Physics 142 - September 15, 2005

Curvilinear Coordinates, Gauss' Law, Electric Potential

PS 2 solns on web later Today

PS2 hand in today

PS1 ready to hand Back (Box)

Last Time —

Gauss' Law $\oint \vec{E}\cdot\hat{n}\,dA = \int \vec{E}\cdot d\vec{A} = \frac{Q_{enclose}}{\epsilon_0}$

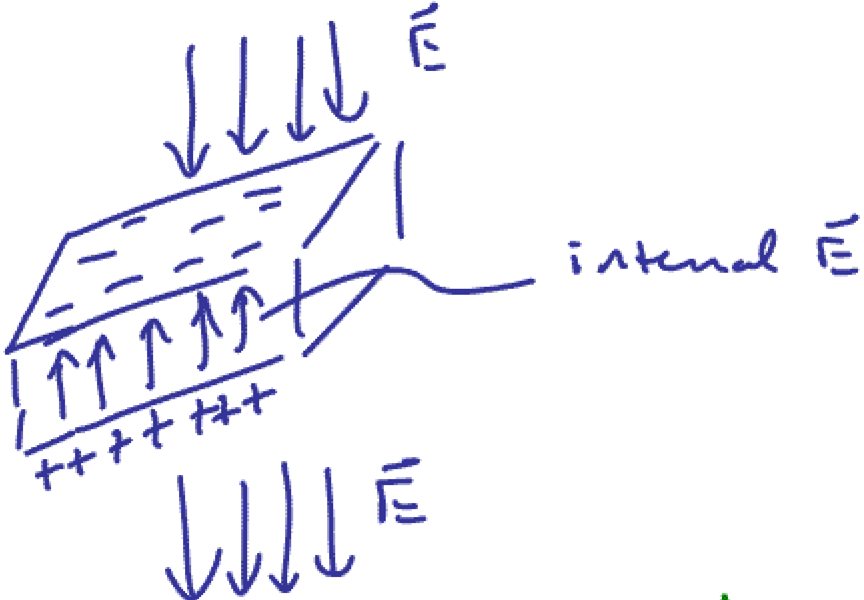
Important

True in general

most useful under selected conditions of Symmetry

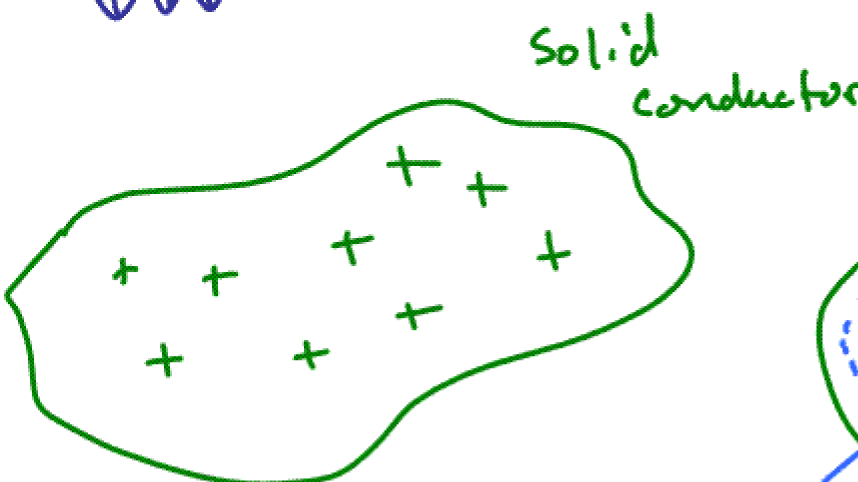
Conductor - charges move freely

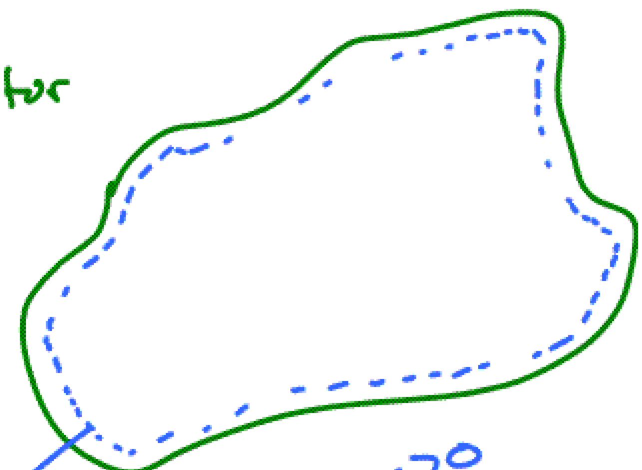
If external $\vec{E}$ is present, charges move until $\vec{E}=0$ inside conductor



$\vec{E} = 0$ inside a conductor

Important





Gaussian surface

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl.}}{\epsilon_0}$$

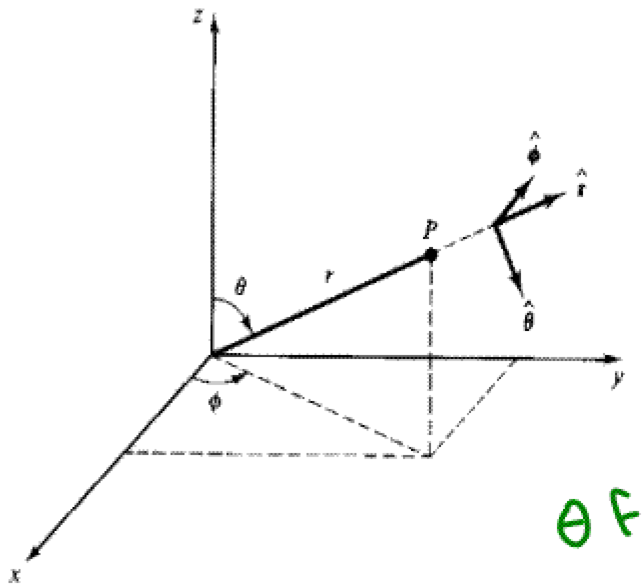
$\rightarrow 0$

$E = 0 \Rightarrow Q_{encl} = 0$

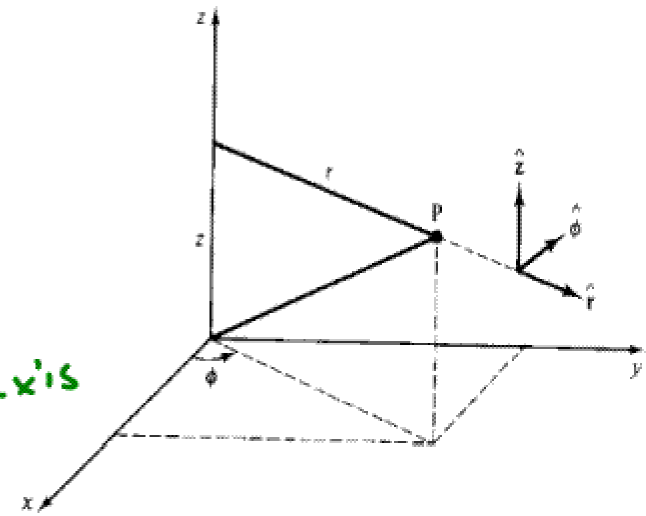
Charge resides on outside surface of conductor

Curvilinear Coordinates

Spherical polar



Cylindrical

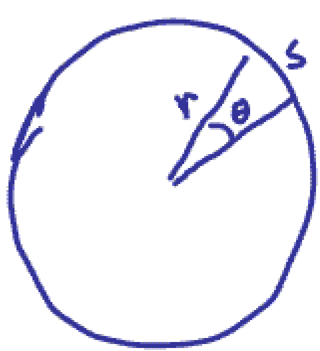


+z axis

θ from 0 to π

-z axis

ϕ from $0 \to 2\pi$

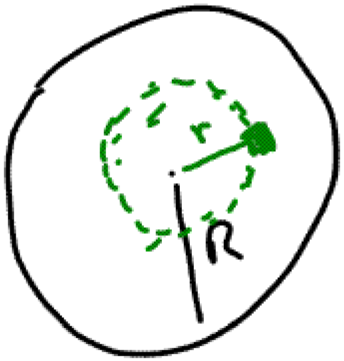


$s = r\theta$

$ds = r\,d\theta$

$da \sim dr\,ds$

$da = r\,d\theta\,dr$

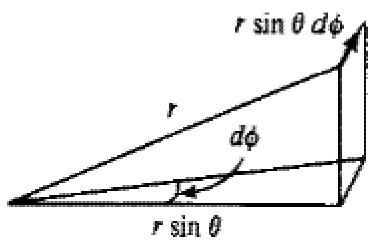


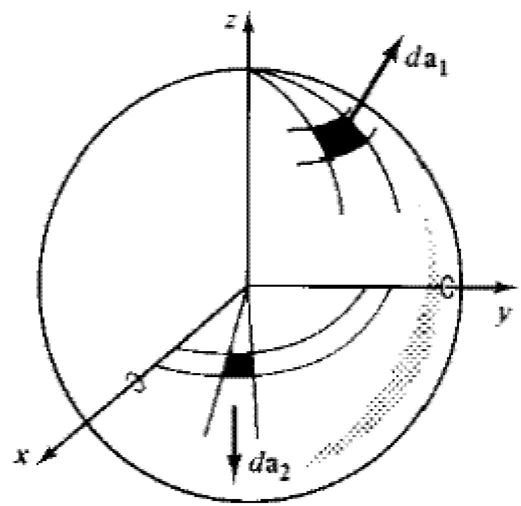
Area of disk

$$\int_0^R \int_0^{2\pi} dA = \int_0^R \int_0^{2\pi} r\, d\theta\, dr$$

$$= 2\pi \int_0^R r\, dr = \pi R^2$$

$dl_\phi = r \sin\theta \, d\phi.$





$da_1 = r^2 \sin\theta \, d\theta \, d\varphi$ $\qquad$ $dv \sim r^2 \sin d\theta \, d\varphi \, dr$

$$\text{Sphere Area} = \int_0^{\pi}\int_0^{2\pi} r^2 \sin\theta \, d\theta \, d\varphi = 2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta$$

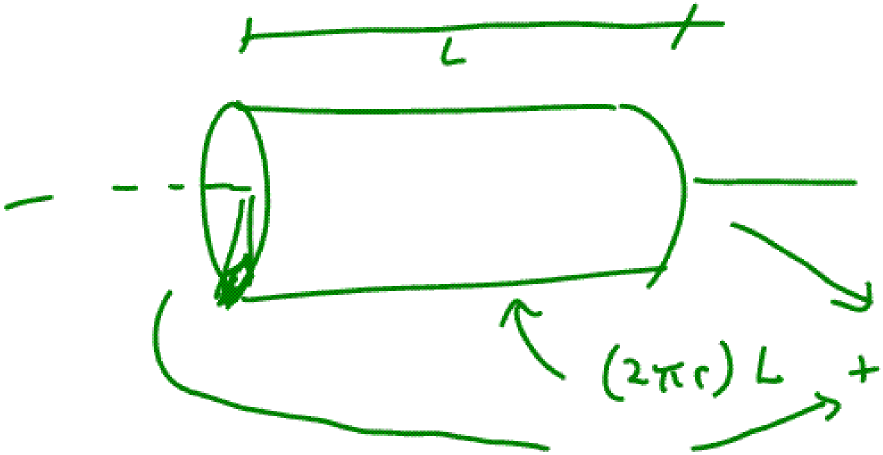
$$= 4\pi r^2$$

$$\text{Vol. Sphere} = \int_{0}^{\pi}\int_{0}^{2\pi}\int_{0}^{R} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$\theta \quad \varphi \quad r$$

$$= 4\pi \int_0^R r^2 dr = \frac{4\pi}{3} R^3$$

Griffiths - Intro to Electrodynamics

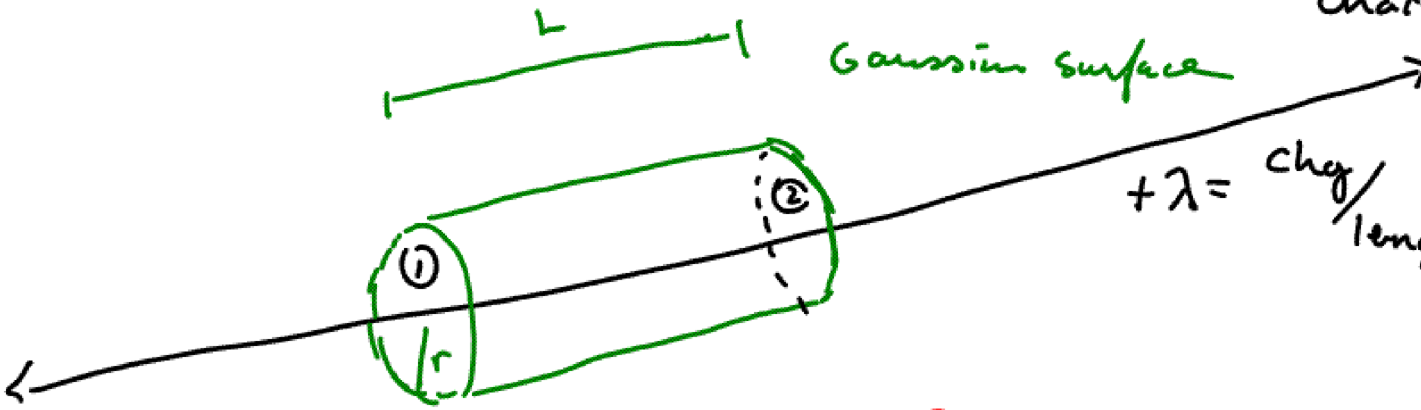


$$Vol_{cyl} = \pi R^2 L$$

$$(2\pi r)L + (2)\pi r^2 = \text{Surf Area}$$

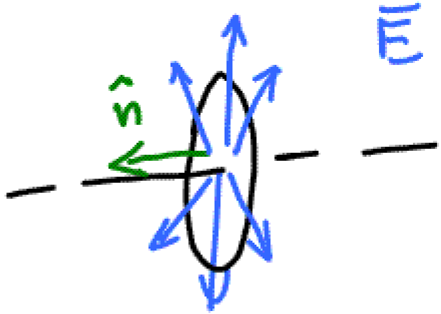
Return to Gauss' Law

∞ line charge



$+\lambda = \text{chg}/\text{length}$

$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{Q_{enc}}{\epsilon_0} = \underbrace{\int \cancelto{0}{\vec{E}\cdot\hat{n}\,dA}}_{\text{endcap 1}} + \underbrace{\int \cancelto{0}{\vec{E}\cdot\hat{n}\,dA}}_{\text{endcap 2}} + \underbrace{\int \vec{E}\cdot\hat{n}\,dA}_{\text{cylinder}}$$



$\vec{E}$ radially outward
By symmetry

$\vec{E} \cdot \hat{n} = 0$ on endcaps

$\vec{E} \cdot \hat{n} = |\vec{E}|$ on cylinder surface

$$\int_{cyl} \vec{E} \cdot \vec{n}\, dA = |\vec{E}|\, 2\pi R L = \frac{Q_{encl}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$|\vec{E}| = \frac{\lambda L}{\epsilon_0} \frac{1}{2\pi R L} = \frac{\lambda}{\epsilon_0 2\pi R}$$

Saw earlier when we did $L \rightarrow \infty$ limit on line segment $\vec{E}$ calc.

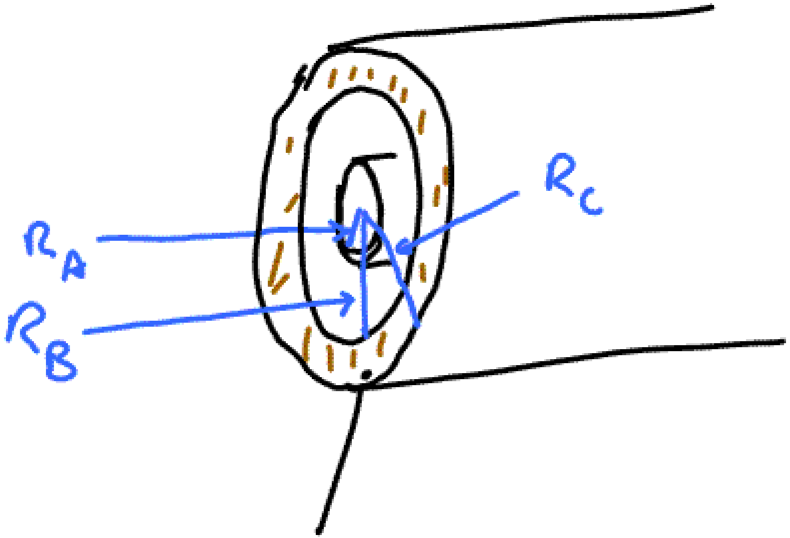
noncondcuting core

radius R_A

$+\lambda$

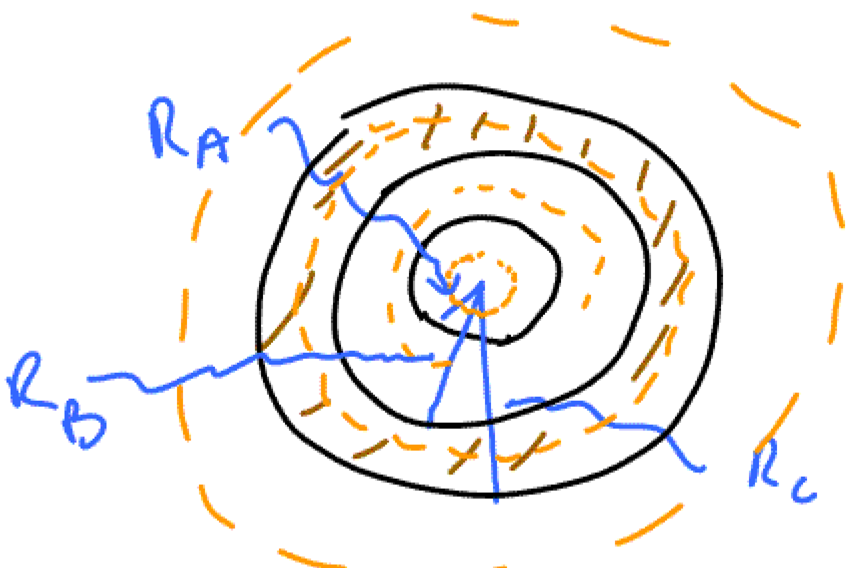
chg distributed

$\rho(r) = ar \quad r < R_A$

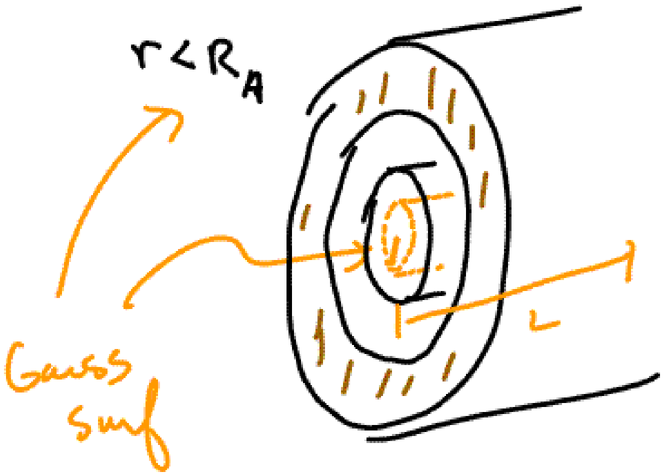


conducting sheath, uncharged

Find $\vec{E}$ in all space



Gaussian surfaces to Examine



$\vec{E}$ radially outward by symmetry

$$\oint \vec{E}\cdot d\vec{A} = |\vec{E}|\,2\pi r L$$

$$\frac{Q_{encl}}{\epsilon_0} = \int_{\substack{\text{vol}\\ \text{Gauss}\\ \text{surf}}} \rho\, dv$$

$dv = 2\pi r' dr\, L$

$$Q_{encl} = \int \rho\, dv$$

$$= \int_0^r a r' \, 2\pi r' L\, dr$$

$$= a 2\pi L \frac{r^3}{3}$$

Gauss

$$|\vec{E}|\, 2\pi r L = \frac{a\, 2\pi L}{\epsilon_0} \frac{r^3}{3}$$

$$\boxed{|\vec{E}| = \frac{a r^2}{3\epsilon_0} \qquad r < R_A}$$

Can use this

Gaussian surface

For $R_A < r < R_B$ region

$$\oint \vec{E} \cdot \hat{n}\, dA = \frac{Q_{encl}}{\epsilon_0}$$

$$|\vec{E}|\, 2\pi r L = \frac{2}{3} \frac{\pi a L R_A^3}{\epsilon_0}$$

R_A Because chg only extends to R_A

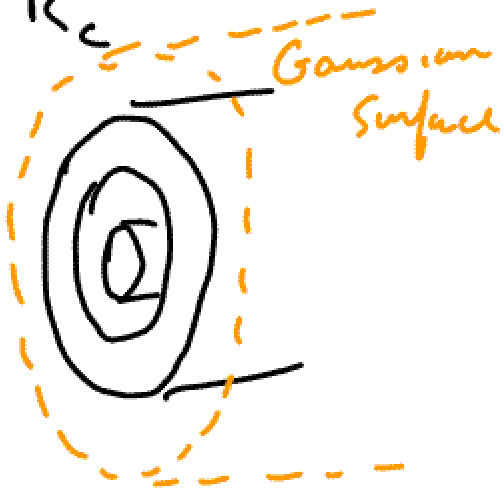
$$|\vec{E}| = \frac{a R_A^3}{3 \epsilon r}$$

radially out (if chg is +)

For $R_A < r < R_B$

$R_B < r < R_C$ $\quad |\vec{E}| = 0$ inside conductor

$r > R_C$



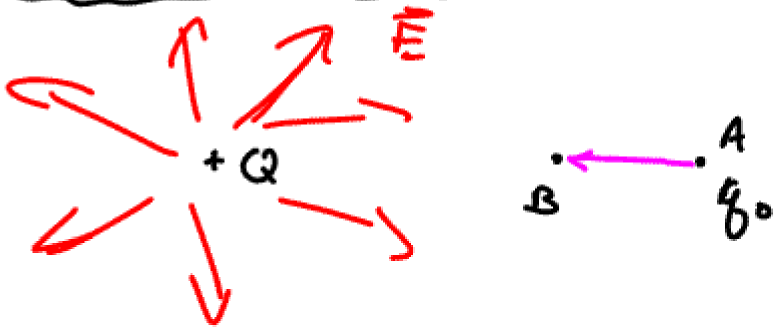
soln is the same as for $R_A < r < R_B$ region

Why?

↳ Q_{encl} is the same

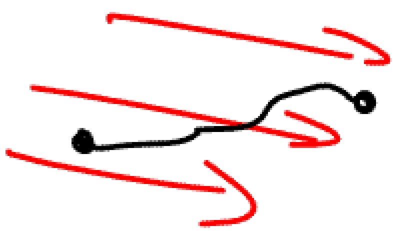
$\oint \vec{E} \cdot \hat{n}\, dA$ is the same

Electric Potential



$$W = \int_A^B \vec{F} \cdot d\vec{s} = -\int q_0 \vec{E} \cdot d\vec{s} = -\int_A^B q_0 E \, dr$$

$$W = -q_0 Q k \int_A^B \frac{1}{r^2} dr = k q_0 Q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$



$$\boxed{\frac{\text{Work}}{q_0} \equiv \text{Potential difference}}$$

$$\Delta V = V_B - V_A = V_{AB}$$