Physics 142 - November 10, 2005

Maxwell's equations cont'd

Review session

5-7 Sunday B+L 106

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$$\int_S \vec{E} \cdot dA = q/\varepsilon_0$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0}$$

$\vec{v}$ is arbitrary vector field

$$\int_V (\vec{\nabla} \cdot \vec{v})\, dv = \int_S \vec{v} \cdot d\vec{A}$$

$$\int_S \vec{B} \cdot d\vec{A} = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

No Magnetic Monopole

$$\int_c \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt}$$

Stokes Thm

$$\int_S (\vec{\nabla}\times\vec{V})\cdot d\vec{A} = \int_c \vec{V}\cdot d\vec{l}$$

$\vec{V} \equiv$ Arbitrary vector field

curl

$$\int_c \vec{E}\cdot d\vec{l} = \int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{A} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$$

$$= -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

$$\int_c \vec{E}\cdot d\vec{l} = -\frac{d\Phi_M}{dt}$$

$$\int_c \vec{B}\cdot d\ell = \mu_0 i + \mu_0\epsilon_0 \frac{d\Phi_E}{dt} \longrightarrow \vec{\nabla}\times\vec{B} = \frac{\partial \vec{E}}{\partial t}$$ NO currents

$$\int_c \vec{B}\cdot d\vec{\ell} = \int_S (\vec{\nabla}\times\vec{B})\cdot d\vec{A} = \mu_0 i + \mu_0\epsilon_0\frac{d\Phi_E}{dt} = \mu_0\int_S \vec{J}\cdot d\vec{A} + \mu_0\epsilon_0\frac{d}{dt}\int_S \vec{E}\cdot d\vec{A}$$

STOKES

$\vec{J}$ = current density

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}}$$

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Differential form of Maxwell's eqns

region w/ No currents

$$\vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Take curl of Both sides

$$\vec{\nabla}\times(\vec{\nabla}\times B) = \mu_0\epsilon_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t} = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\underbrace{\cancel{\vec{\nabla}\cdot\vec{B}}}_{0}) - \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\vec{B})}_{\text{Known as Laplacian}}$$

$$\nabla^2\vec{B}$$

$$\nabla \vec{B} = \frac{\partial B_x}{\partial x}\hat{i} + \frac{\partial B_y}{\partial y}\hat{j} + \frac{\partial B_z}{\partial z}\hat{k}$$

Laplacian of a vector defined as

$$\nabla^2 \vec{V} = (\nabla^2 V_x)\hat{i} + (\nabla^2 V_y)\hat{j} + (\nabla^2 V_z)\hat{k}$$

This is a wave equation

$$\nabla^2 B = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 B_x = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t}$$

$$\nabla^2 B_y = -\mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = -\mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

3 scalar wave equations

Wave soln propagates w/ velocity $\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$