Physics 142 - October 25, 2005

Amperes' Law Example, Induction, Lentz's Law

Last Time -

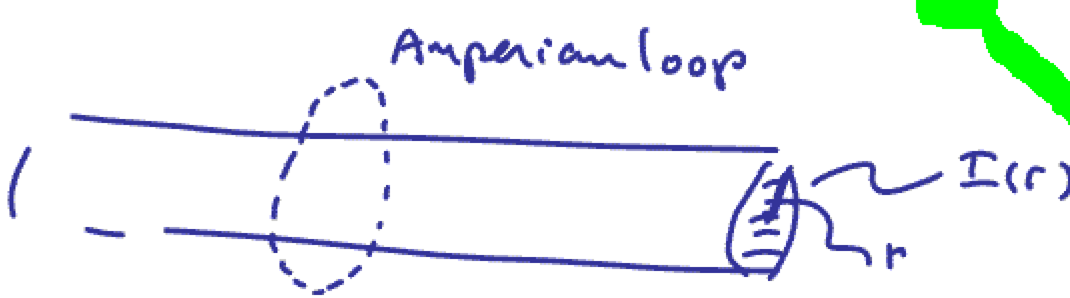
Ampere's Law:

Important

$$\int_{\text{closed loop}} \vec{B} \cdot \vec{dl} = \mu_0 I_{enclosed}$$

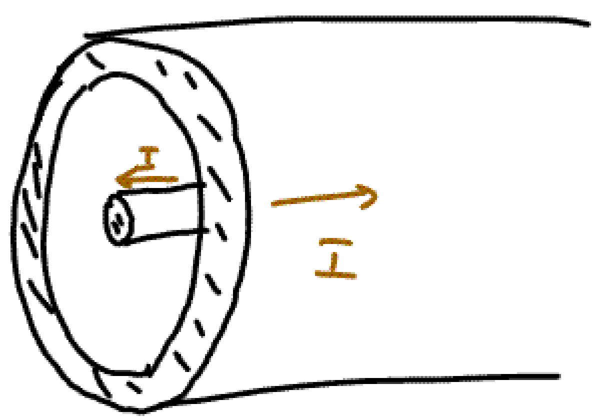
sometimes see $\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{encl}$

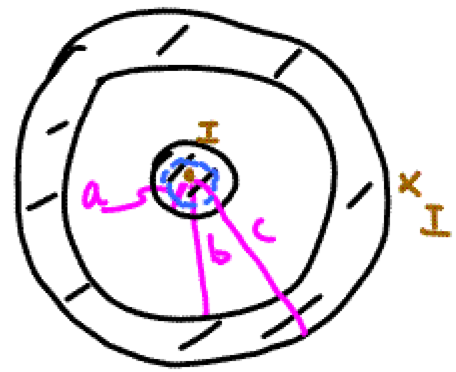
choose loop so
$\vec{B} \cdot \vec{dl}$ simple
$|\vec{B}| = \text{constant}$



AT End of class on 10/20 we started example

Long coaxial cable





Assume I uniform across both conductors

Find $\vec{B}$ in all space —

$r<a$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$\int_0^{2\pi r} |\vec{B}|\,dl = \mu_0 I_{encl}$$

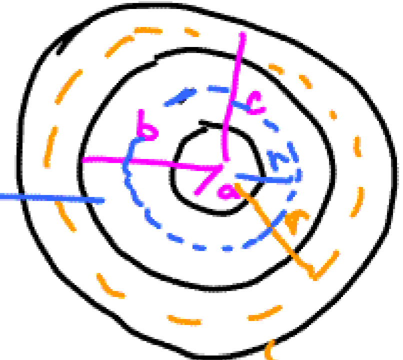
current density

$$I/\pi a^2 = j$$

$$|\vec{B}|2\pi r = \mu_0 I_{encl} = \mu_0 j \pi r^2$$

$$|\vec{B}| = \mu_0 \frac{I}{\pi a^2}\frac{\pi r^2}{2\pi r} = \frac{\mu_0 I r}{2\pi a^2}$$

$a<r<b$

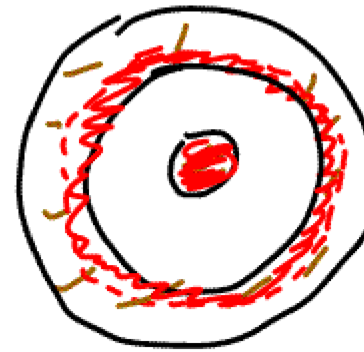


$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$|\vec{B}|\, 2\pi r = \mu_0 I$$

$$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r}}$$

$b<r<c$



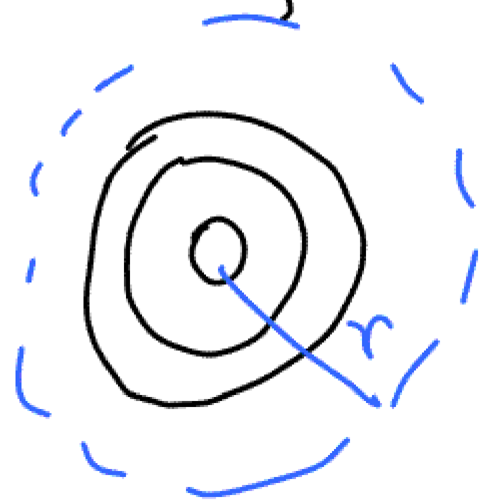
$$|\vec{B}|\, 2\pi r = \mu_0 I_{encl}$$

$$I_{enc} = I - I\left(\text{fraction of Area of outer conductor w/in Amperian loop}\right)$$

$$I_{encl} = I - I \frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r} \left\{ 1 - \left[\frac{r^2-b^2}{c^2-b^2}\right] \right\}$$

$r > c$



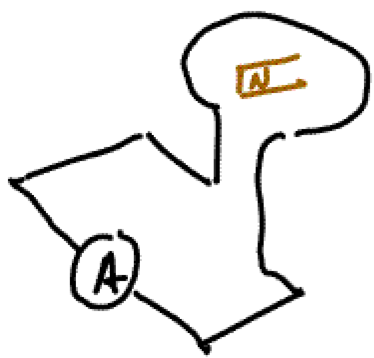
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

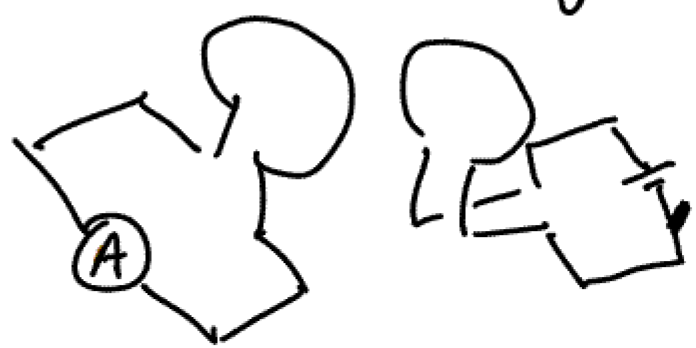
$$|\vec{B}|\, 2\pi r = \mu_0 I_{encl} = 0$$

$$|\vec{B}| = 0$$

Magnetic Induction

1830's ~ Michael Faraday (England)
Joseph Henry (US)





Induction ≡ A changing magnetic field
Induces a changing Electric field
an EMF

No changing fields

magnetostatics

Kirchoff

$$\sum V \Big|_{\text{closed loop}} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Statement of Kirchoff's Law in free space

$$E = -\frac{dV}{dx}$$

$$\int dV \sim \int E\,dx$$

changing Magnetic field

$$\boxed{\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_M}{dt} \qquad \phi_M \equiv \int_{\text{loop}} \vec{B} \cdot d\vec{A} \qquad \text{Faraday's Law}}$$

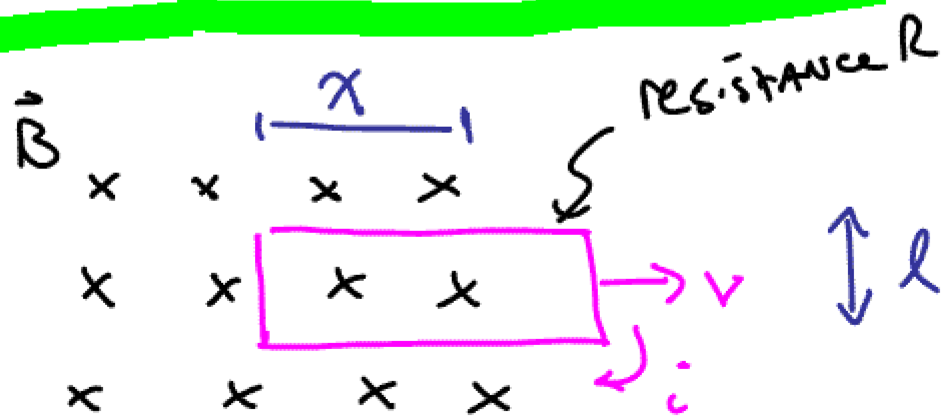
Important

True in wires
Materials
Freespace

$$\mathcal{E}(t)$$

Lenz's Law – An induced current in a closed conducting loop will appear in such a way to oppose the change that creates it.

Important



What is i?

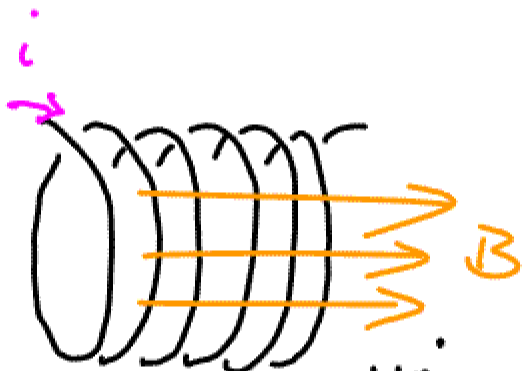
$$\mathcal{E} = -\frac{d\Phi_m}{dt} = -B\ell\frac{dx}{dt} = -B\ell v$$

$$|i| = \frac{B\ell v}{R}$$

R = loop resistance

Motional EMF

Eddy currents



$$|\vec{B}| = \mu_0 n i$$

Turns/length

$\mu n i$

$$\phi_m = \boxed{(BA)}$$

x-sectional area of solenoid loop

single loop

$$\phi_m \propto i$$

proportionality const depends
on Geometry Alone

$BA = \mu_0 n i A$ single loop

$n \cdot l$ = # loops

ϕ_M thru l $= \mu_0 n^2 i A l$

$$\phi_M = L i$$

CONSTANT of self inductance

if $\Delta i \rightarrow \frac{di}{dt} \rightarrow \frac{d\phi_M}{dt} \rightarrow \mathcal{E}$

$$\boxed{\varepsilon = \frac{-d\Phi_M}{dt} = -L\frac{di}{dt}}$$

Important