Physics 142 - September 8, 2005

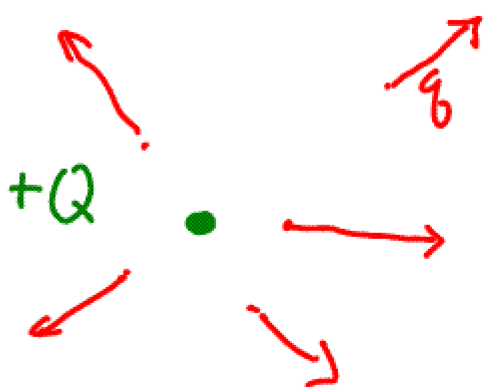
Electric Fields, Flux, Gauss' Law

- P.S. 1 due at end of class
- P.S. 2 plus solns to P.S. 1 will be posted on class web site Tomorrow sometime.

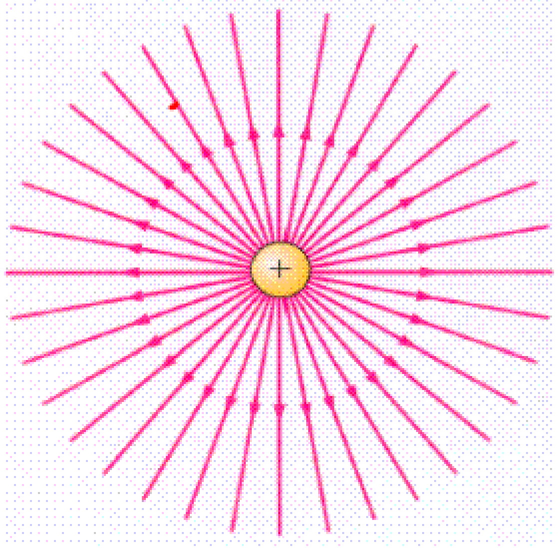
Last time:

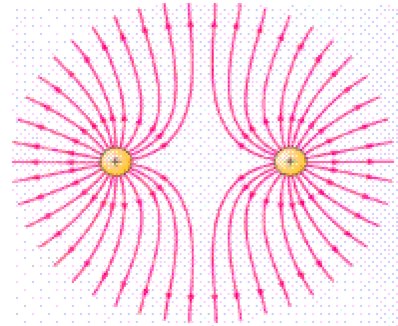
Coulomb's Law $\longrightarrow E_{ionization}$ for H

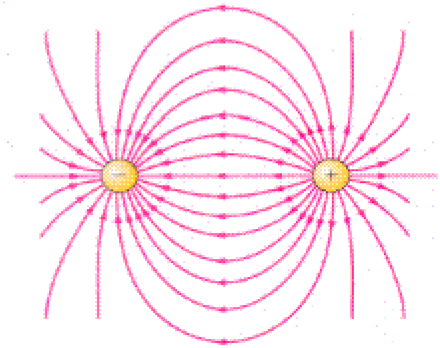
Electric Field



$$\vec{E}_p \equiv \frac{\vec{F}_{q \text{ at } p}}{q}$$







Lines of force:

Never cross

go from $\oplus \longrightarrow \ominus$ or ∞

$\vec{E}$ for pt. charge

q_0 P

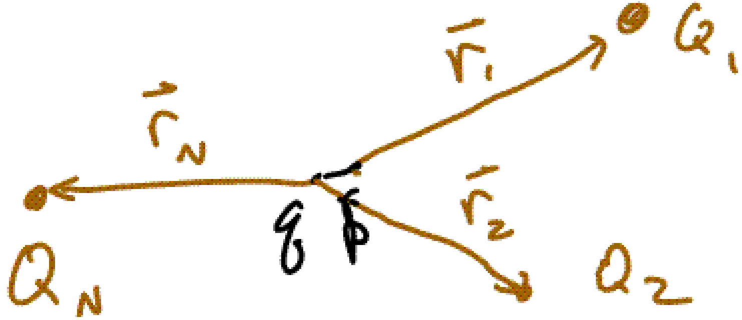
$\vec{r}$

$\oplus$

Q

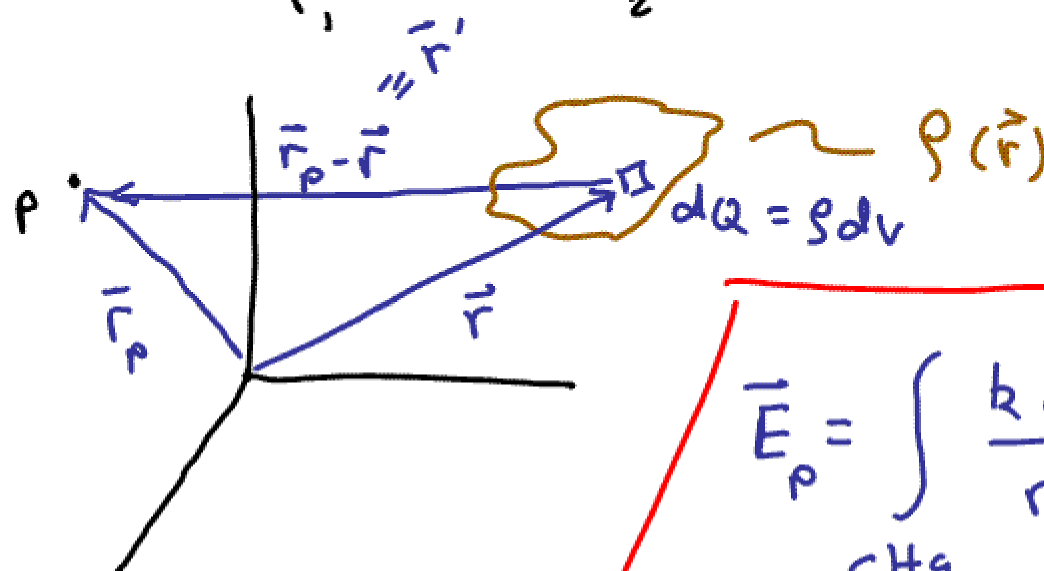
$$\vec{F}_{q,P} = \frac{kQq}{r^2}\hat{r}$$

$$\vec{E}_P = \frac{kQ}{r^2}\hat{r}$$



$$\frac{\vec{F}}{q} = \frac{kQ_1 q}{r_1^2}\hat{r}_1 + \frac{kQ_2 q}{r_2^2}\hat{r}_2 + \cdots + \frac{kQ_N q}{r_N^2}\hat{r}_N$$

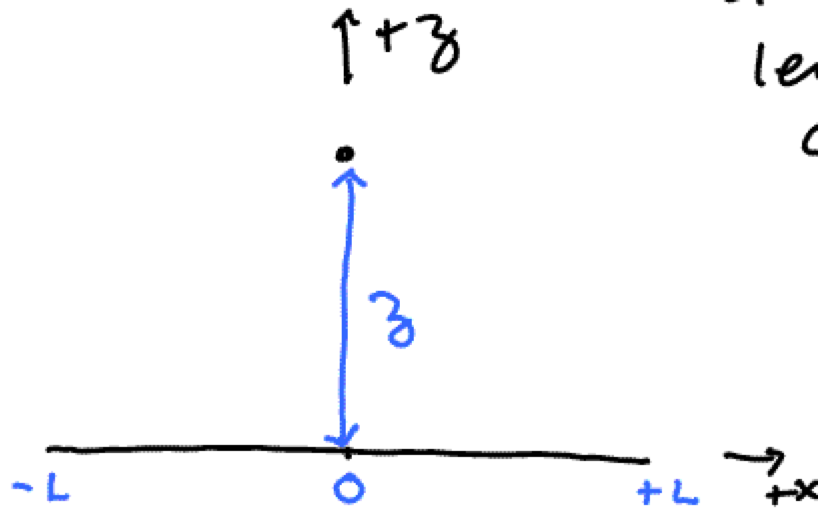
$$\vec{E}_p = \frac{kQ_1}{r_1^2}\hat{r}_1 + \frac{kQ_2}{r_2^2}\hat{r}_2 + \cdots + \frac{kQ_N}{r_N^2}\hat{r}_N$$

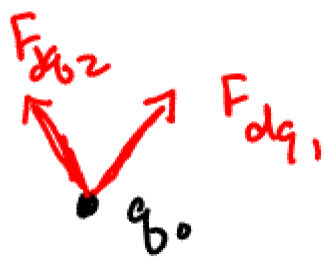


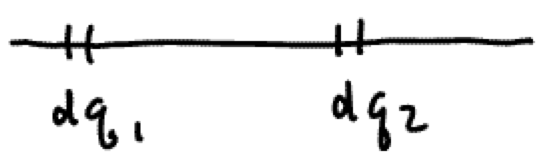
$$\boxed{\vec{E}_p = \int_{\text{cHg dist}} \frac{k\,dQ}{r'^2}\hat{r}'}$$

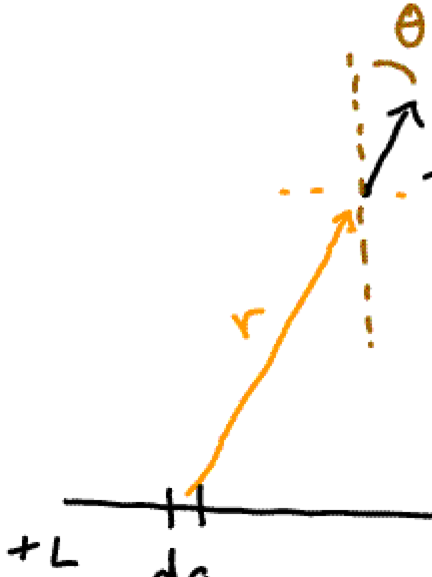
Example

Find $\vec{E}$ at a distance z above the midpoint of a line segment of length $2L$ which carries a uniform line charge of $+\lambda$.









$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

$$\vec{E} = \int_{+L}^{-L} \frac{k\,dq}{r^2}\hat{r}$$

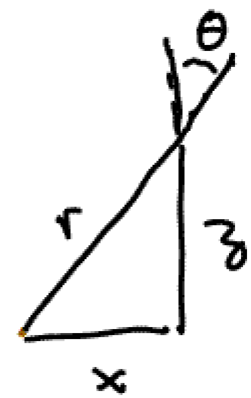
$$dq = \lambda\,dx$$

→ vector superposition

→ symmetry

$$d\vec{E}_{\bullet} = d\vec{E}_z = dE\cos\theta\,\hat{z}$$

$$\vec{E} = 2\int_0^L \frac{k\,dQ\cos\theta}{r^2}\,\hat{z}$$



$$r^2 = x^2 + z^2$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2+z^2}}$$

$$\vec{E} = 2\int_0^L \frac{k\,dQ \qquad z}{(x^2+z^2)(x^2+z^2)^{1/2}}$$

$$\vec{E} = \lambda 2 k \int_0^L \frac{z\,dx}{(x^2+z^2)^{3/2}}$$

$z \equiv$ const in This problem

$$\vec{E} = 2kz\lambda \left[\frac{x}{z^2(x^2+z^2)^{1/2}}\right]_0^L \hat{z}$$

$$\boxed{\vec{E} = \frac{2k\lambda L}{z(L^2+z^2)^{1/2}}\hat{z}}$$

Does Answer Make sense? Units okay

Check with limiting cases

look at limit as

$$z \to \infty \qquad \vec{E} \sim \frac{k\lambda}{z^2}$$ Same form as Point charge

look at limit as

$$L \to \infty$$

$$\vec{E} \to \frac{\infty}{\infty}$$

$$\vec{E} = \frac{2k\lambda}{z}$$

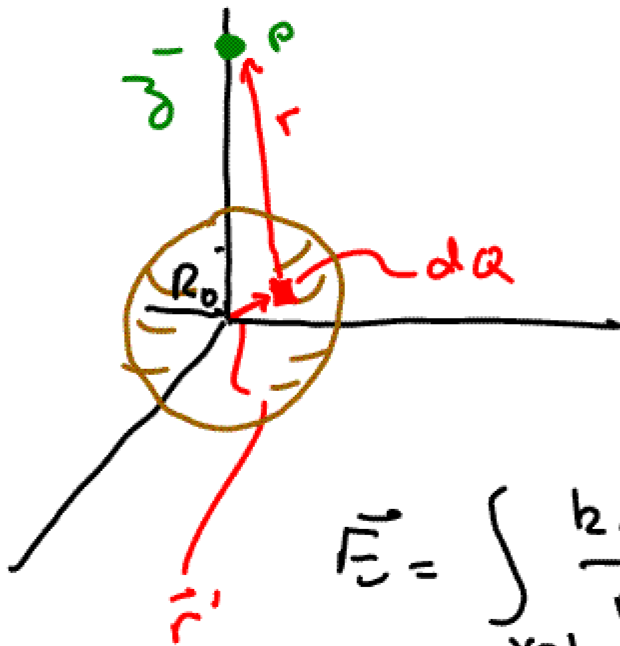
Use L'Hospital's rule

TAKE limit of top and bottom

After taking derivative w/ respect to L

$$\frac{d(\text{Top})}{dL} = \frac{d(2k\lambda L)}{dL} \sim 2k\lambda \qquad \frac{d(\text{bott})}{dL} = \frac{d(zL)}{dL} \sim z$$

$$\vec{E} = \frac{2k\lambda}{z}\hat{z} \text{ as } L \to \infty$$ We will see that this is the field of an ∞ line charge



uniform spherical dist of charge

$$\rho = \frac{Q_{TOT}}{\frac{4}{3}\pi R_0^3}$$

$$\vec{E} = \int_{vol} \frac{k\,dq}{r^2}\,\hat{r}$$

$$= \int_{vol} \frac{k\,dv}{r^2}\,\hat{r} = \int_0^{R_0}\int_0^{\pi}\int_0^{2\pi} \frac{k\rho}{r^2}\, r^2 \sin\theta \cdot d\theta\, d\phi\, dr$$

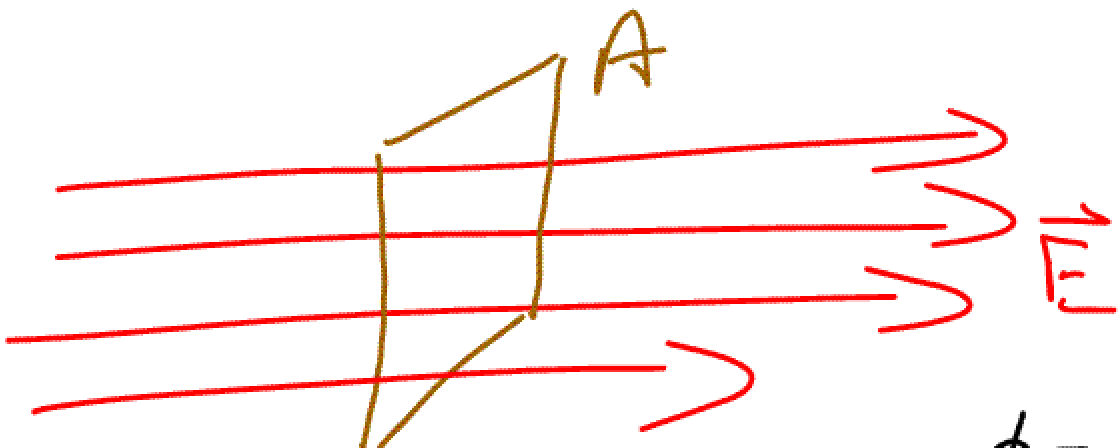
Holy Crap!

But Remember This
We'll come back to it

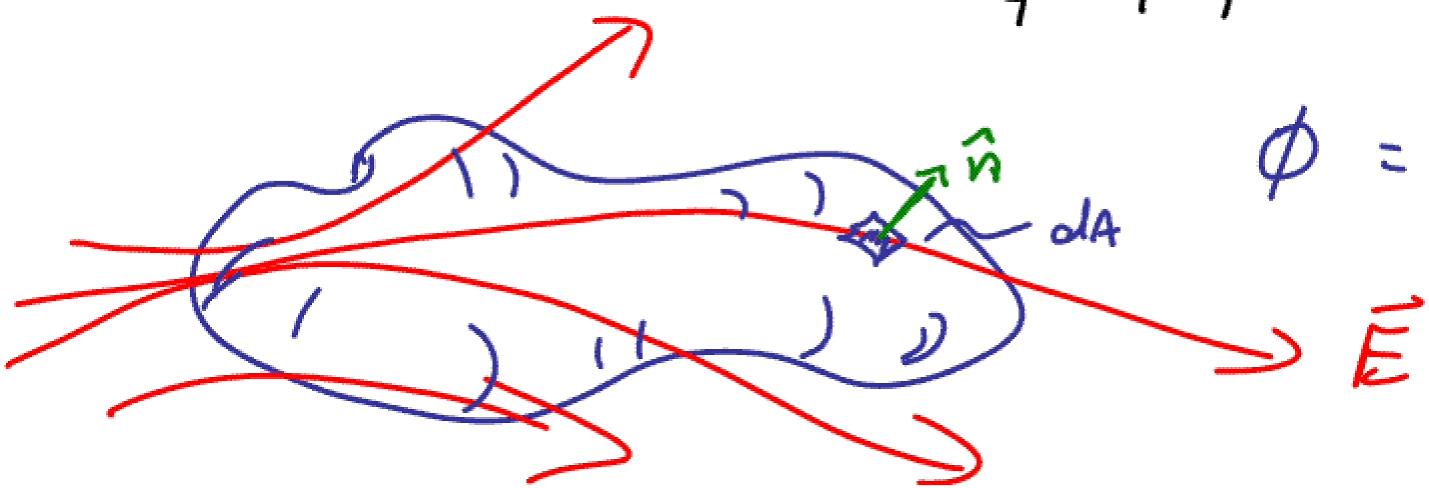
This is very ugly. There are easier ways to calculate Electric fields under certain conditions of symmetry. We'll look at that now . . .

Electric Flux

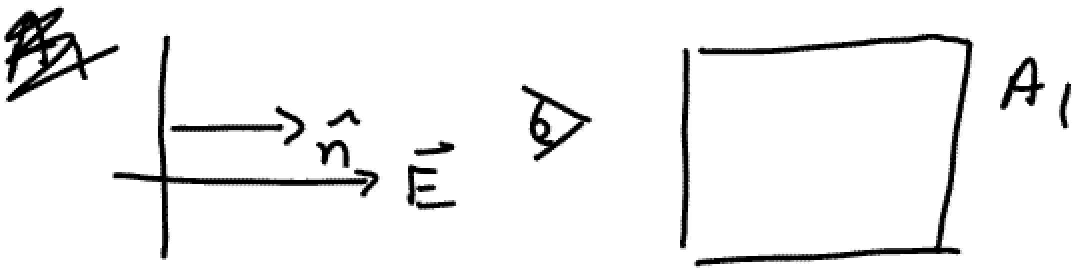
Electric Flux thru a surface $\equiv \phi$
net # of field lines crossing that surface



$$\phi = |\vec{E}| A$$

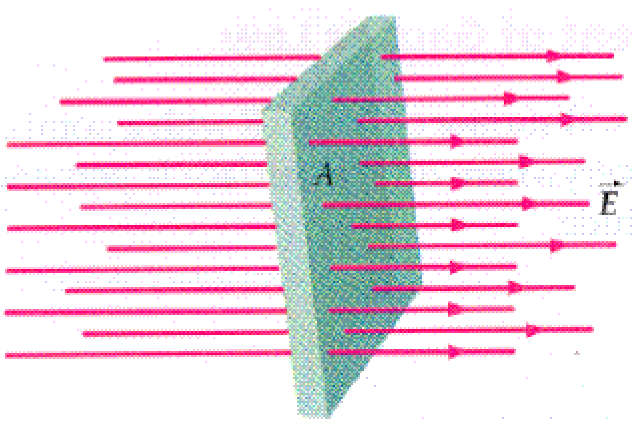


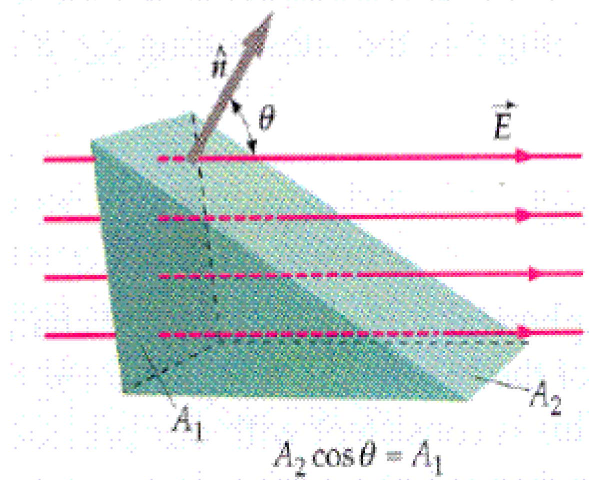
$$\phi = \oint_s \vec{E} \cdot \hat{n} \, dA$$

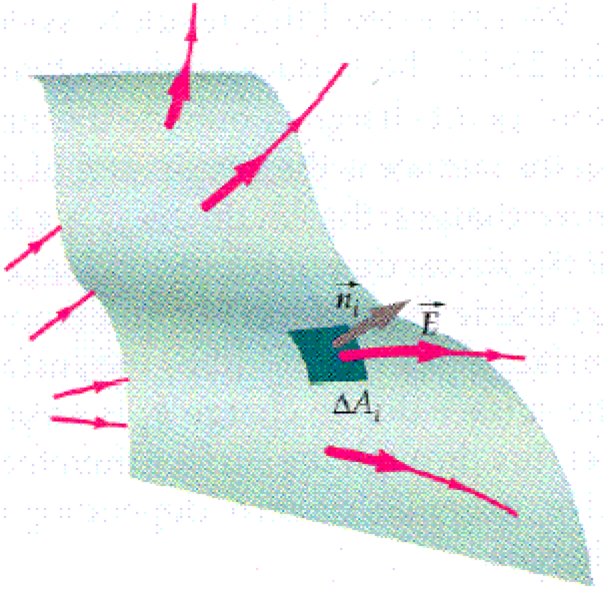


$$A_2 = A_1 \cos\theta$$

$$\phi = \vec{E} \cdot \hat{n} \; A$$







Figures are from Tipler's text "physics" 4th edition W.H.Freeman

maybe they are a bit more clear than my drawings.