Physics 123 - Spring 2013 - Solutions to P.S. 2

1. 36-15

Since the number of particles passing per second is reduced from N to $N/2$, a time T_0 must have elapsed in the particles' rest frame. The time T elapsed in the lab frame will be greater, according to Eq. 36-1a. The particles moved a distance of $2cT_0$ in the lab frame during that time.

$$T_0 = T\sqrt{1-v^2/c^2} \rightarrow T = \frac{T_0}{\sqrt{1-v^2/c^2}} \;;\; v = \frac{x}{T} = \frac{2cT_0}{\frac{T_0}{\sqrt{1-v^2/c^2}}} \rightarrow v = \sqrt{\tfrac{4}{5}}c = \boxed{0.894c}$$

2. 36-35

(*a*) We compare the classical momentum to the relativistic momentum.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}} = \frac{mv}{\frac{mv}{\sqrt{1-v^2/c^2}}} = \sqrt{1-v^2/c^2} = \sqrt{1-(0.10)^2} = 0.995$$

The classical momentum is about $\boxed{-0.5\%}$ in error.

(*b*) We again compare the two momenta.

$$\frac{p_{\text{classical}}}{p_{\text{relativistic}}} = \frac{mv}{\frac{mv}{\sqrt{1-v^2/c^2}}} = \sqrt{1-v^2/c^2} = \sqrt{1-(0.60)^2} = 0.8$$

The classical momentum is $\boxed{-20\%}$ in error.

3. 36-37

The two momenta, as measured in the frame in which the particle was initially at rest, will be equal to each other in magnitude. The lighter particle is designated with a subscript "1", and the heavier particle with a subscript "2".

$$p_1 = p_2 \rightarrow \frac{m_1 v_1}{\sqrt{1-v_1^2/c^2}} = \frac{m_2 v_2}{\sqrt{1-v_2^2/c^2}} \rightarrow$$

$$\frac{v_1^2}{\left(1-v_1^2/c^2\right)} = \left(\frac{m_2}{m_1}\right)^2 \frac{v_2^2}{\left(1-v_2^2/c^2\right)} = \left(\frac{6.68\times10^{-27}\,\text{kg}}{1.67\times10^{-27}\,\text{kg}}\right)^2 \left[\frac{(0.60c)^2}{1-(0.60)^2}\right] = 9.0c^2 \rightarrow$$

$$v_1 = \sqrt{0.90}\,c = \boxed{0.95c}$$

4. 36-40

We find the loss in mass from Eq. 36-12.

$$m = \frac{E}{c^2} = \frac{(200\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})}{(3.00\times10^8\,\text{m/s})^2} = 3.56\times10^{-28}\,\text{kg} \approx \boxed{4\times10^{-28}\,\text{kg}}$$

5.
36-41

We find the mass conversion from Eq. 36-12.

$$m = \frac{E}{c^2} = \frac{\left(8 \times 10^{19}\ \mathrm{J}\right)}{\left(3.00 \times 10^8\ \mathrm{m/s}\right)^2} = \boxed{900\ \mathrm{kg}}$$

6.
36-43

Each photon has momentum 0.50 MeV/c. Thus each photon has mass 0.50 MeV. Assuming the photons have opposite initial directions, then the total momentum is 0, and so the product mass will not be moving. Thus all of the photon energy can be converted into the mass of the particle. Accordingly, the heaviest particle would have a mass of $\boxed{1.00\ \mathrm{MeV}/c^2}$, which is 1.78×10^{-30} kg.

7
36-52

We let M represent the rest mass of the new particle. The initial energy is due to both incoming particles, and the final energy is the rest energy of the new particle. Use Eq. 36-11 for the initial energies.

$$E = 2\left(\gamma mc^2\right) = Mc^2 \rightarrow M = 2\gamma m = \boxed{\frac{2m}{\sqrt{1 - v^2/c^2}}}$$

We assumed that energy is conserved, and so there was $\boxed{\text{no loss of energy}}$ in the collision. The final kinetic energy is 0, so all of the kinetic energy was lost.

$$K_{\text{lost}} = K_{\text{initial}} = 2(\gamma - 1)mc^2 = \boxed{\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)2mc^2}$$

8.
36-53

Since the electron was accelerated by a potential difference of 28 kV, its potential energy decreased by 28 keV, and so its kinetic energy increased from 0 to 28 MeV. Use Eq. 36-10 to find the speed from the kinetic energy.

$$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2 \rightarrow$$

$$v = c\sqrt{1 - \frac{1}{\left(\frac{K}{mc^2} + 1\right)^2}} = c\sqrt{1 - \frac{1}{\left(\frac{0.028\,\text{MeV}}{0.511\,\text{MeV}} + 1\right)^2}} = \boxed{0.32c}$$

9.

$$\begin{bmatrix} -2 & 4 & 3 \\ 12 & -5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 & -1 \\ 10 & 3 & 8 \\ 7 & 2 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} (-2)(5) + (4)(10) + (3)(7) & (-2)(6) + (4)(3) + (3)(2) & (-2)(-1) + (4)(8) + (3)(1) \\ (12)(5) + (-5)(10) + (1)(7) & (12)(6) + (-5)(3) + (1)(2) & (12)(-1) + (-5)(8) + (1)(1) \end{bmatrix} = \begin{bmatrix} 51 & 6 & 37 \\ 17 & 59 & -51 \end{bmatrix}$$

10.

$$\begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1.7 & -1.36 & 0 & 0 \\ -1.36 & 1.7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\gamma = \frac{1}{\sqrt{1-.8^2}} = 1.7$$

$$\beta = \frac{v}{c} = 0.8$$

$$\begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1.7 & -1.36 & 0 & 0 \\ -1.36 & 1.7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \\ -2 \\ 10 \end{pmatrix} = \begin{pmatrix} (1.7)(7) + (-1.36)(3) \\ (-1.36)(7) + (1.7)(3) \\ -2 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 7.8 \\ -4.4 \\ -2 \\ 10 \end{pmatrix}$$

11.

$$A = (x_0^A = 5,\ x_1^A = 3,\ x_2^A = 6,\ x_3^A = -2)$$

$$B = (x_0^B = 2,\ x_1^B = 300,\ x_2^B = 4,\ x_3^B = -220)$$

$$(B-A) = (\underbrace{x_0 = -3}_{x_0^B - x_0^A},\ \underbrace{x_1 = 297}_{x_1^B - x_1^A},\ x_2 = -2,\ -218) \ \cdots$$

Very Space like

Events A, B are Not causally connected

$$\text{Invariant interval} = (B-A)\cdot(B-A) = -x_0^2 + x_1^2 + x_2^2 + x_3^2$$

$$= -9 + 8.8\times10^4 + 4 + 4.7\times10^4 = 1.3\times10^5$$