Physics 142 - October 27, 2005

Lentz's Law, mutual/self inductance
LR Circuits, LC circuits, AC Circuits

E-Mail

Class Web Server

Problem Set 7

What to do if ...

Last Time –

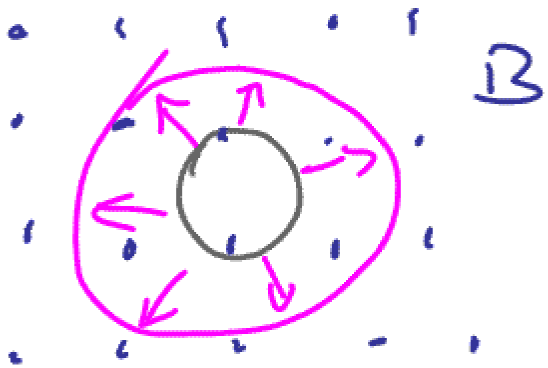
Important

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_M}{dt}$$

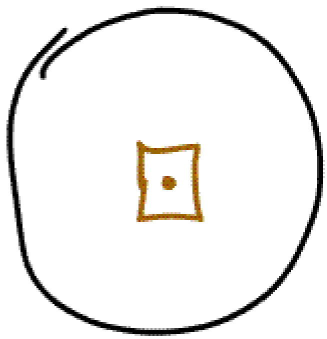
$$\phi_M \equiv \int_{\text{loop}} \vec{B} \cdot d\vec{A}$$

Faraday's Law

Lenz's Law – The induced current opposes the change that creates it

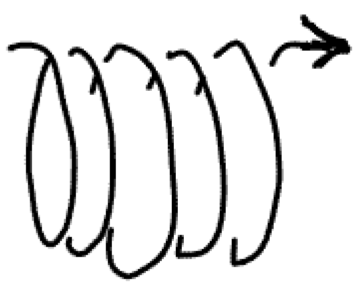


what is direction of induced current?



magnet coming toward you (on other side of board)

unit Henry

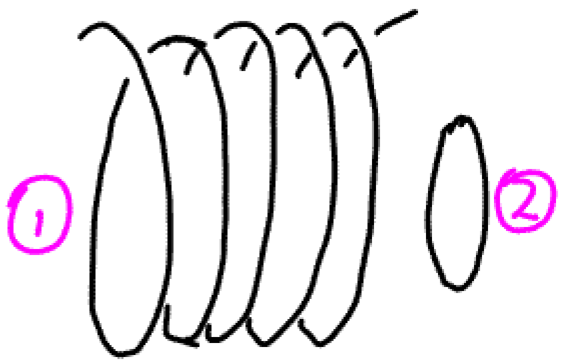


$\Phi_m \propto i$ self inductance

$\Phi_m = \boxed{L}\, i$

$\mathcal{E} = -\frac{d\Phi_m}{dt} = -\boxed{L}\frac{di}{dt}$

Circuit itself fights changes



Δi in 1

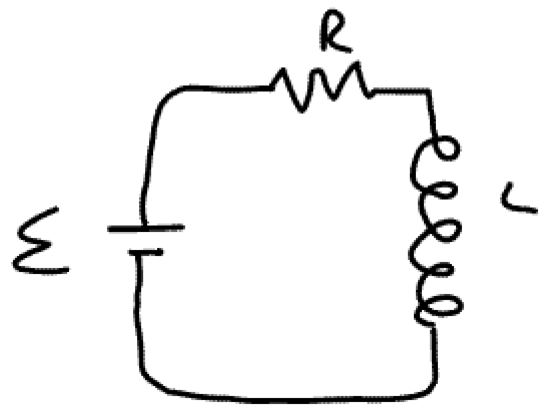
ΔB in 2

↳ induces $\mathcal{E}$ 2

$$\phi_{m_{2}} \propto i_{1} = L\,i$$

Constant of Mutual inductance

Energy and Magnetic field



$$\mathcal{E} = iR + L\frac{di}{dt}$$

$$\mathcal{E}i = i^2R + Li\frac{di}{dt}$$

Power out of EMF

Power dissipated in Resistor

Power in or out of inductor

E stored in B field of inductor

in inductor $\frac{dU_B}{dt} = Li\frac{di}{dt}$

$$dU_B = Li\,di$$

$$U_B = \int_0^I Li\,di = \frac{1}{2}LI^2$$

Analogous $\frac{1}{2}CV^2$ from capacitors

Solenoid w/ i, n turns/length

Find Energy density of B field

$$B_{\text{solenoid}} = \mu_0 n i \quad \text{(inside)}$$

$$= 0 \quad \text{(outside)}$$

$$u_B \equiv \text{Energy density in } B \text{ field}$$

$$= \frac{U_B}{A\ell} = \frac{1}{2} L i^2 / A\ell$$

$$\phi_m = Li \qquad L = \frac{\Phi_m}{i} = \frac{(BA)(n\ell)}{i}$$

$$L = \frac{\mu_0 n i A n \ell}{i} = \mu_0 n^2 A \ell$$

$$u_B = \frac{1}{2}\frac{\mu_0 n^2 A l i^2}{A l} = \frac{1}{2}\mu_0 i^2 n^2 = \frac{B^2}{2\mu_0}$$

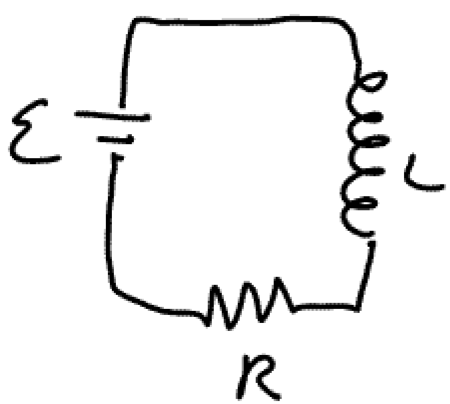
Important

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$

Analogous to

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

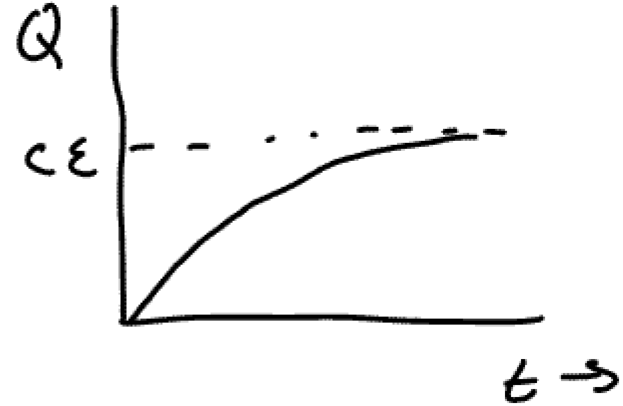
LR Circuit



$$\mathcal{E} - L\frac{di}{dt} - iR = 0$$

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

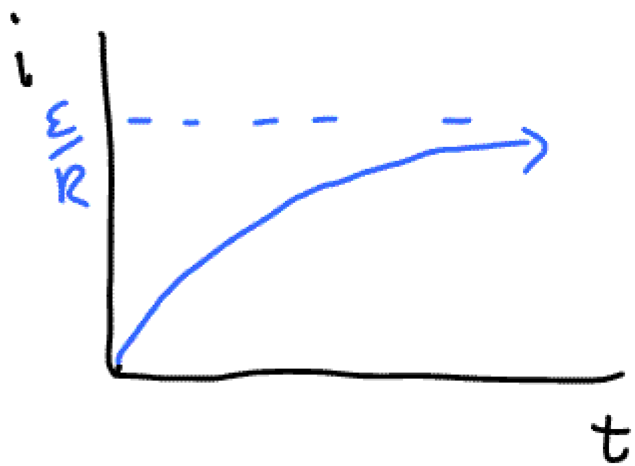
Similar to RC circuit charging



$RC \equiv \tau_c$

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

LR circuit



Inductive Time Constant

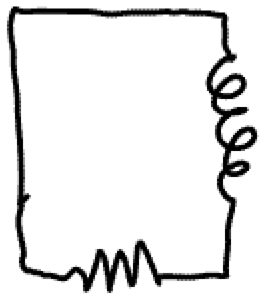
$$\frac{L}{R} = \tau_L$$

$V = iR$

$$V = \mathcal{E}\left(1 - e^{-\frac{tR}{L}}\right) = \mathcal{E}\left(1 - e^{-t/\tau_L}\right)$$

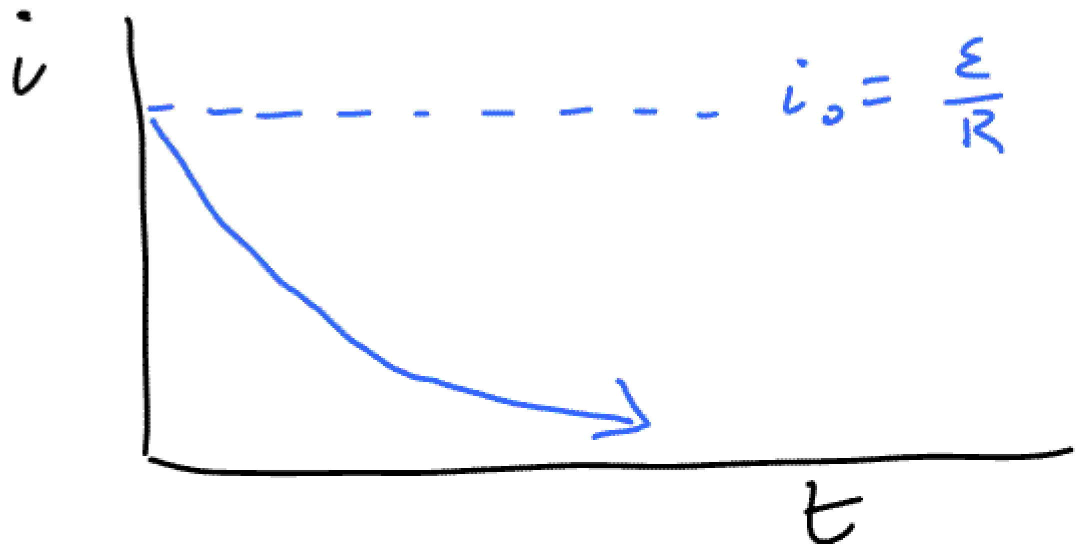
across inductor

Remove EMF, Short L across R

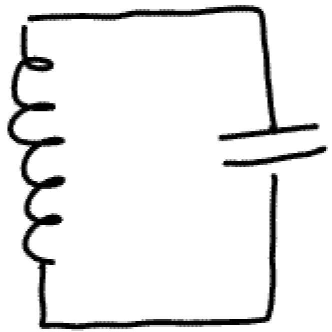


$$0 = iR + L\frac{di}{dt}$$

$$i = \frac{\varepsilon}{R} e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$



LC circuit



$$U = U_B + U_E = \frac{1}{2}Li^2 + \frac{q^2}{2C}$$

R in circuit $\to 0$

$$\frac{dU}{dt} = 0 = \frac{1}{2}L2i\frac{di}{dt} + \frac{2q}{2C}\frac{dq}{dt}$$

$$0 = L\frac{dq}{dt}\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt}$$

$$0 = L\frac{d^2q}{dt^2} + \frac{q}{C}$$

differential Eqn

Soln $\longrightarrow$ $q(t) = Q\cos(\omega t + \phi)$

$$\omega = \frac{1}{\sqrt{LC}}$$

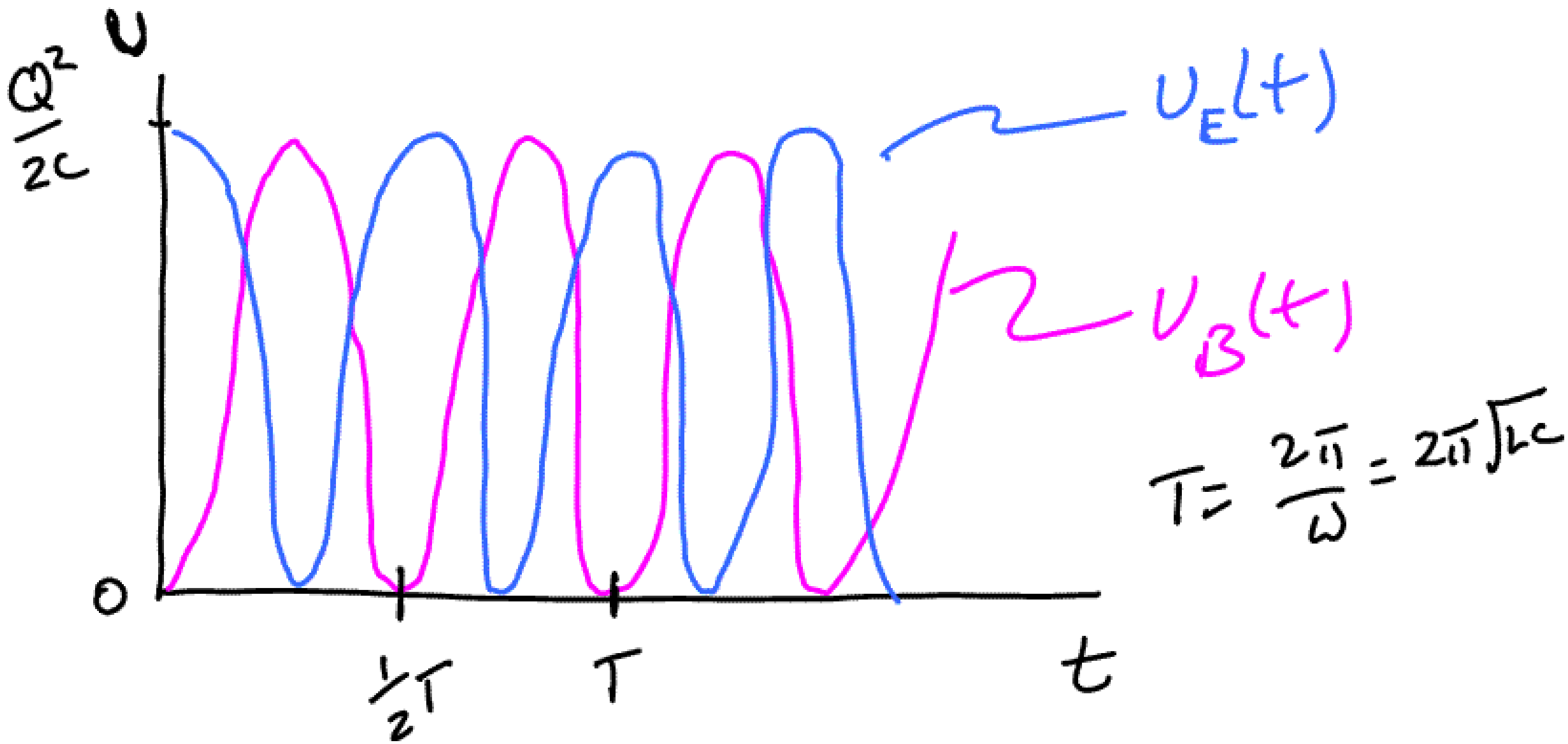
Harmonic dependence

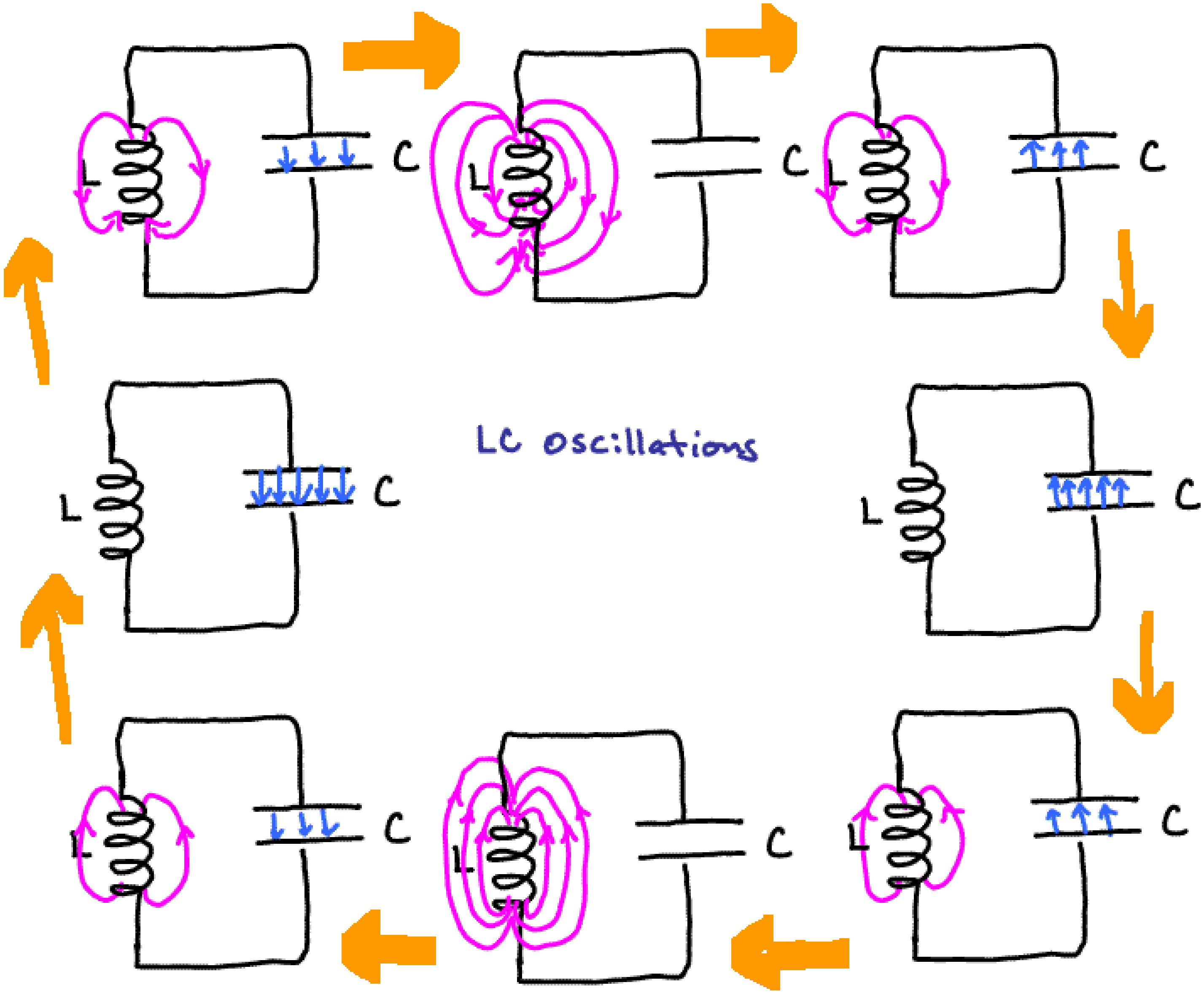
Energy flow

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi)$$

$$\frac{dq}{dt} = i = -Q\omega\sin(\omega t + \phi)$$

$$U_B = \frac{1}{2}Li^2 = \frac{L}{2}Q^2\omega^2\sin^2(\omega t + \phi)$$


U
$\frac{Q^2}{2C}$
0
$\frac{1}{2}T$
T
t
$U_E(t)$
$U_B(t)$
$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$


LC oscillations
L
C