Physics 142 - September 13, 2005

Gauss' Law

- Workshops start this week (yesterday)
- PDF version of P.S. 1 solns okay?
- Is soln set in sufficient detail?
- How goes P.S. 2 ?

Last Time

$$\vec{E}_p = \int_{\text{cHg dist}} \frac{k\,dQ}{r'^2}\,\hat{r}'$$

Calculate electric field (thus electrostatic force) for arbitrary charge distribution

This can be difficult to impossible.

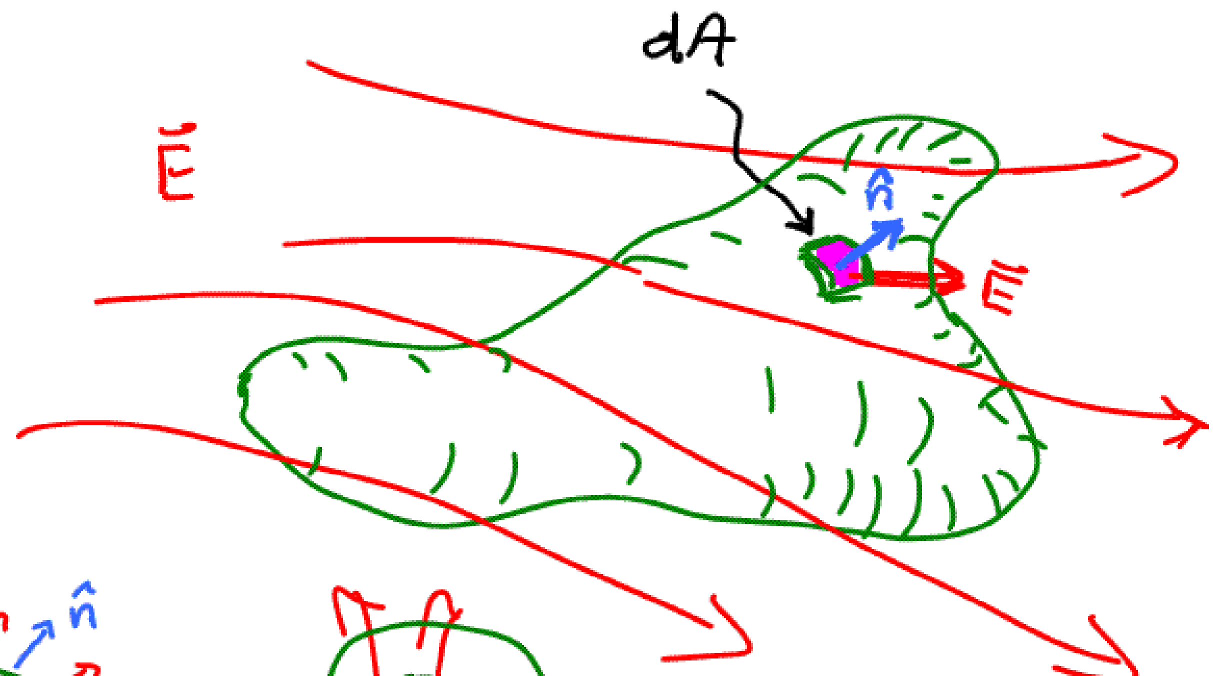
Electric Flux

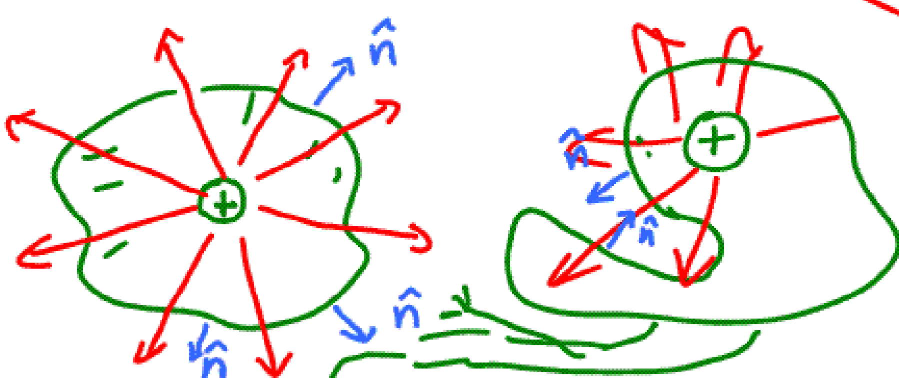
$$\phi \equiv \oint \vec{E} \cdot \hat{n}\, dA$$

sum $\vec{E}$ // to $\hat{n}$ over surface

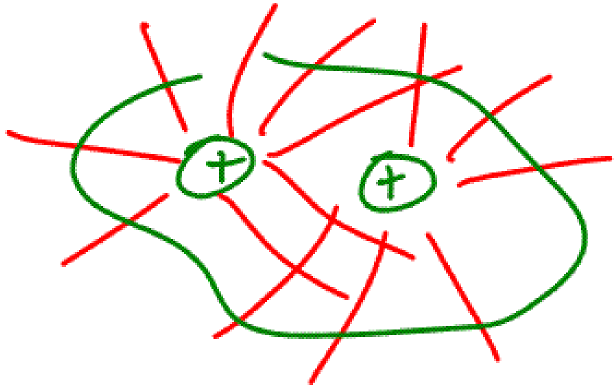
$\oplus$ if along $\hat{n}$

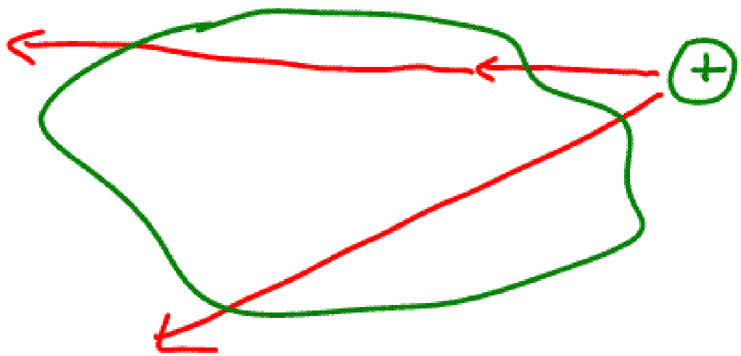
$\ominus$ if against

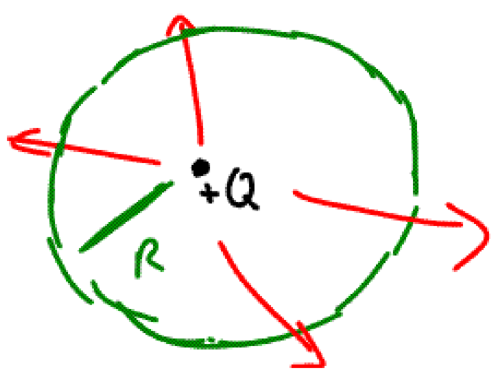




Net + flux







pt chg Q at origin
of spherical surface
(Radius R)

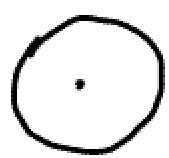
$$\phi_{\text{surface}} = \int_S \vec{E}\cdot d\vec{A} = |\vec{E}| \int_S dA = |\vec{E}|\, 4\pi R^2$$

SAme as $\vec{E}\cdot\hat{n}\,dA$

$$\phi = \frac{kQ}{\cancel{R^2}}\, 4\pi \cancel{R^2} = kQ4\pi \cancel{R^2} = \frac{1}{4\pi\epsilon_0}\, Q4\pi \cancel{R^2}$$

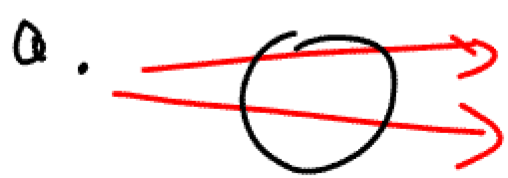
$|\vec{E}|$ at spherical surf

$$= Q/\epsilon_0$$



This case

$$\oint \vec{E}\cdot d\vec{A} = Q/\epsilon_0$$



Gauss' Law

$$\boxed{\oint \vec{E}\cdot\hat{n}\,dA = \frac{Q_{enclosed}}{\epsilon_0}}$$

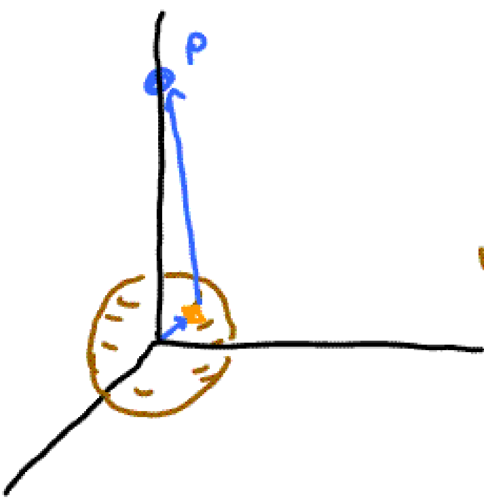
$$\oint \vec{E}\cdot d\vec{A}$$

General
All
closed
surfaces

Important

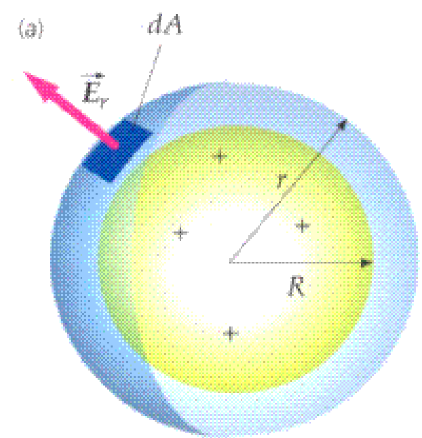
why do I care?

What is $\vec{E}$ at pt p.

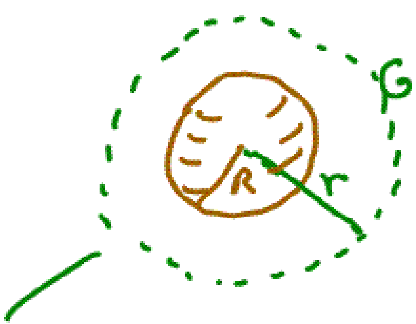


uniform sphere of charge
Total chg Q

Find $\vec{E}$ in all space



Spherical
Gaussian Surface



$r > R$ outside sphere

Apply Gauss' law on Gaussian surface.

$$E \cdot \hat{n} \;?= |\vec{E}|$$

radially outward

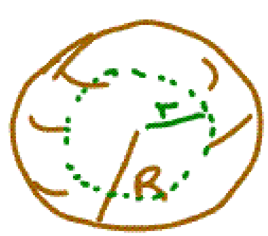
Chg enclosed inside Gaussian surface

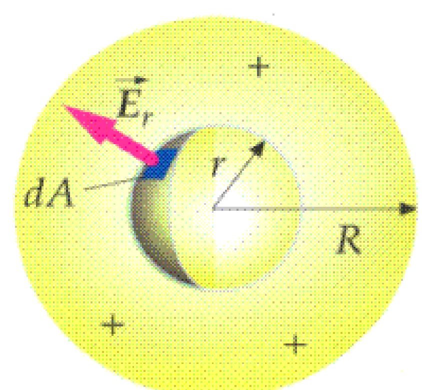
$$\oint \vec{E} \cdot dA = |\vec{E}| \underbrace{\oint dA}_{4\pi r^2} = \frac{\boxed{Q_{encl}}}{\epsilon_0} = Q/\epsilon_0$$

— Area Gaussian surface

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2} \quad \text{at } r \text{ for } r > R$$

$r < R$





$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

What is Q_{encl}?

$$|\vec{E}|\, 4\pi r^2 = \frac{Q_{encl}}{\epsilon_0}$$

ρ vol. chg. density is CONSTANT

$$Q_{encl} = \rho \text{ vol of Gauss surface}$$

$$Q_{encl} = \frac{Q}{\frac{4}{3}\pi R^3} \frac{4}{3}\pi r^3$$

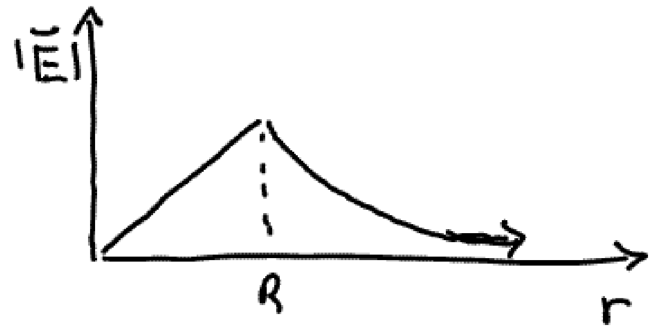
$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

vol of Big Sphere

$$|\vec{E}|\, 4\pi r^2 = \frac{Q\, r^3}{\epsilon_0 R^3}$$

$$|\vec{E}| = \frac{Q}{4\pi\epsilon_0} \frac{r}{R^3} \quad \text{for all } r < R$$

direction radially out



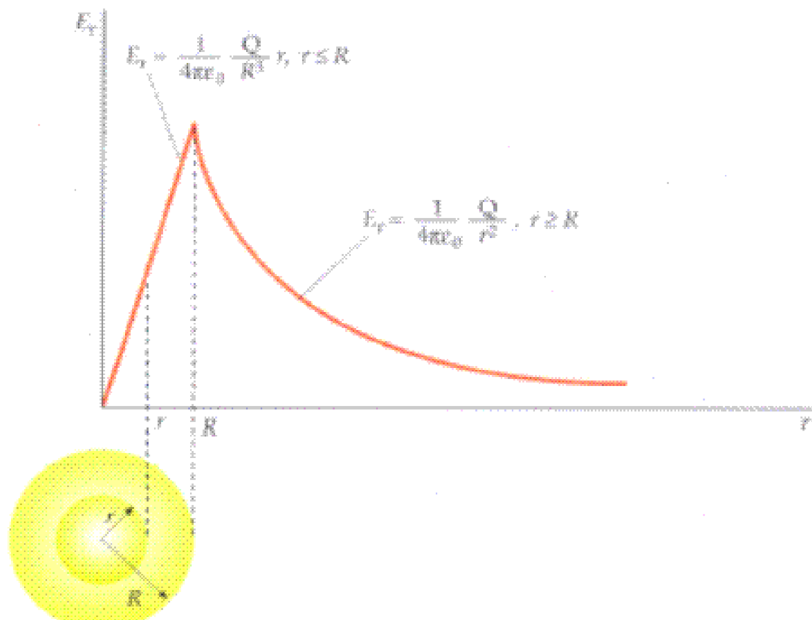


Figure from Tipler's text

Will Come Back to Gauss' Law problems shortly

Let's look at an application of Gauss' Law –

conductor – Charge move freely

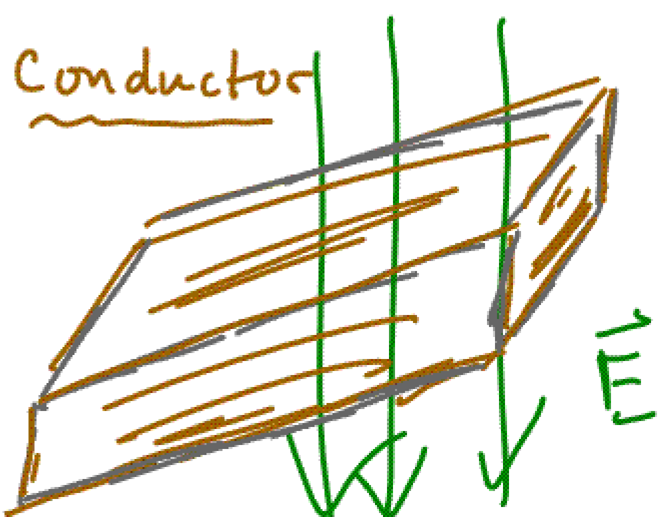
metals typically

insulator – chg cannot move freely

Rubber, glass

semiconductor – "conductivity" depends on environment

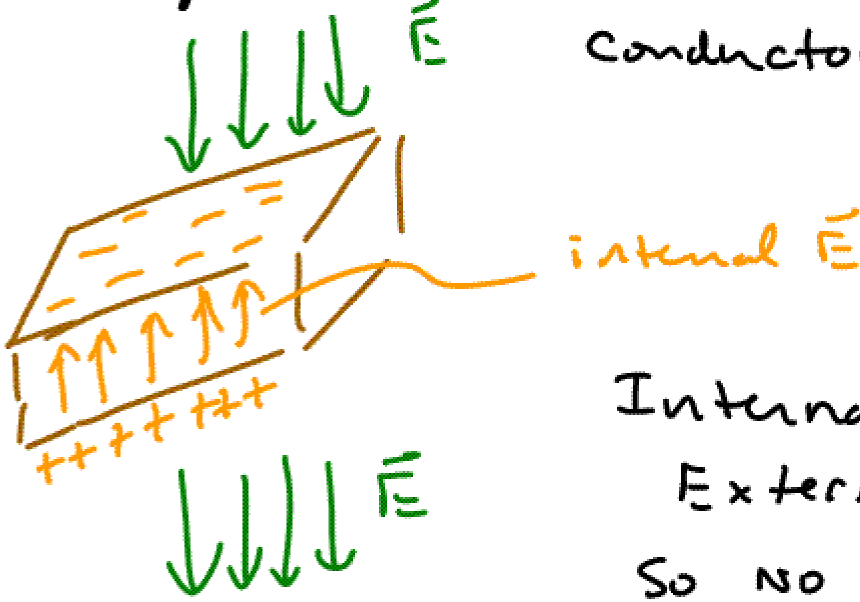
Silicon



Conductor in $\vec{E}$

charges move in response to $\vec{E}$ because they can

Charges move until there is no $\vec{E}$ inside conductor



Internal $\vec{E}$ cancels out External $\vec{E}$

So no $\vec{E}$ inside conductor

If not true → charges move until it is true.

Basis for Electrostatic Shielding

$\boxed{\vec{E} = 0 \text{ inside conductor}}$

Important