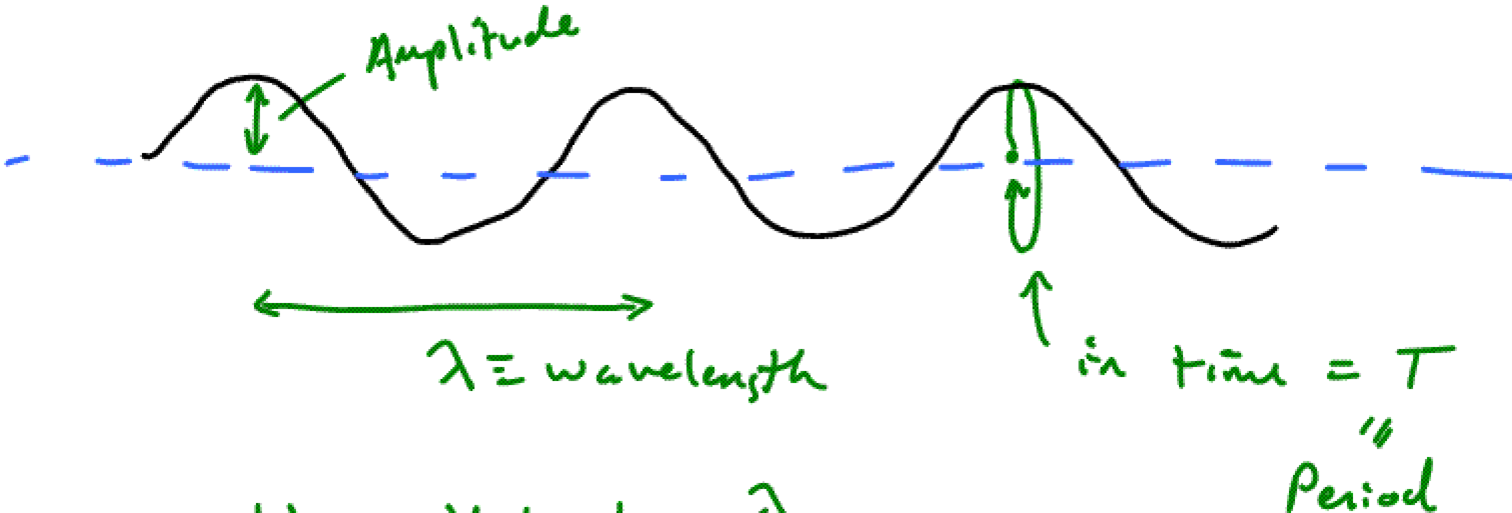
Physics 142 - November 17, 2005

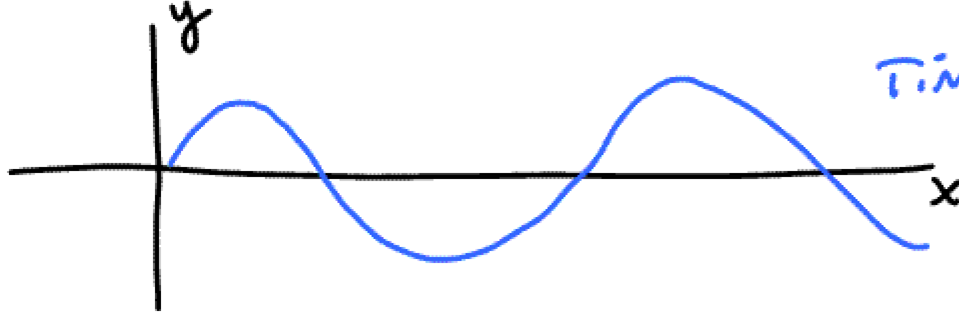
Waves (Blistering Review)



Wave Velocity $= \dfrac{\lambda}{T}$

$$\frac{1}{T} \equiv \underset{f}{\nu} = \text{frequency}$$

$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

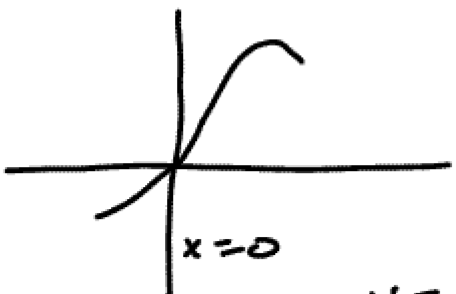


Time frozen

$$y(x) = A \sin kx$$

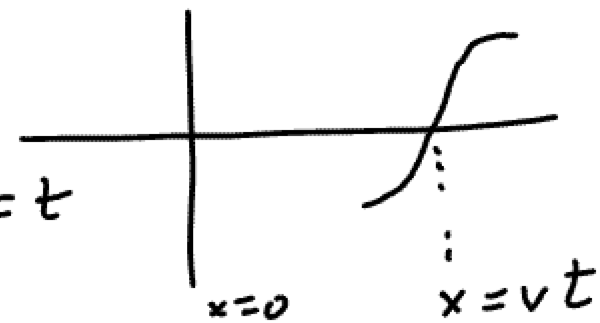
$\frac{2\pi}{\lambda}$ enforces periodicity

wave #



$y = f(x)$

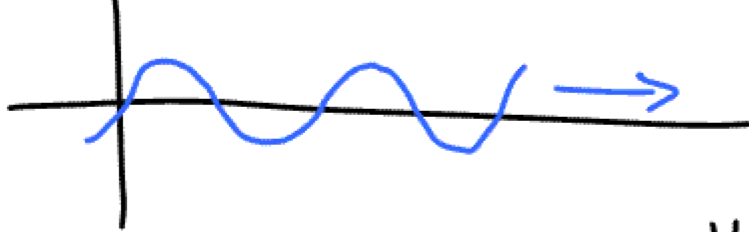
time = t



$$y = f(x - vt)$$

wave traveling to right

$x + vt$ → left



$$y(x,t) = A\sin[k(x - vt)]$$

$$y(x,t) = A\sin(kx - kvt)$$

$$k = \frac{2\pi}{\lambda} \qquad v = \frac{\lambda}{T} \qquad kv = \frac{2\pi}{T} = \omega$$

$$y(x,t) = A\sin(kx - \omega t)$$

starting conditions

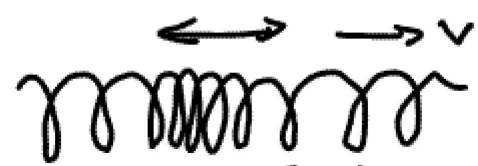
$$y(x,t) = A\sin(kx - \omega t + \varphi)$$

phase Angle

↳ of the wave

TRANSVERSE WAVE

String, plane wave (EM)

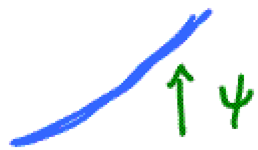


longitudinal wave

slinky, sound

segment of string

- Small displacements (in one plane)

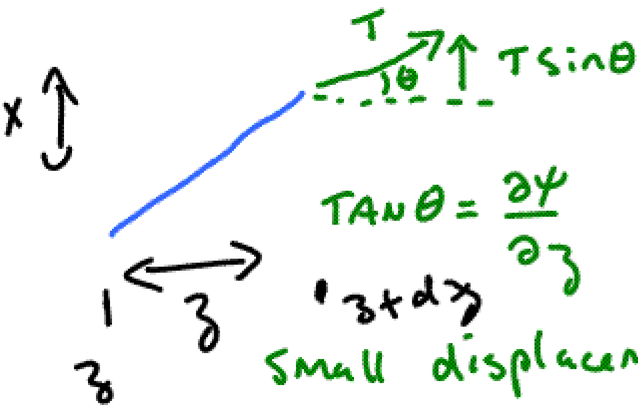


- v string $\frac{\partial \psi}{\partial t}$

T = Tension

- mass segment dm

$dm/dz = \sigma$ — linear mass density



- Slope at given pt $\frac{\partial \psi}{\partial z}$

$\tan\theta = \frac{\partial \psi}{\partial z}$

small displacements $\tan\theta \approx \sin\theta \approx \theta$

$$dF_x = T\left(\frac{\partial \psi}{\partial z}\right)_{z+dz} - T\left(\frac{\partial \psi}{\partial z}\right)_z = \frac{\partial}{\partial z}\left(T\frac{\partial \psi}{\partial z}\right)dz$$

$$dF = d(ma) = \sigma\, dz \frac{\partial^2 \psi}{\partial t^2}$$

$$\sigma\, dz \frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial z}\left(T \frac{\partial \psi}{\partial z}\right) dz$$

ASSUME T CONSTANT

Wave equation

$$\boxed{\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial z^2}}$$

$$c = \sqrt{\frac{T}{\sigma}}$$

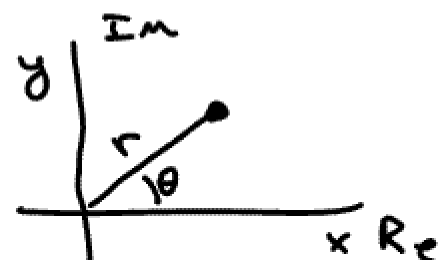
Tension

linear mass density

Complex representation of Waves

complex # $\quad z = x + iy$

x, y both real



$$z = x + iy = r(\cos\theta + i\sin\theta)$$

Euler formula $\quad e^{i\theta} = \cos\theta + i\sin\theta$

$$z = re^{i\theta}$$ θ = phase angle of complex #

Modulus

complex conjugate

$$z^* = re^{-i\theta} = x - iy = r(\cos\theta - i\sin\theta)$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$$

$$|z| = (zz^*)^{1/2} = \sqrt{x^2+y^2}$$

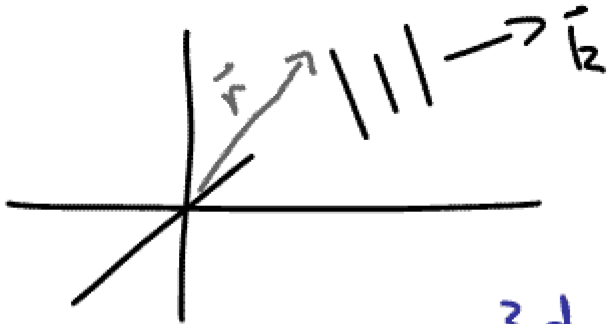
$$z = \mathrm{Re}(z) + i\,\mathrm{Im}(z)$$

$$\mathrm{Re}(z) = r\cos\theta \qquad \mathrm{Im}(z) = r\sin\theta$$

$$\psi(x,t) = A\cos(kx-\omega t) = \mathrm{Re}\left[Ae^{i(kx-\omega t)}\right]$$

$$= Ae^{i(kx-\omega t)}$$

where Re part is "understood"



plane $\vec{k} \cdot \vec{r} = a$

3d plane wave

$$\psi(\vec{r},t) = A e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

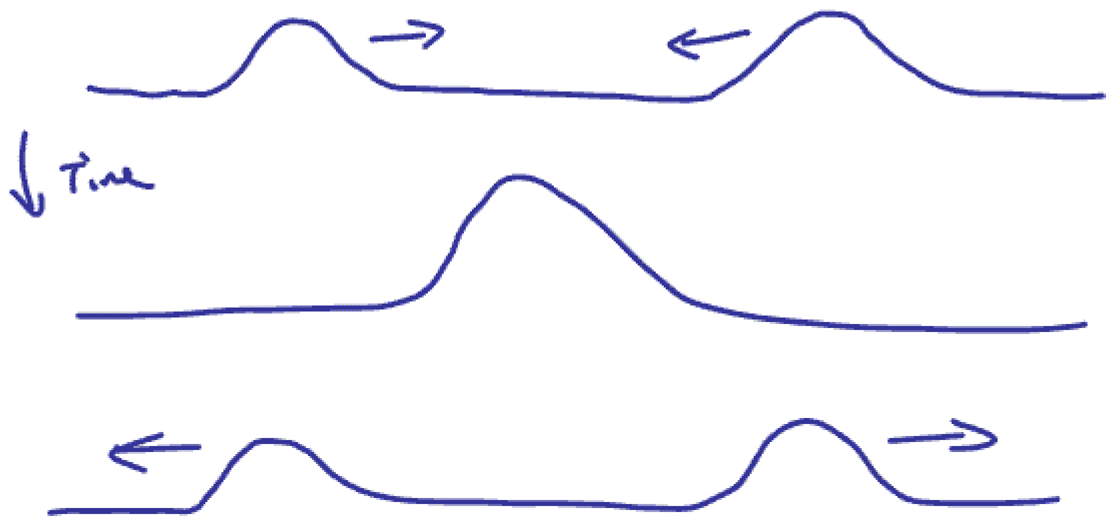
wave eqn becomes

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

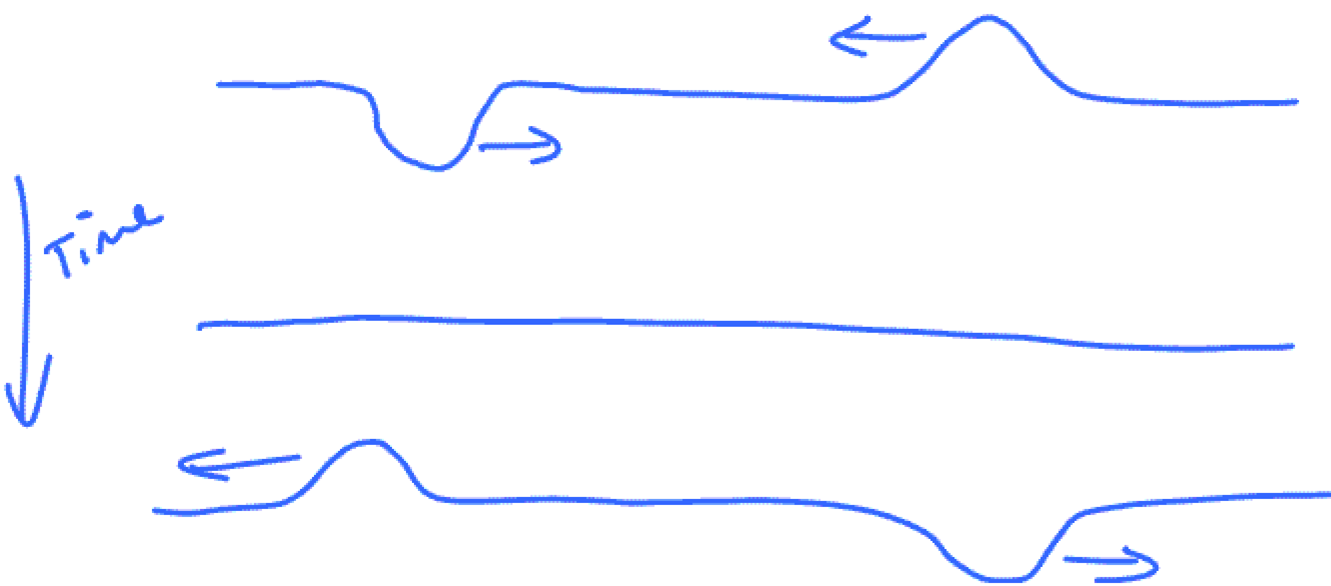
Principle of Superposition

when 2 or more waves pass a given pt simultaneously, the resultant displacement is the sum of the individual displacements

interference



constructive
Interference



destructive
Interference

2 Harmonic waves, same ω

Traveling in opposite directions on string

$E \sim \psi$

Not electric field

$$E_1 = E_0 \sin(kx - \omega t)$$

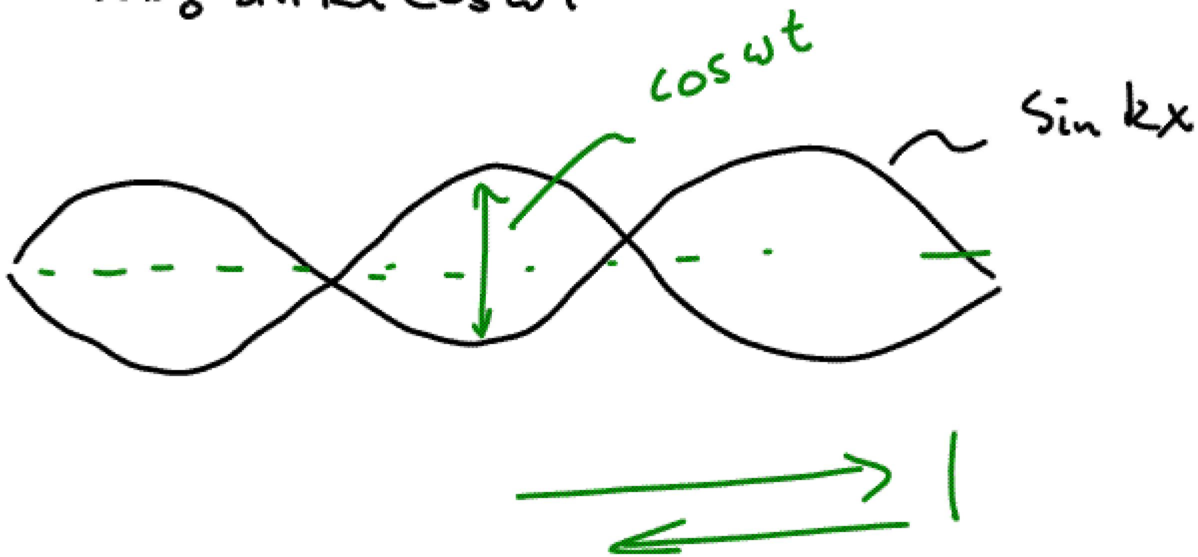
$$E_2 = E_0 \sin(kx + \omega t)$$

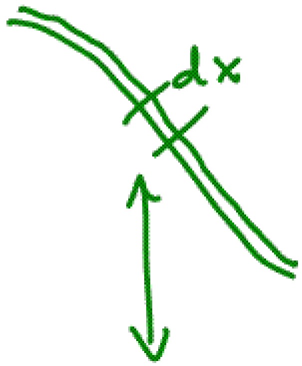
Superposition

$$E = E_1 + E_2 = E_0\left(\sin(kx - \omega t) + \sin(kx + \omega t)\right)$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$E = 2E_0 \sin kx \cos \omega t$$





$\mu =$ mass/length

dm
mass $= \mu \, dx$

$$E = \frac{1}{2} k x^2$$

$$\frac{k}{m} = \omega^2$$

$$dE = \frac{1}{2} \mu \, dx \, \omega^2 A^2$$

Moves w/ $v = \frac{dx}{dt}$

$$dE = \frac{1}{2} \mu \omega^2 A^2 v \, dt$$

$$\frac{dE}{dt} = \frac{1}{2} \mu \omega^2 v A^2$$

Rate of energy transfer thru pt

power = watts

$$\frac{dE}{dt} \propto A^2 v$$

$$\text{Intensity} \equiv \text{Power}/m^2 = \text{watts}/m^2$$

Addition of waves of different frequencies

$$E_1 = E_{01}\cos(k_1x - \omega_1t)$$

$$E_2 = E_{02}\cos(k_2x - \omega_2t)$$

$$E_{01} = E_{02}$$

$$E = E_1 + E_2 = E_{01}\left[\cos(k_1x - \omega_1t) + \cos(k_2x - \omega_2t)\right]$$

use $\cos\alpha + \cos\beta = 2\cos\left[\tfrac{1}{2}(\alpha+\beta)\right]\cos\left[\tfrac{1}{2}(\alpha-\beta)\right]$

$$E = 2E_{01}\cos\left\{\tfrac{1}{2}\left[(k_1+k_2)x - (\omega_1+\omega_2)t\right]\right\}\cos\left\{\tfrac{1}{2}(k_1-k_2)x - (\omega_1-\omega_2)t\right\}$$

$$E = 2E_{01}\cos(\bar{k}x - \bar{\omega}t)\cos(\Delta k x - \Delta\omega t)$$