
e) dB
f) 240 dB

$I(\text{in dB}) = 10 \log I/I_0$

$10 \log I_1/I_0 = 60\text{ dB}$

$$10 \log \frac{4I_1}{I_0} = \overbrace{10 \log \frac{I_1}{I_0}} + \overbrace{10 \log 4}^{6}$$

Problem 3 (4 pts, show work):

It takes one year for our planet to orbit the sun. How much time would that orbit appear to take to an observer on a spaceship passing our solar system at a constant speed of 0.95c?

Time of 1 year is in proper Frame t_{Earth}

$\gamma t_{earth} = t_{spaceship} = \boxed{3.2 \text{ years}}$

$\gamma = \frac{1}{\sqrt{1-(.95)^2}} = 3.2$

Problem 4 (4 pts, not necessary to show work):

a) List the electron configuration for a ground state nitrogen atom (Z=7).

$1s^2 2s^2 2p^3$

b) How many unpaired electrons are there in the L=1 state for this atom.

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Problem 5 (6 pts, show work):

The radius of the n=1 orbit in the H atom is 0.053 nm. Approximately what radius would you expect to see for the n=3 orbit of a lithium (Z=3) atom?

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2 Z} \sim 0.053 \frac{n^2}{Z}$$

In Li: 3s e^- Shielded
So $Z=1$ Appropriate

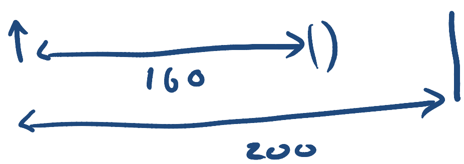
$(0.053)n^2 = 0.053(9) = \boxed{0.48\text{ nm}}$

if said $n^2/Z = 9/3 = 0.16$ nm $\rightarrow$ 3 pts

Problem 6 (6 pts, show work):

An object is placed 200 cm from a screen. When a converging lens is placed between the object and screen at a distance of 160 cm from the object, an image is formed on the screen. Without moving the object or screen, an image can also be formed on the screen when the distance from the object to the lens is

a) 8 cm
b) 16 cm
c) 20 cm
d) 32 cm
e) 40 cm



$$\frac{1}{f} = \frac{1}{160} + \frac{1}{40} \rightarrow f = 32$$

$$\frac{1}{32} = \frac{1}{O} + \frac{1}{200 - O}$$

$$O^2 - 200\,O + 200(32) = 0$$

Solve quadratic

$O = 160$ or 40

Problem 7 (5 pts, show work):

The ^{137}Cs released during the recent Fukishima nuclear accident has a half-life of 31 years. How long will it be before the ^{137}Cs activity in dirt near Fukishima falls by a factor of 30 relative to what it is presently.? (Neglect any form of environmental dispersal in this calculation.)

$$31\text{ yrs} \times 365 \frac{\text{days}}{\text{yr}} \times 24 \frac{\text{hr}}{\text{day}} \times 60 \frac{\text{m}}{\text{hr}} \times 60 \frac{\text{s}}{\text{min}} = 9.78 \times 10^8\text{ s}$$

$$\lambda = \frac{0.693}{t_{1/2}} = 7.09 \times 10^{-10}\text{ s}^{-1}$$

$$N = N_0 e^{-\lambda t}$$

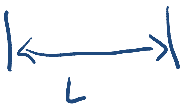
$$\frac{1}{30} = e^{-\lambda t}$$

$$\frac{\ln \frac{1}{30}}{-7.09 \times 10^{-10}} = t$$

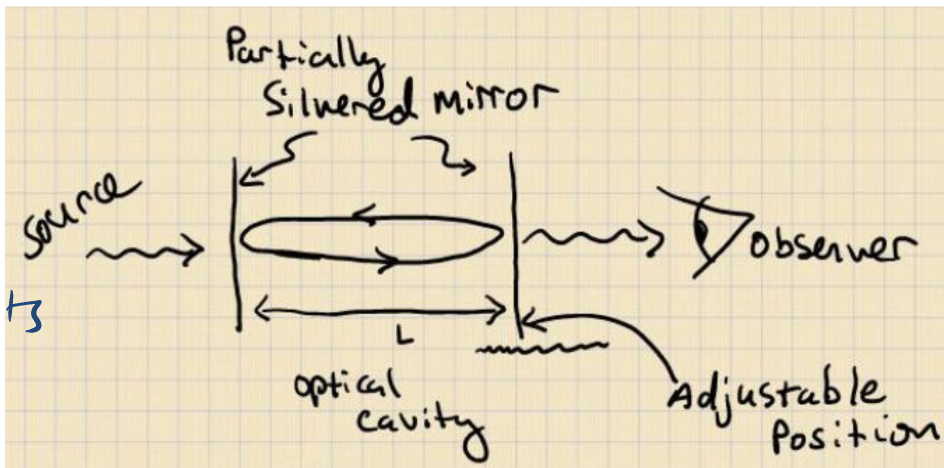
$t = 4.8 \times 10^9$ s or ~ 152 years

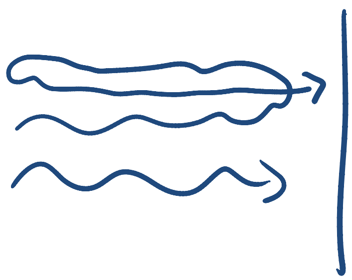
Problem 8 (6 pts, show work):

Laser light with a wavelength of 600 nm is incident on an optical cavity with partially silvered mirrors as shown in the sketch (you might call this a plane-parallel Fabry-Perot resonator). Some of the laser light that is incident on the mirror at the left enters the cavity. That light traverses the cavity and encounters the mirror on the right where some of the light is reflected and some of the light is transmitted. The reflected light moves to the left where some of it is reflected from the left mirror and some is transmitted. And so forth. The distance between the mirrors is adjustable using a very precisely machined screw that supports the right mirror. As the distance between the mirrors, L, is increased, the observer sees the intensity of the light leaving the cavity to vary in intensity. Suppose that L is such that the observer sees a maximum in intensity of the light. Then the distance L is increased slowly by 2600 nm. During that time how many intensity maxima did the observer see ignoring the starting maximum. *(For simplicity, assume all the light moves in horizontal rays only.)*



incr of L by 1 unit increases path by 2 units





2 reflections → 2 phase changes of 180° ≡ 0 phase change

0 phase change

So get max when $2L = m\lambda$

Start w/ a maximum → Every $\Delta L = \frac{1}{2}\lambda$ get another maximum

$(2)\dfrac{2600\text{ nm}}{600\text{ nm}} = 8.6\,\lambda$

So as mirror moves 2600 nm will see 8 intensity maxima

Problem 9 (6 pts, show work):

Sound travels at 340 m/s in air and 1500 m/s in water. A sound of 256 Hz is made under water.

a) What is the wavelength of the sound waves in the water?

$v = \lambda f \quad 1500 = \lambda_w 256 \quad \lambda_w = 5.8\ m$

Suppose this sound passes into the air above the water.

b) What is the wavelength and frequency of the sound waves in the air?

$F_{air} = 256\ Hz$ (unchanged) $\quad 340 = \lambda_a 256 \quad \lambda_{air} = 1.3\ m$

Problem 10 (5 pts, show work):

An object moves through a medium causing a disturbance. Briefly describe the circumstances under which this disturbance will take the form of a shock wave.

a shock wave will form when the object moves faster than the wave disturbance can travel, i.e. plane traveling faster than speed of sound.

Problem 11 (5 pts, show work):

The total nuclear binding energy of ${}^3_1H$ is 8.5 MeV and of ${}^3_2He$ is 6.7 MeV. Briefly give a plausible explanation of this difference.

The ${}^3He$ nucleus is less tightly bound because there are 2 protons instead of the 1 in H.

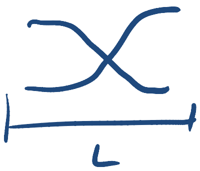
So, the Coulomb force must be overcome to bind the nucleons, reducing the BE

If you just calculate the B.E.'s you are finding the numbers but not giving an explanation for why they are what they are → 2 pts awarded for this.

Problem 12 (6 pts, show work):

Jimmy Jones walks across the kitchen at a pace of 2 steps per second holding a glass of milk that is approximately 10 cm in width. The milk sloshes higher and higher in the glass until it spills over the top of the glass, much to his mother's displeasure. What is the maximum speed of waves in the milk?

Standing waves



Fundamental resonance

$V_{wave} = \lambda f = 2L\,2$

$= 4(0.1) = 0.4\ m/s$

$L = \frac{1}{2}\lambda \rightsquigarrow \lambda = 2L$

Frequency = 2Hz

Problem 13 (8 pts, show work):

A nuclear reaction where two particles join to form a single excited nucleus which decays to its ground state by photon emission is known as "radiative capture". Suppose a beam of alpha particles are incident on a target of ^{7}Li nuclei. Assume the beam of alphas has a minimal kinetic energy, just sufficient to allow a nuclear reaction to take place. You might find it useful to know the conversion factor: 1 u = 931.494 MeV/c^2

Z	Name	Symbol	Mass of Atom (u)
1	Hydrogen	^{1}H	1.007825
	Deuterium	^{2}H	2.014102
	Tritium	^{3}H	3.016049
2	Helium	^{3}He	3.016029
		^{4}He	4.002603
3	Lithium	^{6}Li	6.015122
		^{7}Li	7.016004
4	Beryllium	^{9}Be	9.012182
5	Boron	^{10}B	10.012937
		^{11}B	11.009305
6	Carbon	^{12}C	12.000000
		^{13}C	13.003355
		^{14}C	14.003242
7	Nitrogen	^{14}N	14.003074
		^{15}N	15.000109
8	Oxygen	^{16}O	15.994915
		^{17}O	16.999132
		^{18}O	17.999160

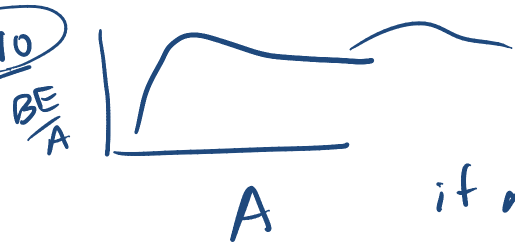
(a) If a ^{7}Li nucleus radiatively captures an alpha particle in this beam, what is the energy of the emitted photon in MeV?

$^7_3Li + ^4_2\alpha \rightarrow ^{11}_5B + \gamma$

$E_\gamma = -11.009305 + 7.016004 + 4.002603$
$= 0.0093\ u \rightsquigarrow \approx 8.66\ MeV$

(b) Would you ever expect to observe the radiative capture of an alpha particle by a very heavy nucleus? Why or why not?

NO



Heavy nuclei live here

if α captured, the BE/A is decreased (takes energy)

So, No excess energy to go into a γ

Problem 14 (9 pts, show work):

For a one-dimensional particle in a box, i.e., in an infinite square well potential, What is the fractional energy difference between adjacent energy eigenvalues (states), $\frac{\Delta E_n}{E_n}$. What happens in the "classical limit" where n gets large?

$$\frac{\Delta E_n}{E_n} = \frac{E_{n+1} - E_n}{E_n} \qquad E_n = n^2\pi^2\hbar^2 / L^2 2m \quad \text{for } \infty \text{ square well}$$

$$\frac{\Delta E_n}{E_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{n^2 + 2n + 1 - n^2}{n^2} = \frac{2n+1}{n^2}$$

$$\lim_{n\to\infty} \frac{2n+1}{n^2} = 0 \qquad \frac{\Delta E_n}{E_n} \to 0$$

This is what you'd expect classically $\rightsquigarrow$ quantization disappears for large n.

Problem 15 (5 pts, show work):

According to the hot big bang model, the universe is approximately 13.7 billion years old. Some people have speculated that the universe is a quantum fluctuation in energy where the kinetic energy of the expansion of the universe is almost offset by the increasingly negative gravitational potential energy brought about during the expansion. In this case, the total energy of the universe is close to zero. In such a theory, the total energy of the universe must be no bigger than what value?

$\Delta E \Delta t < \hbar$ ← ok if h or h/2 or ħ/2

$$13.7\times10^9\, yr \times 365\frac{d}{yr} \times 24\frac{hr}{d} \times 60\frac{m}{hr} \times 60\frac{s}{m} = 4.3\times10^{17}\, s$$

$$\frac{6.62\times10^{-34}\, J\cdot s}{(2\pi)(4.3\times10^{17}\, s)} \sim \Delta E \sim 2.45\times10^{-52} \text{ Joules}$$

Problem 16 (9 pts, show work):

A particle of mass m is incident on a 'reverse' step potential at x=0. The particle moves from negative values of x toward positive values of x. The potential for x<0 is a positive constant, V. The potential for x>0 is zero. The particle has an energy, E, where E>V.

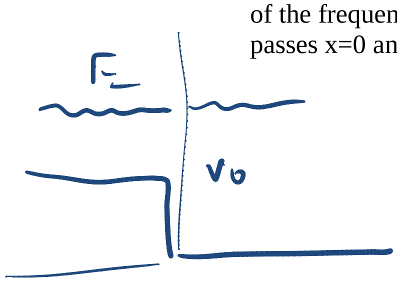
a) (1 pt) In terms of the variables in this problem, what is the spatial frequency for the wave function describing the particle for x<0?

$$k_1 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

b) (1 pt) In terms of the variables in this problem, what is the spatial frequency for the wave function describing the particle for x>0?

$$k_2 = \sqrt{2mE/\hbar^2}$$

c) (7 pts) Determine the transmission probability (transmission coefficient) for this particle in terms of the frequencies you listed in parts (a) and (b). That is to say, what is the probability this particle passes x=0 and moves on to positive x?



$$\psi(x)\big|_{x<0} = Ae^{ik_1x} + Be^{-ik_1x}$$

$$\psi(x)\big|_{x>0} = Ce^{ik_2x} + De^{-ik_2x}$$

Particle coming in from $-\infty \longrightarrow D=0$

$$\psi(0)\big|_{x<0} = \psi(0)\big|_{x>0} \leadsto A+B=C$$

$$\frac{d\psi(0)}{dx}\bigg|_{x<0} = \frac{d\psi(0)}{dx}\bigg|_{x>0} \leadsto ik_1A - ik_1B = ik_2C \leadsto k_1(A-B) = k_2C$$

$C = A+B$

$k_1(A-B) = k_2(A+B)$

$A(k_1-k_2) = B(k_2+k_1)$

$B = A\frac{k_1-k_2}{k_2+k_1}$

$$\Rightarrow C = A\left(1+\frac{k_1-k_2}{k_2+k_1}\right)$$

$$T = \frac{k_2C^*C}{k_1A^*A} = \frac{k_2\left(1+\frac{k_1-k_2}{k_2+k_1}\right)^2}{k_1}$$

$$T = \frac{2k_2}{(k_2+k_1)}$$

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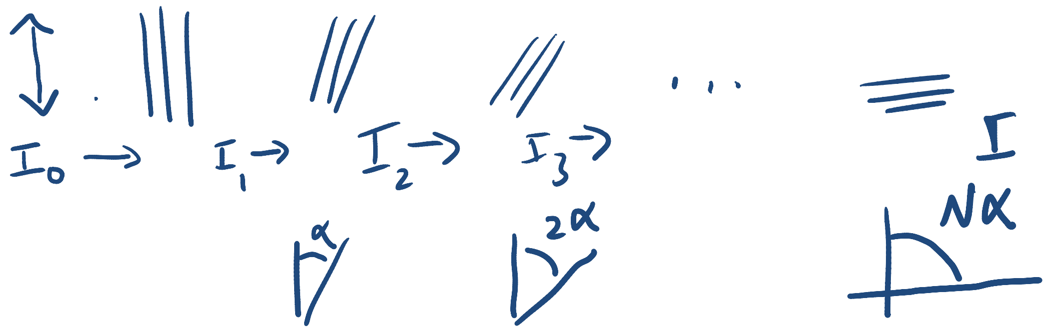
Problem 17 (9 pts, show work):

A Polaroid film is a thin film containing long molecules that are all linearly oriented. This film acts as a linear polarizer to light that passes through it. Suppose that a very large number (N) of thin Polaroid film are arranged in a 'sandwich' topology. Also suppose that the axis of linear polarization of each Polaroid is at a constant angle α greater than that of its immediate predecessor in the sandwich. Thus the last Polaroid is at an angle $\theta = N\alpha$ from the first. Neglecting any losses due to reflection at the many surfaces, and supposing that linearly polarized light of intensity I_o is incident on the first Polaroid film with its polarization along the axis of polarization of the first Polaroid film, find the output intensity (relative to the input intensity) as a function of θ for very large N.



$I_0 = I_1$ in this problem

after 2 layers $\quad I_2 = I_1 \cos^2\alpha = I_0 \cos^2\alpha$

after 3 layers $\quad I_3 = I_2 \cos^2\alpha = I_1 \cos^2\alpha \cos^2\alpha = I_0 \cos^2\alpha \cos^2\alpha$

$\vdots$

after N layers $\quad I_N = I_0 \cos^{2N}\alpha$

as N goes large, α goes small ... expand $\cos^{2N}\alpha$ as Taylors series about $\alpha = 0$

$$I_N = I_0 \left\{ \cos^{2N}(0) + \underbrace{2N\cos^{2N-1}(0)\sin(0)\alpha}_{0} + \frac{(-2N)\cos^{2N}(0)\alpha^2}{2!} + \underbrace{\frac{(2N)(2N-1)\cos^{2N-2}(0)}{2!}\sin^2(0)\alpha^2}_{0} \cdots \right\}$$

$I_N = I_0(1 - N\alpha^2 \ldots)$

$\theta = N\alpha \leadsto I_N = I_0\left(1 - \frac{\theta^2}{N} \ldots\right)$ as $N \rightarrow$ large $I_N \rightarrow I_0$

No loss in intensity!!

1)	/3
2)	/4
3)	/4
4)	/4
5)	/6
6)	/6
7)	/5
8)	/6
9)	/6
10)	/5
11)	/5
12)	/6
13)	/8
14)	/9
15)	/5
16)	/9
17)	/9
tot	/100