Physics 142 - November 15, 2005

Implications of Maxwell's Equations

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad \vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

$$\oint_S \vec{B}\cdot d\vec{A} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl} + \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A} \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_M}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A} \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

Integral form | Differential form

Maxwell's Equations

in region of no currents

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

mult by $\nabla \times \ldots$

Vector Calc Identities

$$\boxed{\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Vector wave equation

3 scalar wave equations

$$\boxed{\nabla^2 B_x = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}}$$

$$\nabla^2 B_y = -\mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2}$$

$$\nabla^2 B_z = -\mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

Laplacian

Laplacian of Scalar

$$\nabla^2 T = \vec{\nabla} \cdot (\vec{\nabla} T) = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \left(\hat{i} \frac{\partial T}{\partial x} + \hat{j} \frac{\partial T}{\partial y} + \hat{k} \frac{\partial T}{\partial z} \right)$$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

Laplacian of a vector $\nabla^2 \vec{v}$

$$\nabla^2 \vec{v} = (\nabla^2 v_x)\hat{i} + (\nabla^2 v_y)\hat{j} + (\nabla^2 v_z)\hat{k}$$

$$\nabla^2 B_x = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} = -\mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2}$$

Similarly $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial(\vec{\nabla} \times \vec{B})}{\partial t} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

wave eqn for $\vec{E}$ prop. w/ velocity $\sqrt{\frac{1}{\mu_0 \epsilon_0}} = c$

$\rho = 0$ $\vec{J} = 0$

region no charge no current

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$= \hat{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{j}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$ $\hat{i}$ part of $\vec{\nabla} \times \vec{E}$ eqn

$$-\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) = -\frac{\partial B_y}{\partial t}$$ $\hat{j}$ part

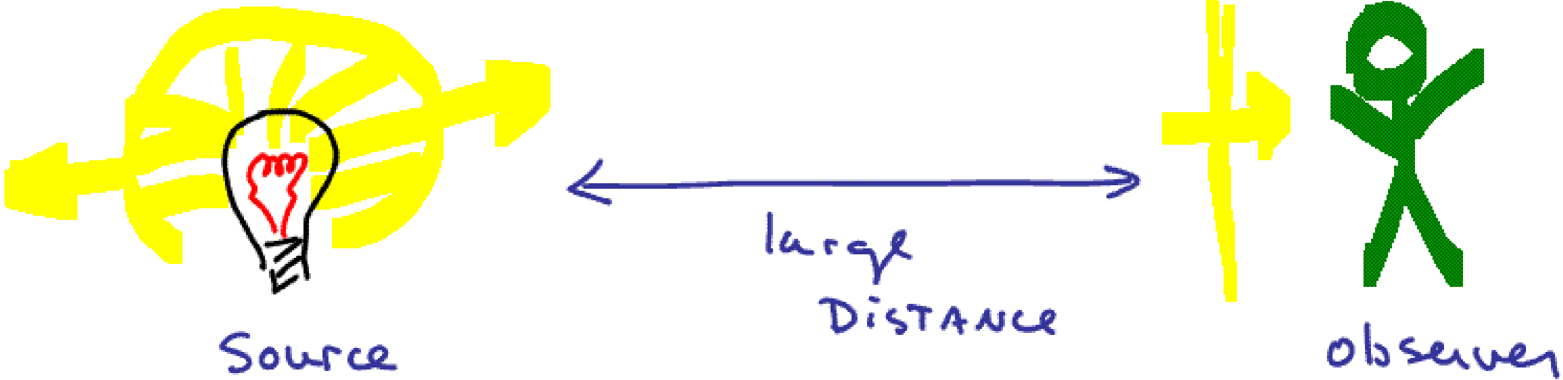
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t}$$ $\hat{k}$ part

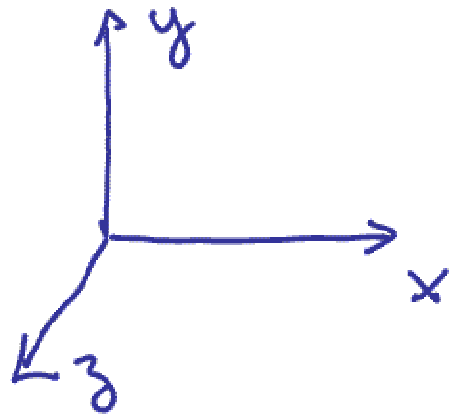
Similarly $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ implies

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

$$-\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right) = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \mu_0 \epsilon_0 \frac{\partial B_z}{\partial t}$$





$$\vec{E} = \vec{E}(x,t)$$

Boundary Conditions

$$\vec{\nabla}\cdot\vec{E} = 0 \qquad \frac{\partial E_x}{\partial x} + \underbrace{\cancel{\frac{\partial E_y}{\partial y}}}_{0} + \underbrace{\cancel{\frac{\partial E_z}{\partial z}}}_{0} = 0$$

E_x const for all x $\rightarrow$ $E_x = 0$

E_y E_z might not be zero

E is transverse to direction of propagation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Impose

$\vec{E}$ is along y direction

$\vec{E} = E(x,t)\,\hat{j}$

"polarization"

$$\cancel{\frac{\partial E_z}{\partial y}}^{0} - \cancel{\frac{\partial E_y}{\partial z}}^{0} = -\frac{\partial B_x}{\partial t} \Rightarrow B_x \text{ const}$$

$$-\left(\cancel{\frac{\partial E_z}{\partial x}}^{0} - \cancel{\frac{\partial E_x}{\partial z}}^{0}\right) = -\frac{\partial B_y}{\partial t} \Rightarrow B_y \text{ const}$$

$$\frac{\partial E_y}{\partial x} - \cancel{\frac{\partial E_x}{\partial y}}^{0} = -\frac{\partial B_z}{\partial t}$$

If E_y Δ'es w/ x
B_z Δ'es

$\Rightarrow$ Time dependent
B field
$\perp$ to E field

$\vec{E}, \vec{B}$ are transverse, mutually $\perp$

$$E_y(x,t) = E_{oy}\cos(\underbrace{k}_{\frac{2\pi}{\lambda}}x - \underbrace{\omega}_{\frac{2\pi}{T}}t + \underbrace{\varphi}_{\text{phase (initial condition)}})$$

$$B_z = -\int \frac{\partial E_y}{\partial x}\,dt$$

Assume for zero now

$\phi = 0$

$$B_z = \int k E_{oy} \sin(kx - \omega t)\,dt$$

$$B_z = \frac{k}{\omega} E_{oy} \cos(kx - \omega t)$$

$$\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{2\pi/T} = \frac{T}{\lambda} = \frac{1}{c}$$

$$\frac{\lambda}{T} = \text{wave speed}$$

$$B_z = \frac{1}{c} E_y$$