Physics 142 - November 22, 2005

Electromagnetic waves, Energy flow, Momentum, Polarization

Exam 2 graded — remind me to hand back at end of class $\langle \text{grade} \rangle \sim 73$

Workshops are happening this week

Last Time — we reviewed Mechanical waves
11/17

→ do demos

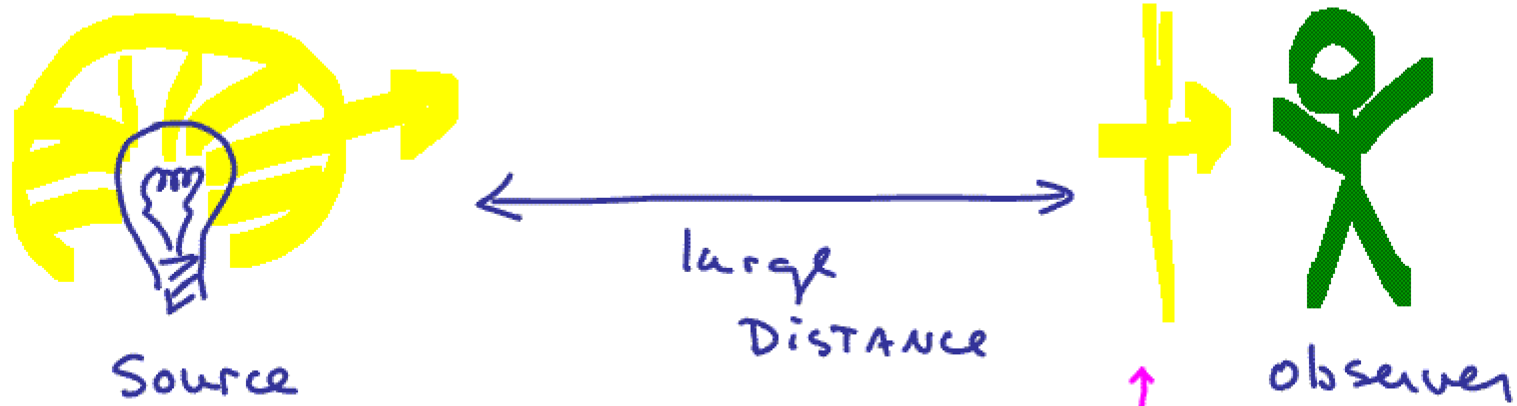
Time Before (11/15) —

We expanded differential form of Maxwell's equations and found

$$\nabla^2 \vec{B} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \qquad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

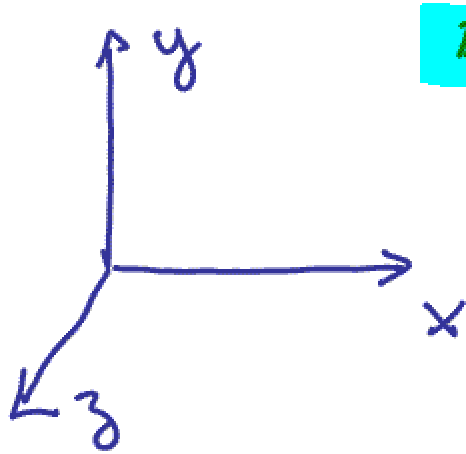
⇒ These are wave equations completely analogous to what we've seen for mechanical waves.

Further expansion of Maxwell's equations:



Boundary Conditions

get plane wave at observer



$$\vec{E} = E_0(x,t)\,\hat{y}$$

in region of no charge or currents

$$\vec{\nabla}\cdot\vec{E} = 0 \implies \frac{\partial E_x}{\partial x} = 0 \qquad E_x \text{ CONSTANT}$$

only $E_x = 0$ Makes sense

(otherwise turn on light and universe instantly filled w/ nonzero E_x ?!)

$$\nabla \times \vec{B} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \left.\begin{array}{l} \frac{\partial B_x}{\partial t} = 0 \\ \frac{\partial B_y}{\partial t} = 0 \end{array}\right]$$

B_x, B_y constant only makes sense if $B_x = B_y = 0$

+ similar eqns from $\nabla \times \vec{E}$ eqn

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$\Downarrow$

E, B coupled and time dependent

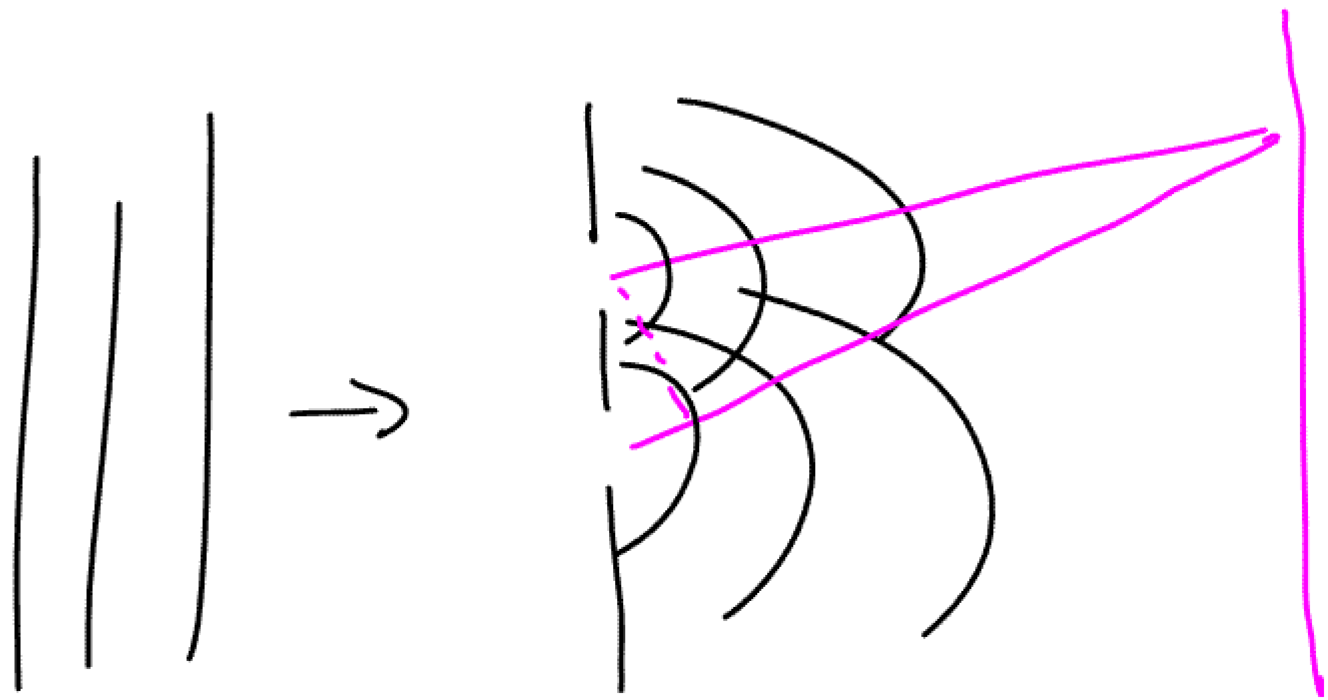
E, B $\perp$ to each other

$\vec{E}, \vec{B}$ in phase

$$E_y(x,t) = E_{oy} \cos(kx - \omega t + \varphi)$$

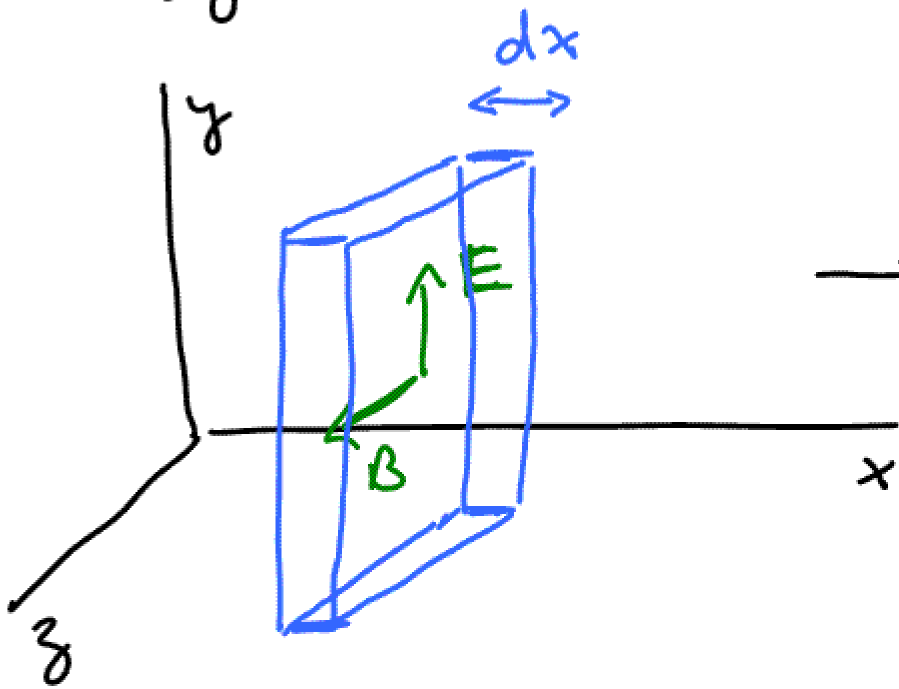
$$B_z(x,t) = \frac{k}{\omega} E_{oy} \cos(kx - \omega t + \varphi) = \frac{1}{c} E_y$$

$$|\vec{E}| = c|\vec{B}|$$



Energy Flow in EM Waves

Ohanian



$\longrightarrow$ plane wave

$$u_E = \frac{\epsilon_0}{2} E^2 \qquad u_B = \frac{1}{2\mu_0} B^2$$

$$dU = \text{Energy in Volume} = (u_E + u_B)\ \text{volume}_{\text{Box}}$$

$$dU = \left(\frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right)(\text{Area})\,dx$$

$$E = cB \qquad \epsilon_0 = \frac{1}{\mu_0 c^2} \qquad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$du = \left[\frac{1}{2\mu_0 c^2} EcB + \frac{1}{2\mu_0} B\frac{E}{c}\right](\text{Area})\,dx$$

dU = differential Energy

moves thru box in time $dt = \frac{dx}{c}$

$$du = \left(\frac{1}{\mu_0 c} EB\right) \text{Area}\,dx$$

$$\frac{dU}{dt} = \frac{EB}{\mu_0} \text{ (Area)}$$

$$\frac{dU}{dt}\frac{1}{\text{Area}} = \frac{EB}{\mu_0} = \frac{\text{Watts}}{m^2} \equiv \text{Intensity}$$

Energy flux

Add direction

$\vec{S} \equiv$ poynting vector $\equiv$ Energy flow

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

$\vec{E}, \vec{B}, \vec{S}$ all fluctuate w/ time

Time Average of $S \equiv \bar{S} = \langle S \rangle$

$\langle |\vec{S}| \rangle$

$$E = E_0 \sin \omega t$$

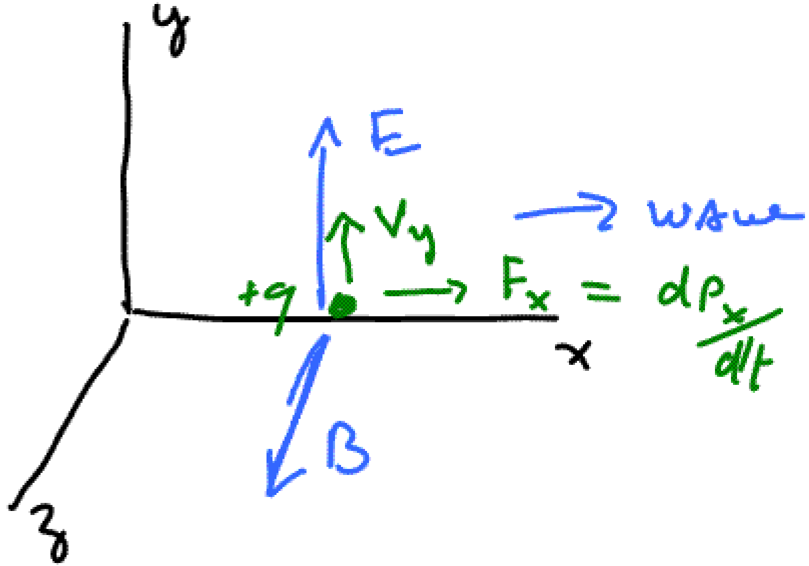
$$B = \frac{E_0}{c} \sin \omega t$$

$$S = \frac{1}{\mu_0 c} E_0^2 \sin^2 \omega t$$

$$\bar{S} = \langle S \rangle = \frac{E_0^2}{2\mu_0 c}$$

CAN write in terms of

$$= \frac{c}{2\mu_0} B^2$$



$$\frac{dp_x}{dt} = F_x = q(\vec{v} \times \vec{B})_x = q(v_y B_z - \cancel{v_z}^{\,0} B_y)$$

$$\frac{dp_x}{dt} = q\, v_y B_z \qquad B_z = \frac{E_y}{c}$$

$$\frac{dp_x}{dt} = \frac{q}{c} v_y E_y$$

$$\frac{d\,w(\text{ork})}{dt} = q\,\vec{E}\cdot\vec{v} = q\, v_y E_y$$

$$\frac{dP_x}{dt} = \frac{1}{c}\frac{dw}{dt}$$

$$dP_x = \frac{1}{c}\,dw$$

$$\boxed{P = \frac{U}{C}}$$

Momentum in EM wave

$=$

$\frac{1}{c}$(Energy in EM wave)

EM wave Absorbed

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$F = \frac{dP}{dt} = \frac{dU}{dt}\frac{1}{c} = \frac{1}{c}\frac{\frac{E}{Area}}{dt}$$

Area

$$F = \frac{1}{c} S (\text{Area})$$

$$\frac{F}{\text{Area}} = \text{pressure} = \frac{S}{c}$$

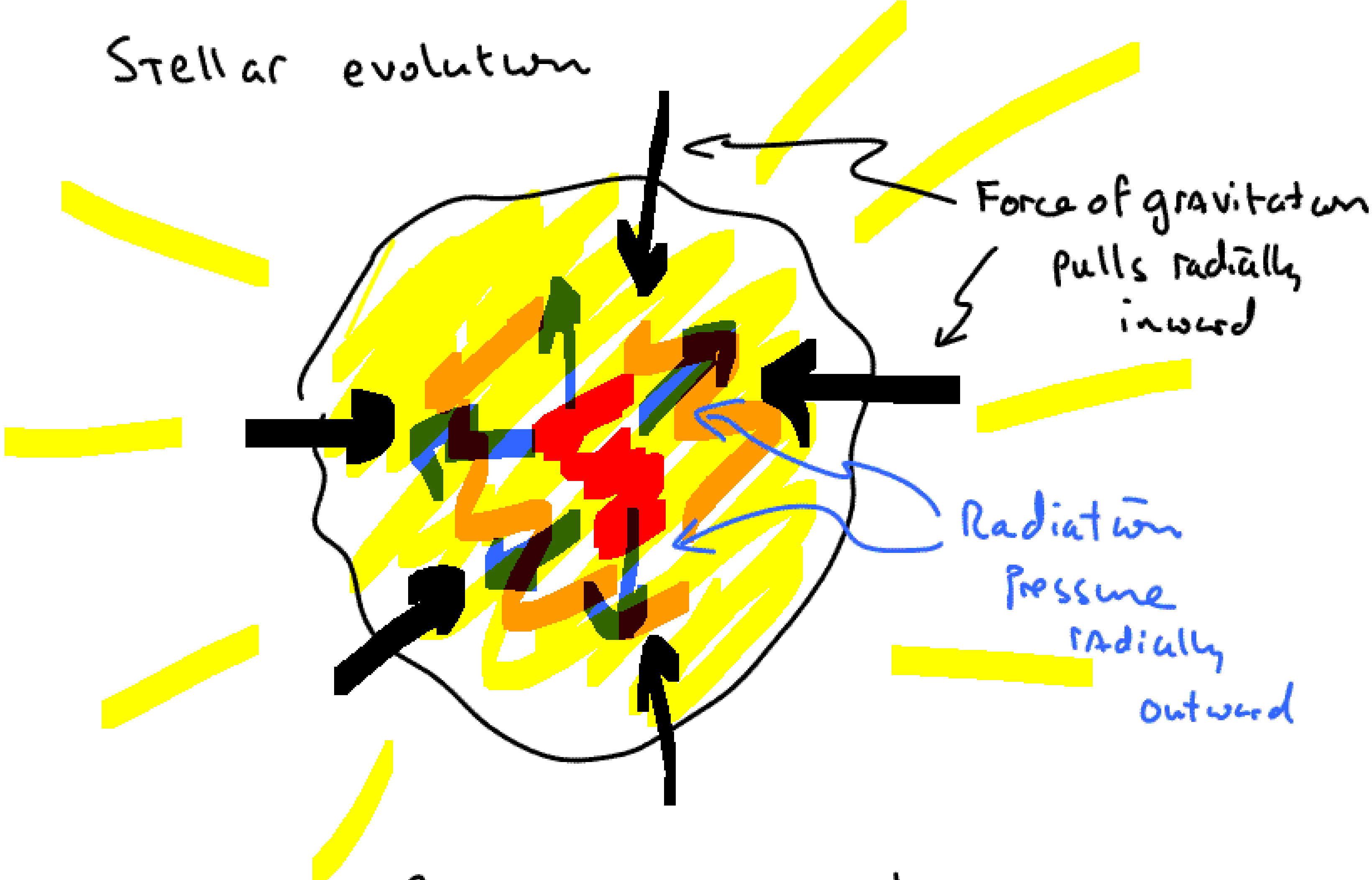
$$\boxed{\text{Radiation Pressure} = \frac{S}{c}}$$

TOTAL Absorption

x2 if reflection

$$\langle P \rangle = \frac{\langle S \rangle}{c}$$

Stellar evolution



Stars are systems where the radiation pressure (from thermonuclear reactions in the core) is in equilibrium with the gravitational attraction of the matter toward the center of the star.