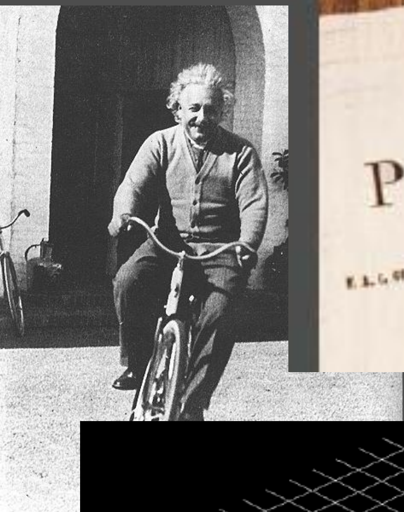
Relativity: the warping of space, time, and minds



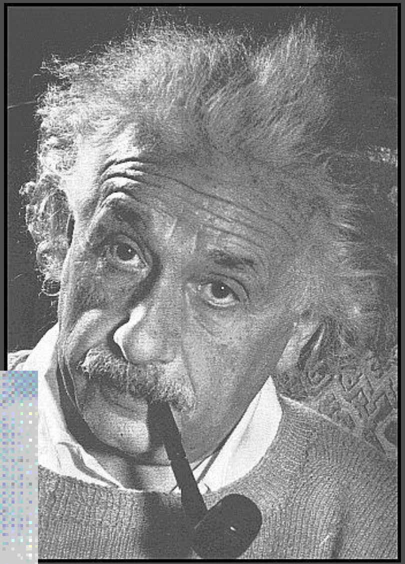

ANNALEN DER PHYSIK.

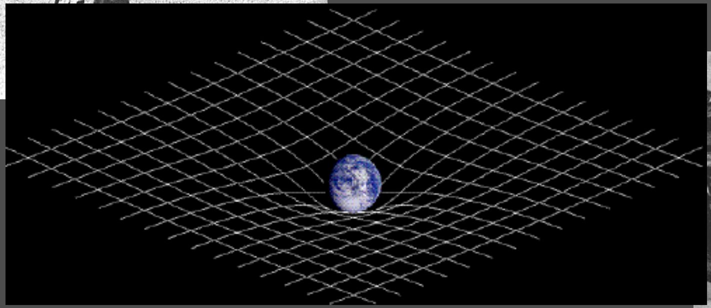
VIERTE FOLGE.

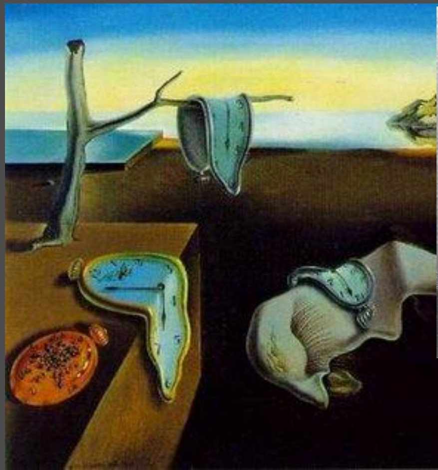
BAND 17.

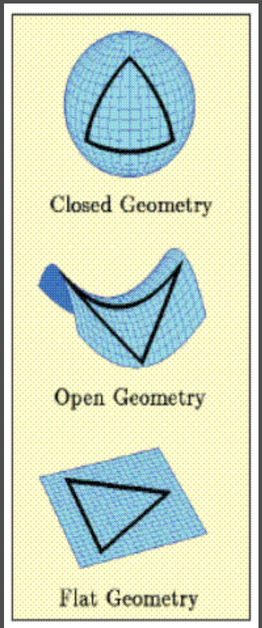
3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet und ruht der Leiter, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an den Orten, wo sich Teile des Leiters befinden, einen Strom erzeugt. Ruht aber der Magnet und bewegt sich der Leiter





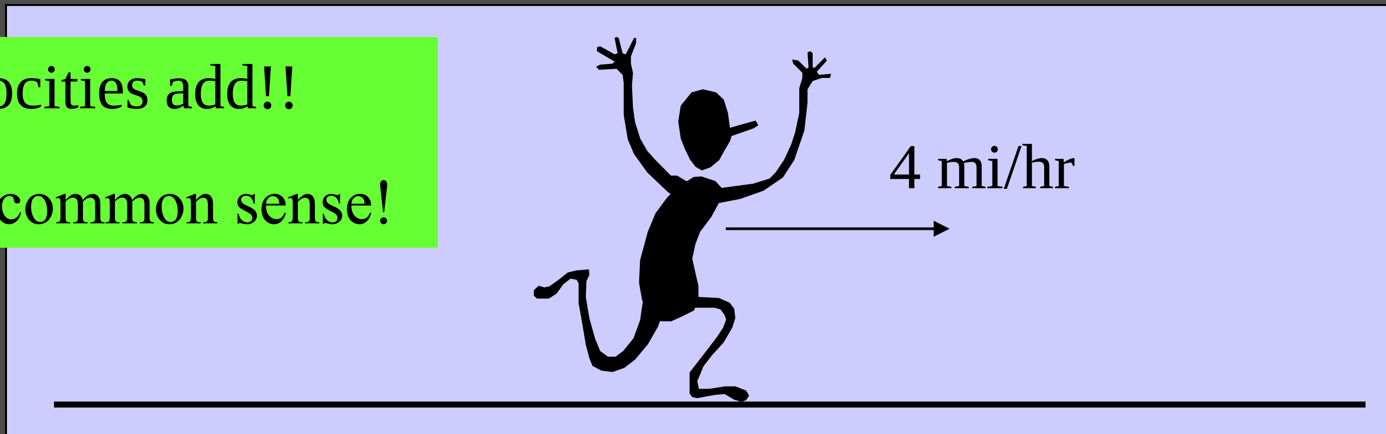




Steve Manly
Department of Physics and Astronomy
University of Rochester

Velocities add!!

It’s common sense!

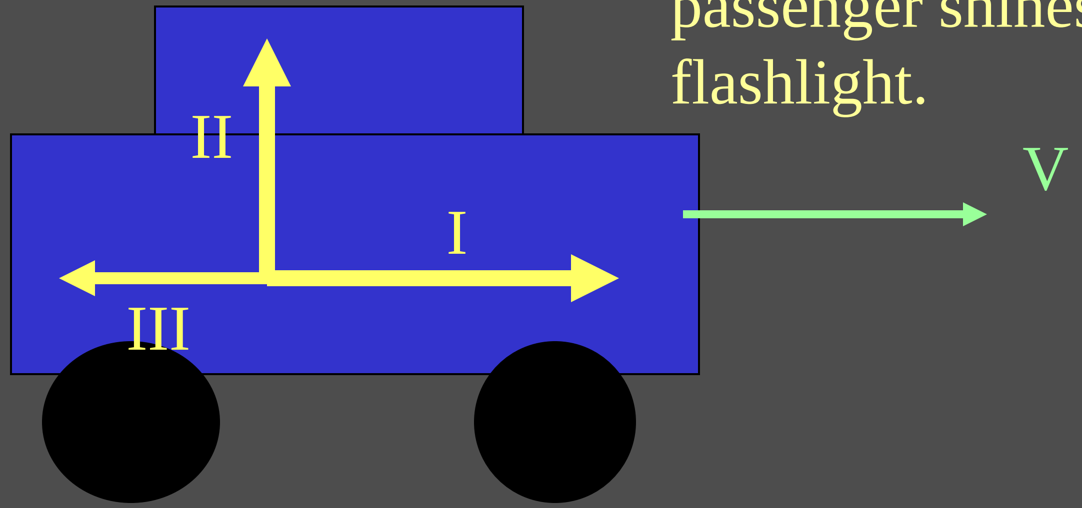


Speed with respect to you is 4 mi/hr

Speed with respect to you is 2 + 4 = 6 mi/hr

The speed of light is greater for beam I, beam II or beam III?

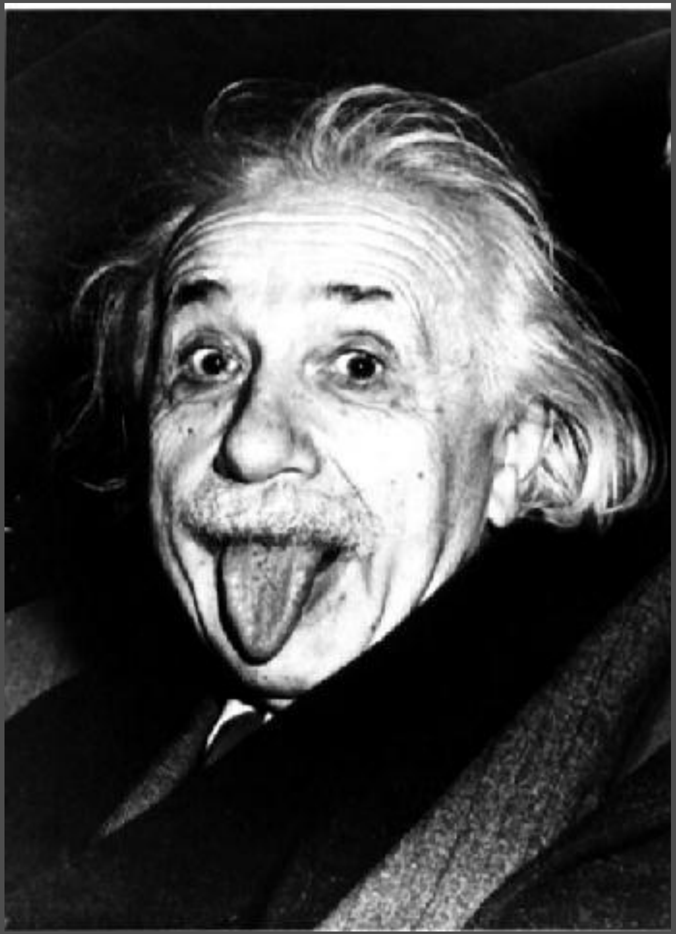
Car moves while passenger shines a flashlight.



Experiment says the speed of light is the same in all directions!!

Weird, huh? What does it mean for the real world?

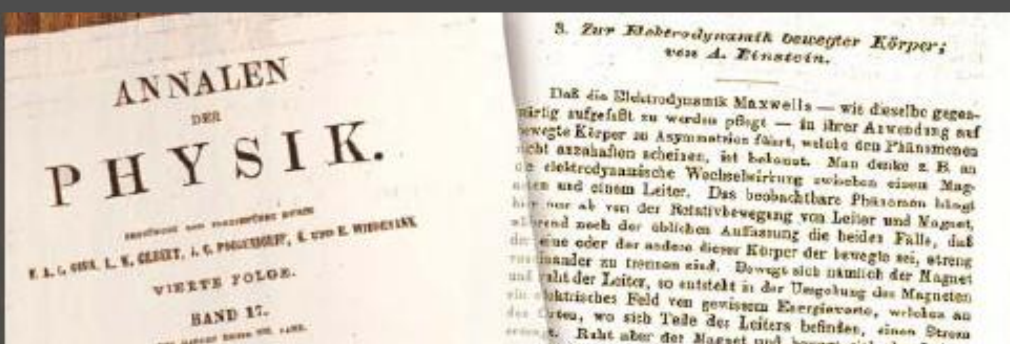
Enter our man Einstein!



Einstein’s 2 postulates:

The velocity of light is the same for observers in all inertial reference frames.

The “physics” is the same for all observers (even if in different inertial reference frames).


ANNALEN
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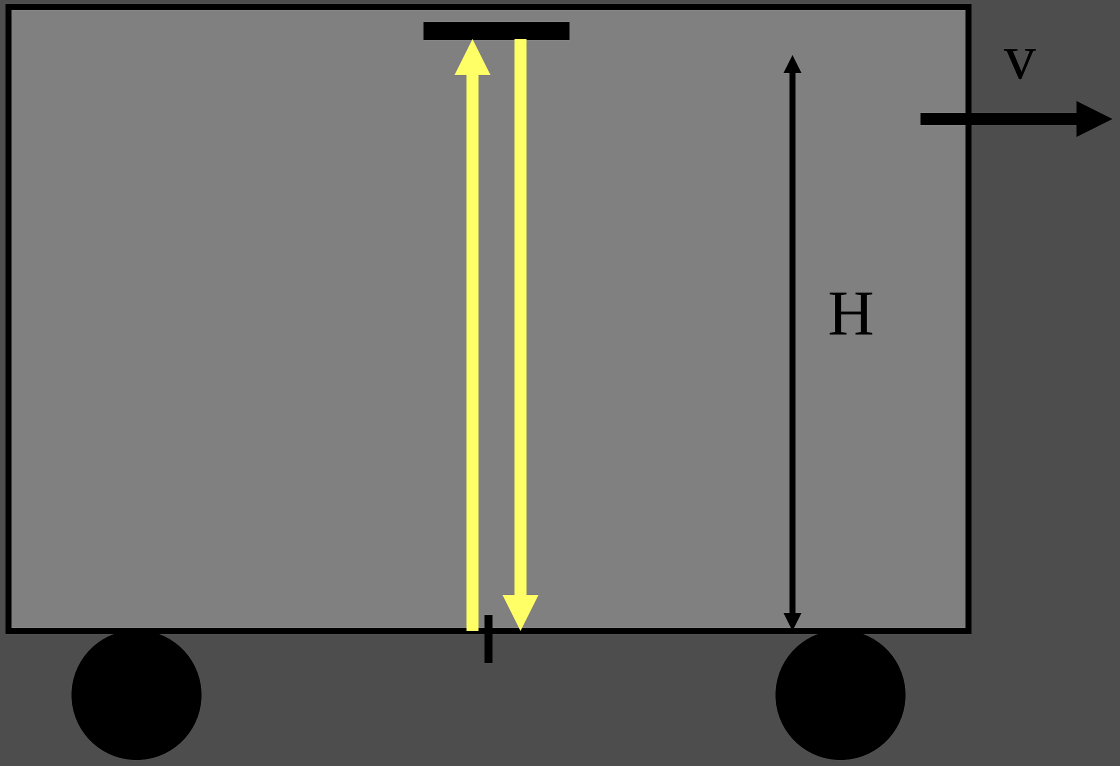
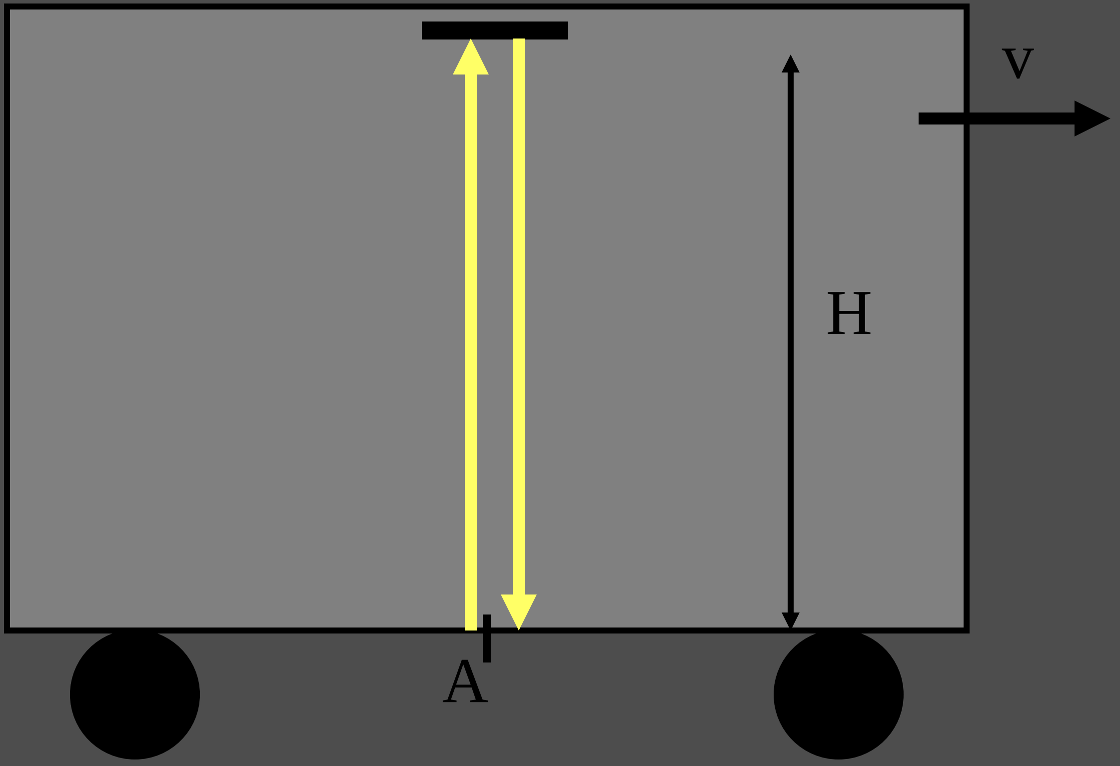
erzeugt. Ruht aber der Magnet und bewegt sich der Leiter,

Einstein thought experiment:
Consider a beam of light that is emitted from the floor of a train that bounces off a mirror on the ceiling and returns to the point on the floor where it was emitted.

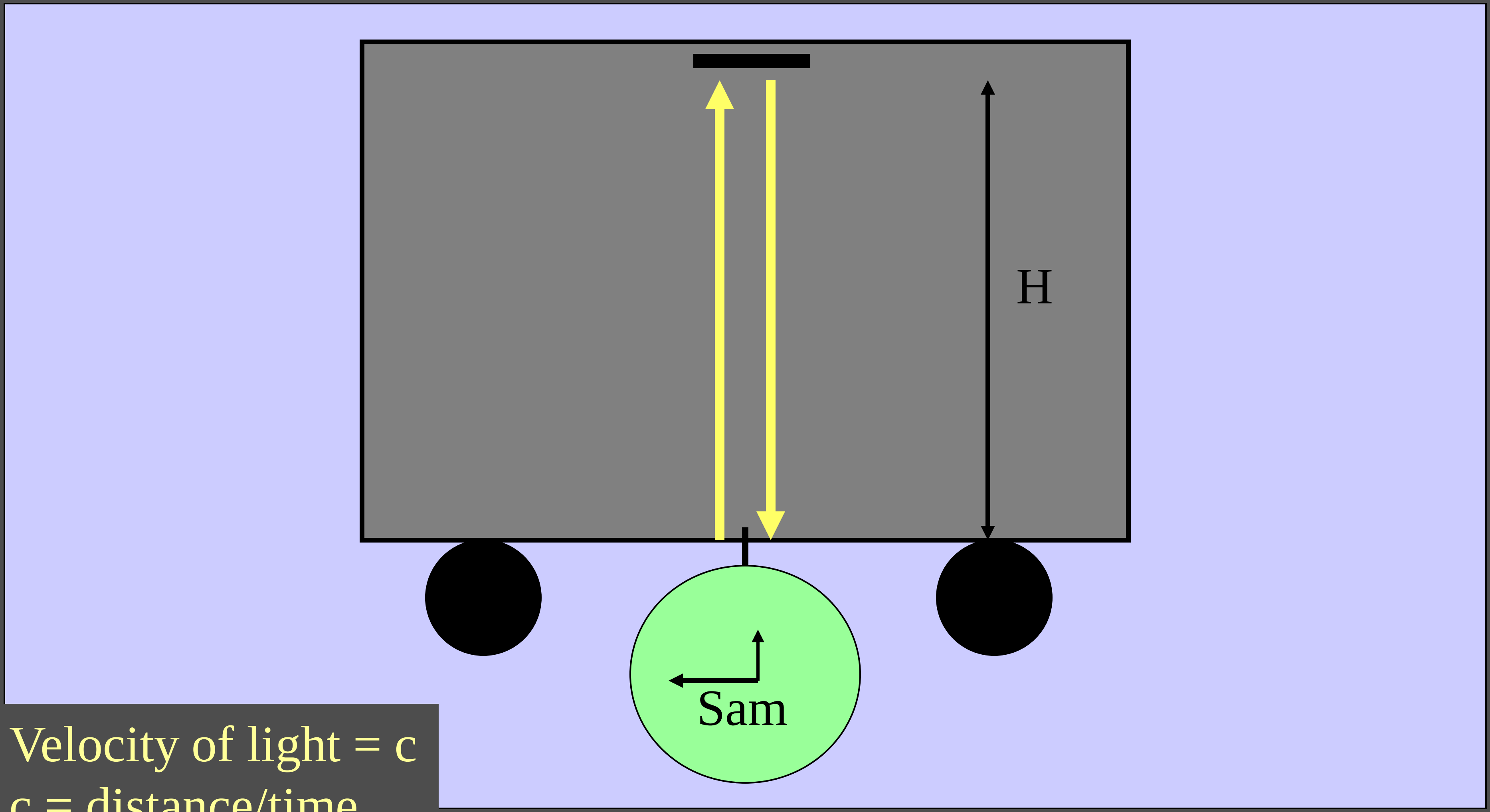


Fact: Light is emitted and detected at point A.

This fact must be true no matter who makes the measurement!!!!



Sam is on the train



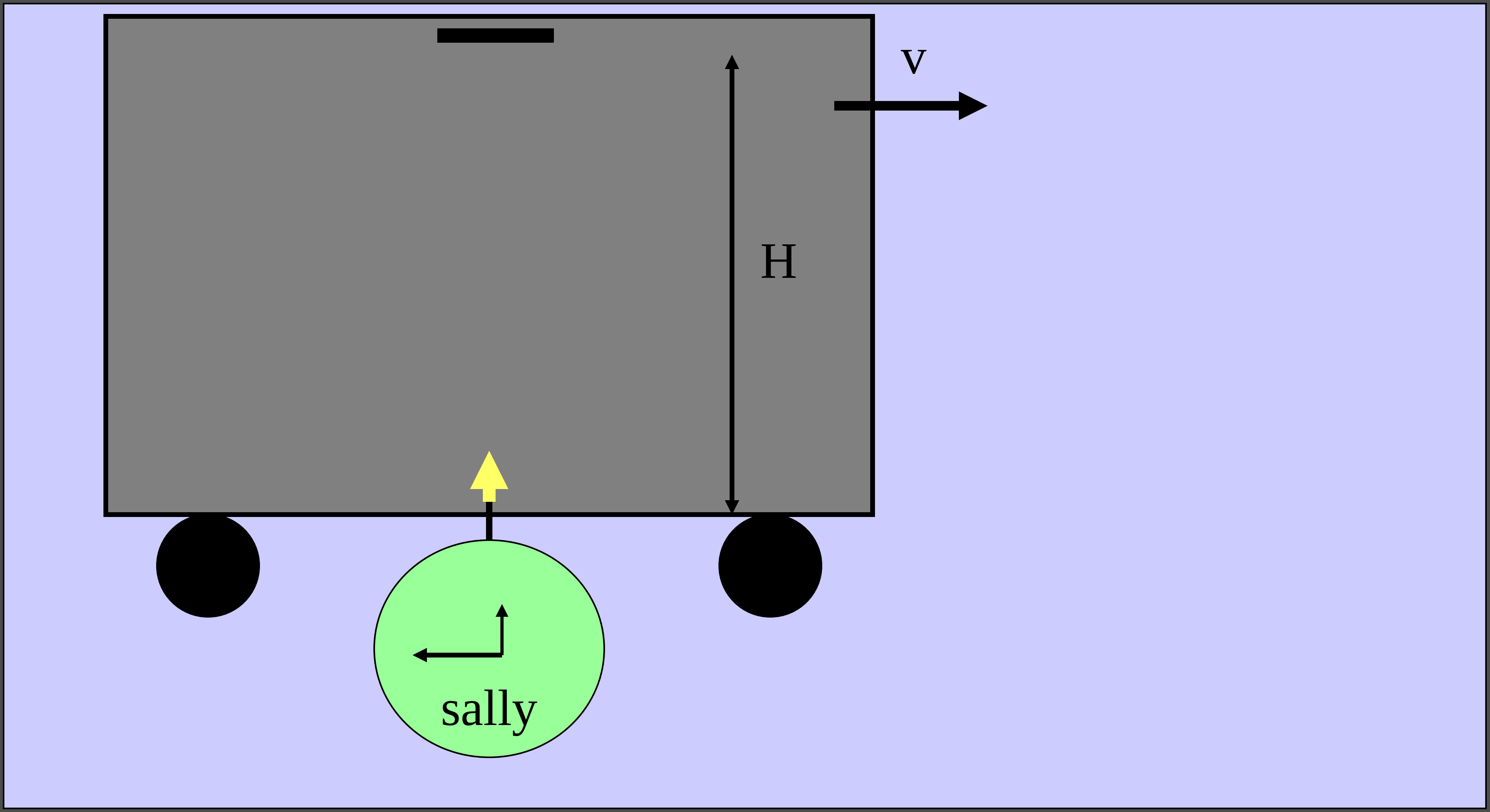
Velocity of light = c

c = distance/time

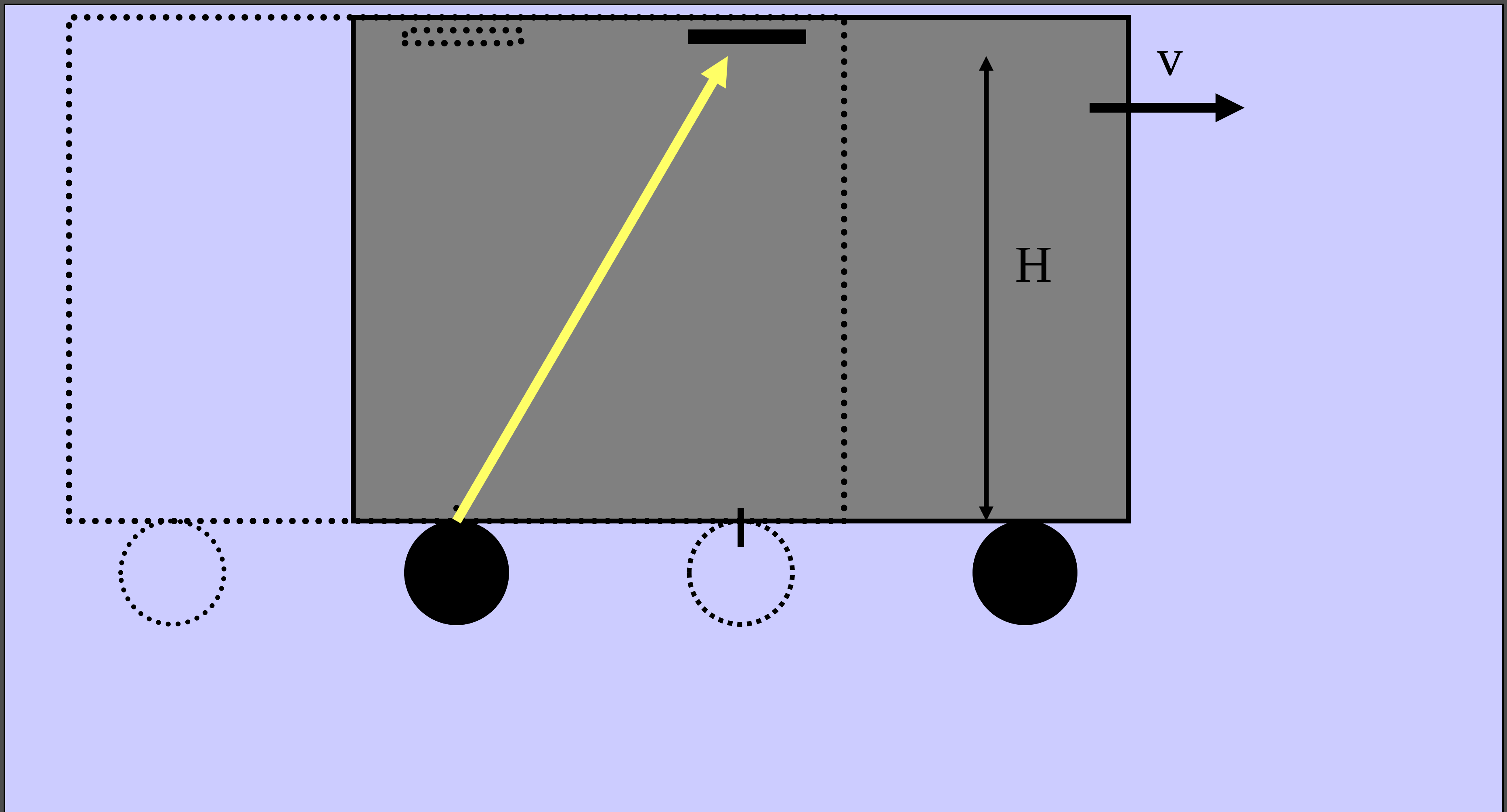
$c = 2H/T_{sam}$

$T_{sam} = 2H/c$

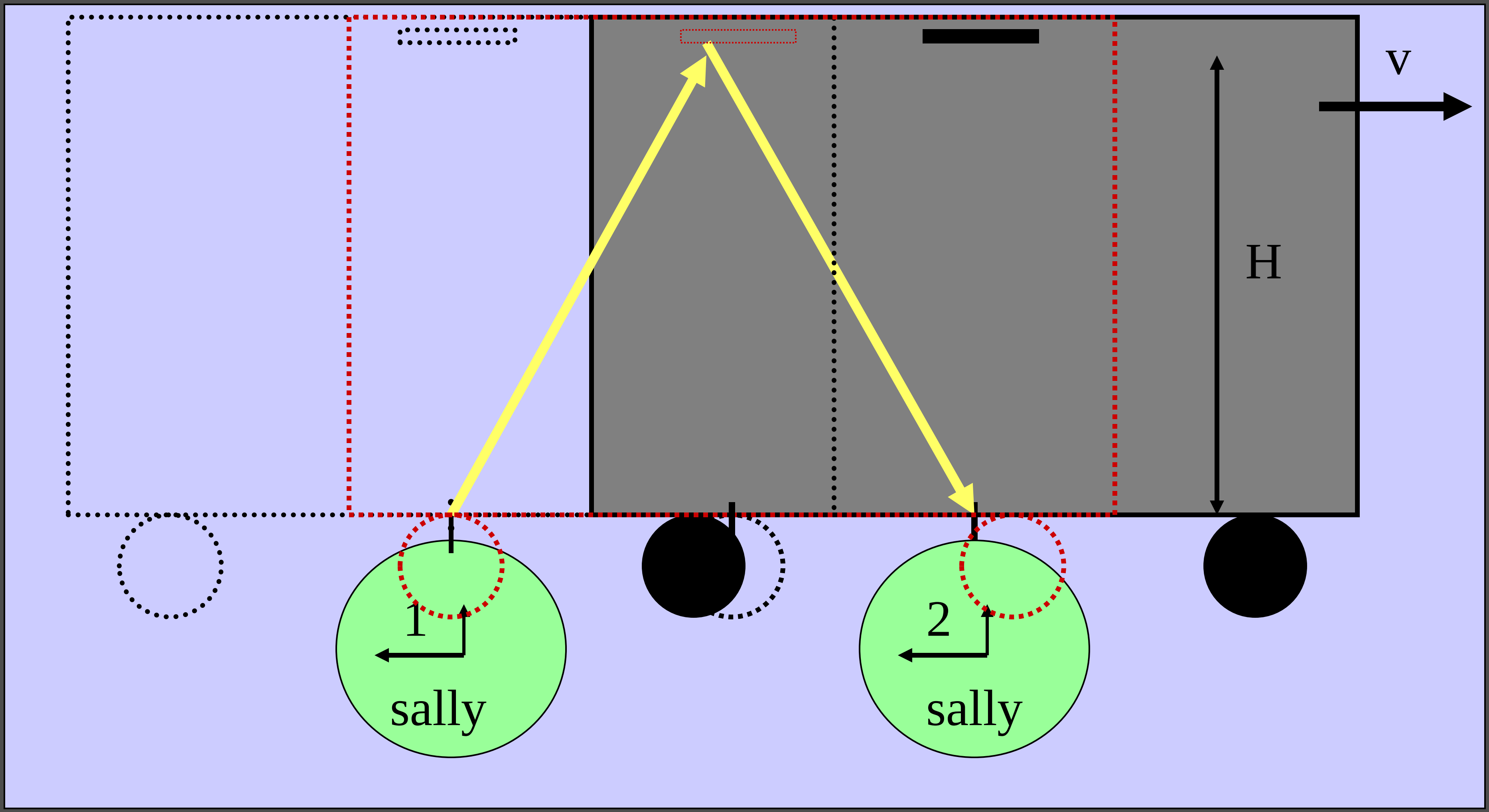
Sally watches the train pass and makes the same measurement.



Light is emitted


v
H

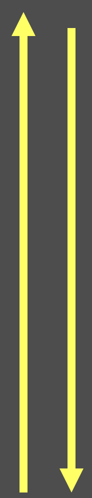
Sally is standing still, so it takes two clocks.



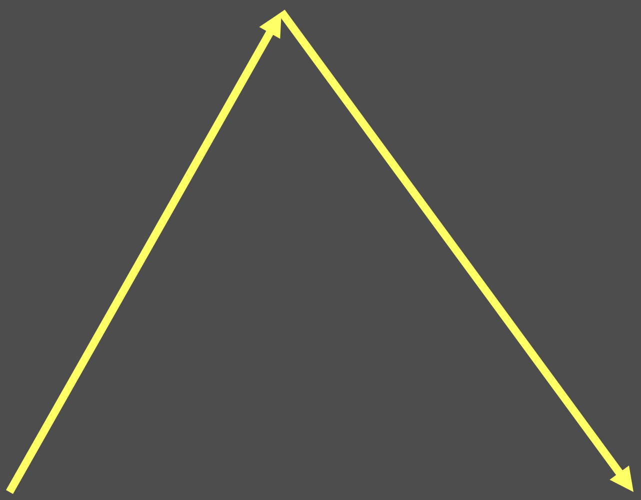
Light is emitted

Light returns

Sam



Sally

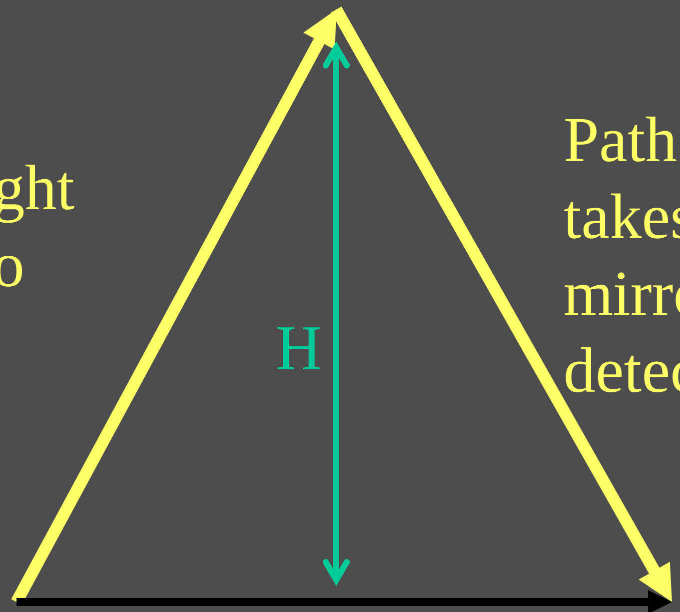


Sally sees the light traveling further. If light travels at a constant speed, the same "event" must seem to take longer to Sally than Sam!

Time is relative … not absolute!!

From Sally's point of view

Path light takes to mirror



Path light takes from mirror to detector

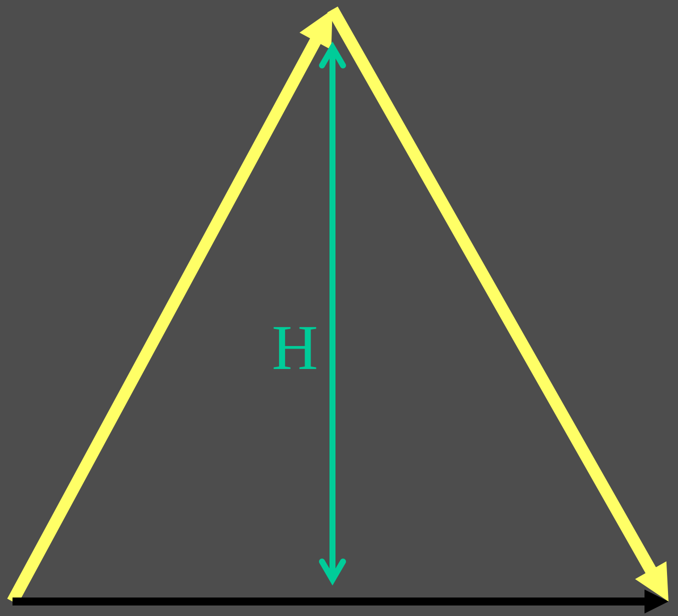
Distance train travels while light is traveling

$= VT_{sally}$

$$D = \sqrt{H^2 + (\frac{1}{2}\mathrm{v}T_{sally})^2}$$

Makes use of Pythagorian theorem

From Sally's point of view



$$c = \text{distance/time} = 2D/T_{sally}$$

$$T_{sally} = 2D/c$$

Sam (on train) Sally (on ground)

$2H/T_{sam} = c$ $c = 2D/T_{sally}$

$$c = \frac{2}{T_{sally}}\sqrt{H^2 + (\frac{1}{2}\mathrm{v}T_{sally})^2}$$

$$\frac{2H}{T_{sam}} = \frac{2}{T_{sally}}\sqrt{H^2 + \left(\frac{1}{2}\mathrm{v}T_{sally}\right)^2}$$

$$\left(\frac{2H}{T_{sam}}\right)^2 = \left(\frac{2H}{T_{sally}}\right)^2 + \left(\frac{2}{T_{sally}}\right)^2\left(\frac{1}{2}\mathrm{v}T_{sally}\right)^2$$

$$\left(\frac{2H}{T_{sam}}\right)^2 = \left(\frac{2H}{T_{sally}}\right)^2 + \mathrm{v}^2$$

$$\left(\frac{1}{T_{sam}}\right)^2 = \left(\frac{1}{T_{sally}}\right)^2 + \frac{\mathrm{v}^2}{(2H)^2}$$

Recall $2H/T_{sam} = c$ or $2H = cT_{sam}$

$$\left(\frac{1}{T_{sam}}\right)^2 = \left(\frac{1}{T_{sally}}\right)^2 + \frac{\mathrm{v}^2}{(cT_{sam})^2}$$

$$c^2 = \frac{c^2 T_{sam}^2}{T_{sally}^2} + \mathrm{v}^2 \quad \longrightarrow \quad \boxed{T_{sally} = \left[\frac{1}{\sqrt{1-\left(\frac{\mathrm{v}}{c}\right)^2}}\right] T_{sam}}$$

Sam (on train)

Sally (on ground)

$2H/T_{sam} = c$

$c = 2D/T_{sally}$

$$c = \frac{2}{T_{sally}}\sqrt{H^2 + (\frac{1}{2}\mathrm{v}T_{sally})^2}$$

A bit of algebra.

$$T_{sally} = \left[\frac{1}{\sqrt{1-\left(\frac{\mathrm{v}}{c}\right)^2}}\right]T_{sam}$$

This number is >1.

It becomes larger as

v approaches c.

Think about it!

Sam and Sally measure the time interval for the same event.

The ONLY difference between Sam and Sally is that one is moving with respect to the other.

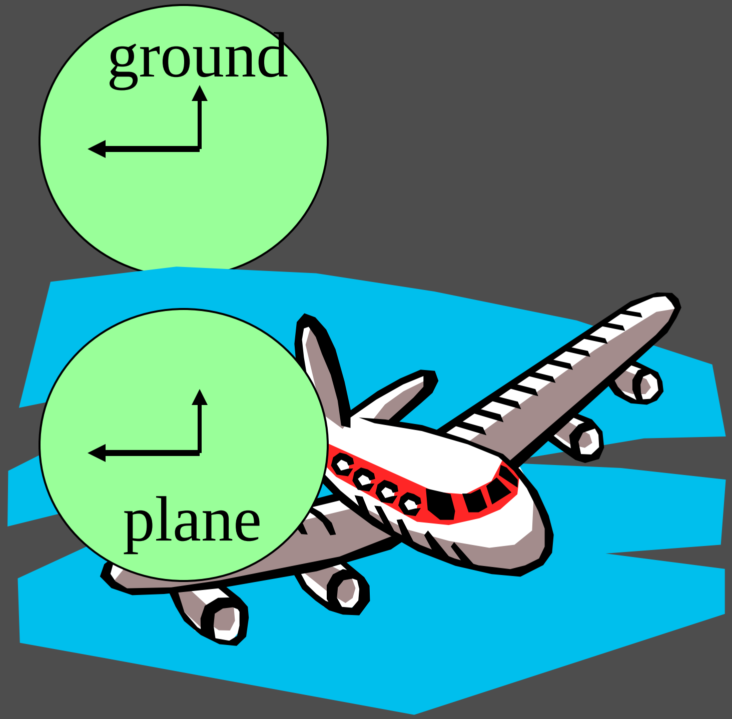
Yet, $T_{sally} > T_{sam}$

The same event takes a different amount of time depending on your "reference frame"!!

Time is not absolute! It is relative!

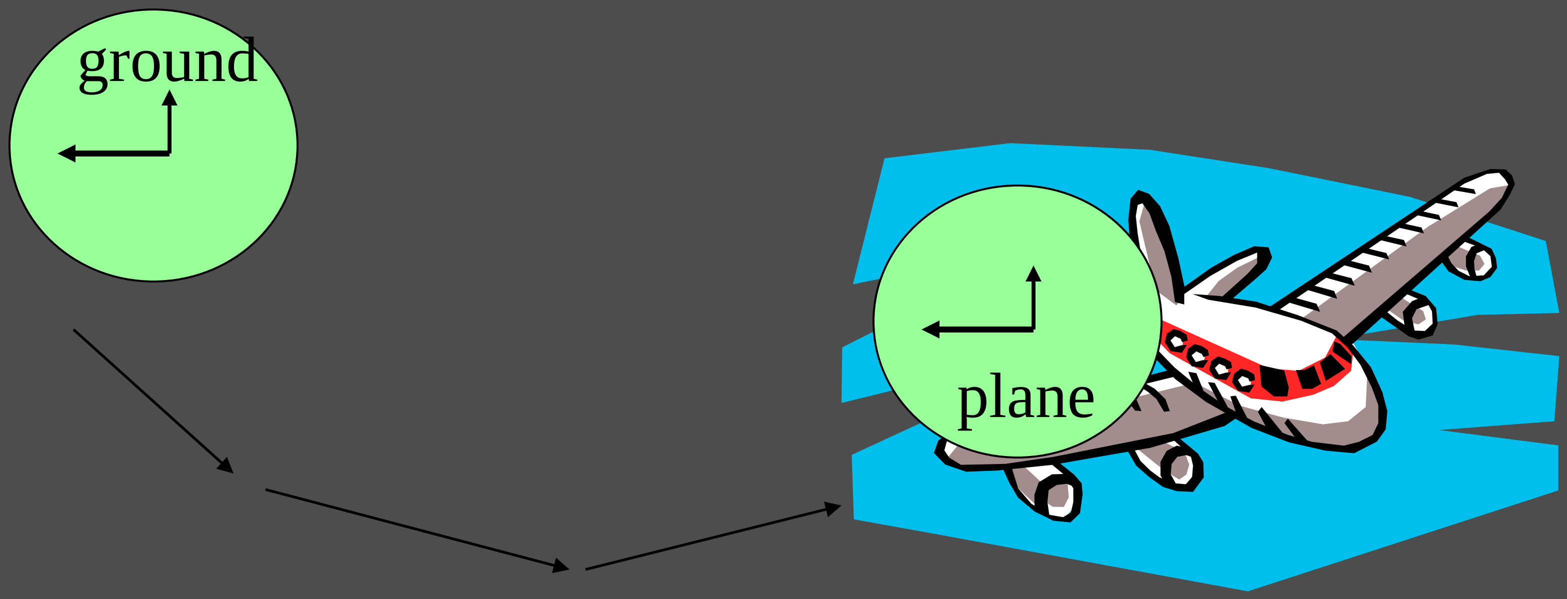
Can this be true??

Experiment says YES!

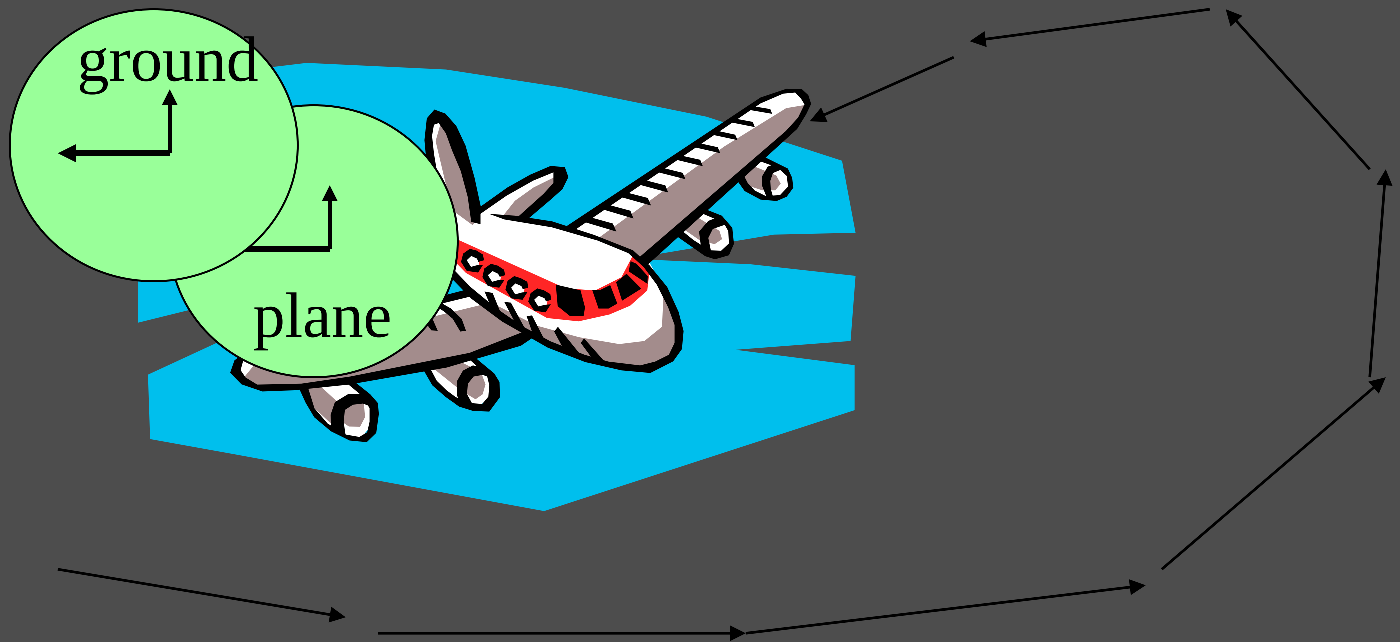


Can this be true??

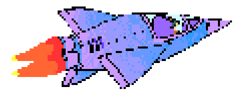
Experiment says YES!



Less time elapsed on the clocks carried on the airplane



V=0.98c



Lifetime=70 years on spaceship

Earth at rest

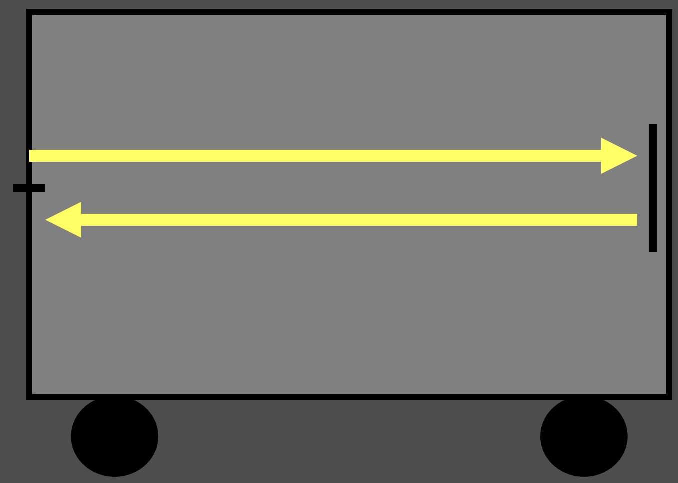
How long does person appear to live to astronomers on earth?

$$t_{earth} = \overbrace{\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}^{\gamma > 1} \underbrace{t_{spaceship}}_{\text{"Proper Time"}}$$

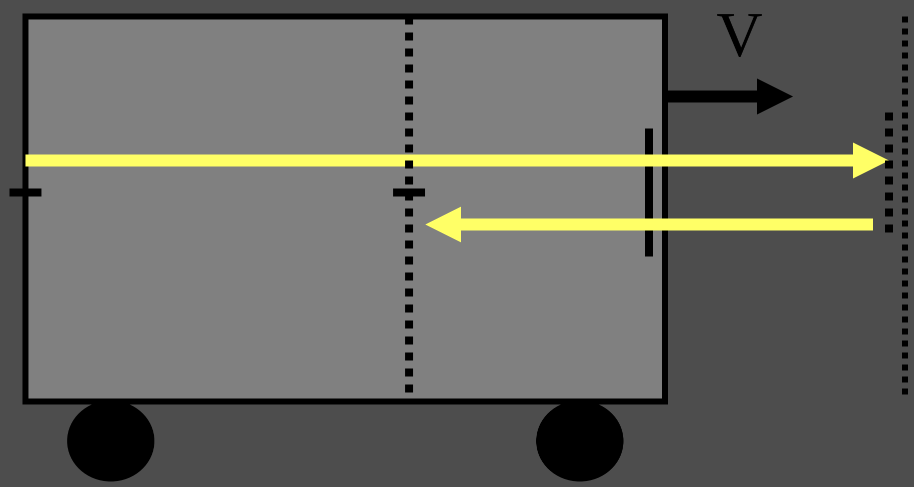
$$t_{earth} = \frac{1}{\sqrt{1-\left(\frac{.98c}{c}\right)^2}}(70 \text{ years})$$

$$t_{earth} = (5)(70 \text{ years})$$

$$t_{earth} = 350 \text{ years!}$$



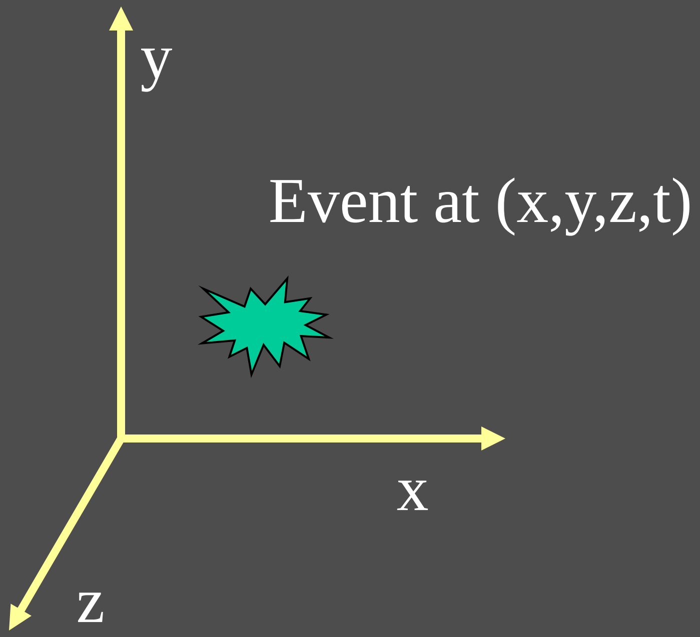
Measure the length of a boxcar where you are on the car.



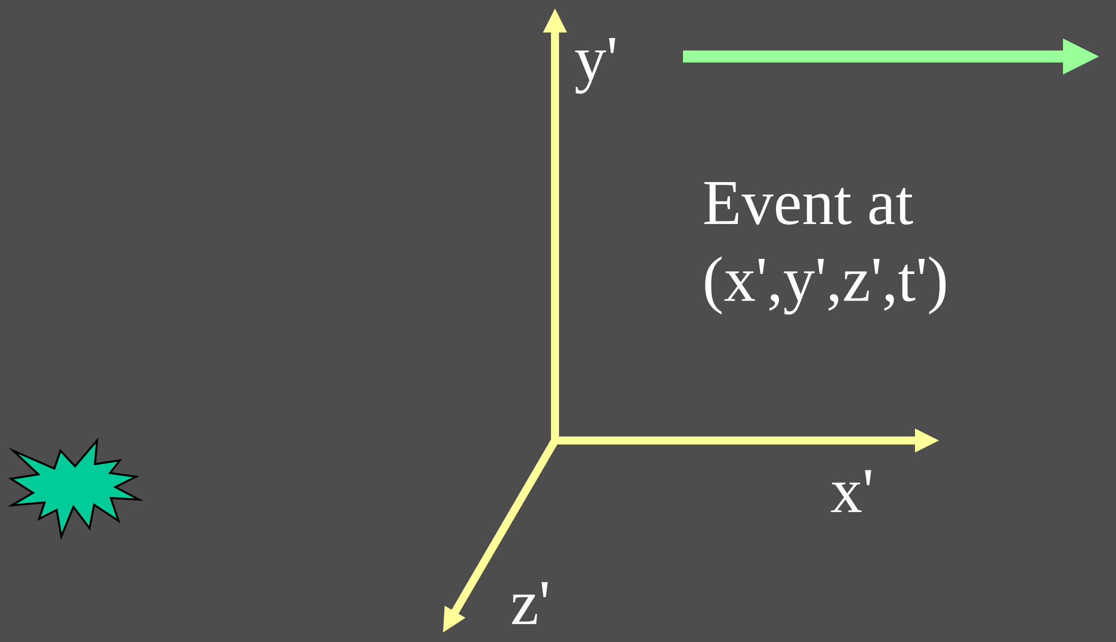
Measure the length of a boxcar moving by you.

Length is relative, too!

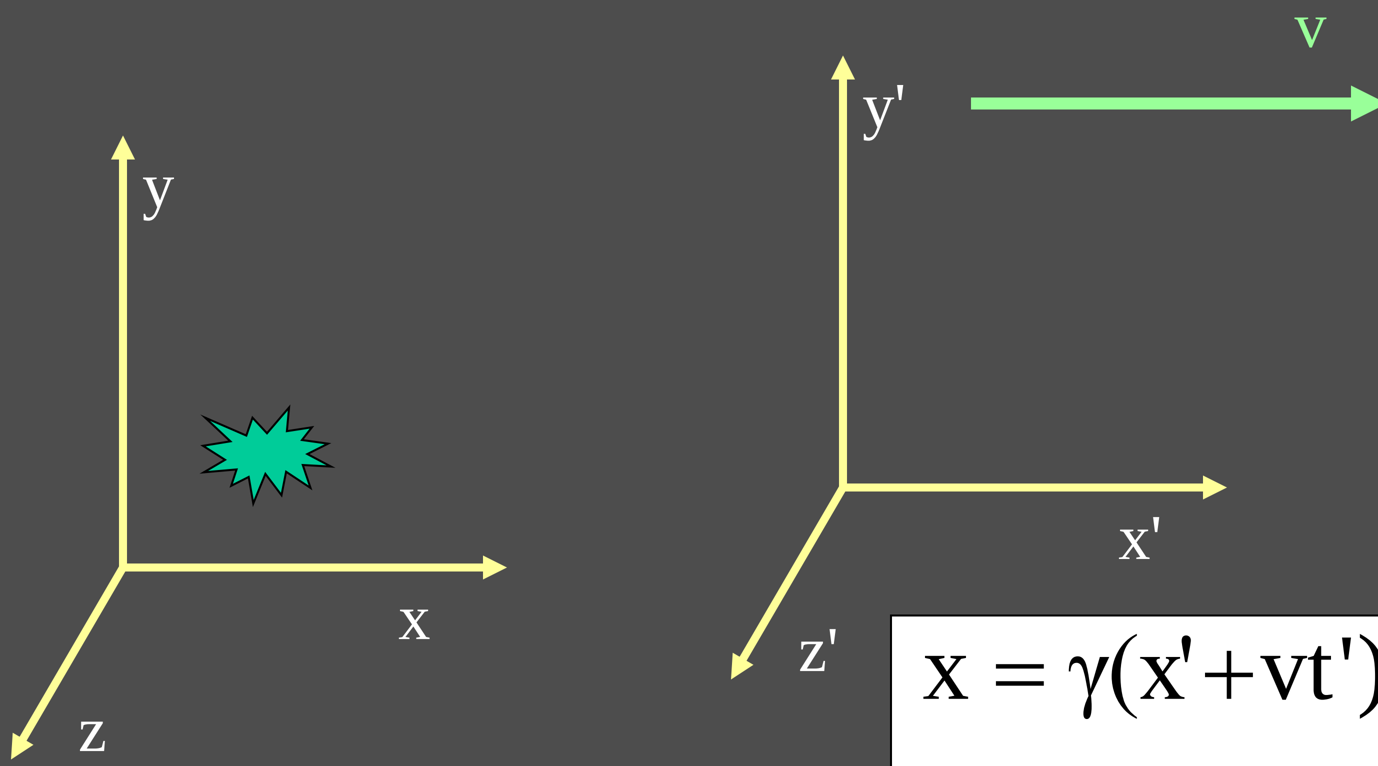
Lorentz transformations



Lorentz transformations



Lorentz transformations



How are (x,y,z,t) related to (x',y',z',t')?

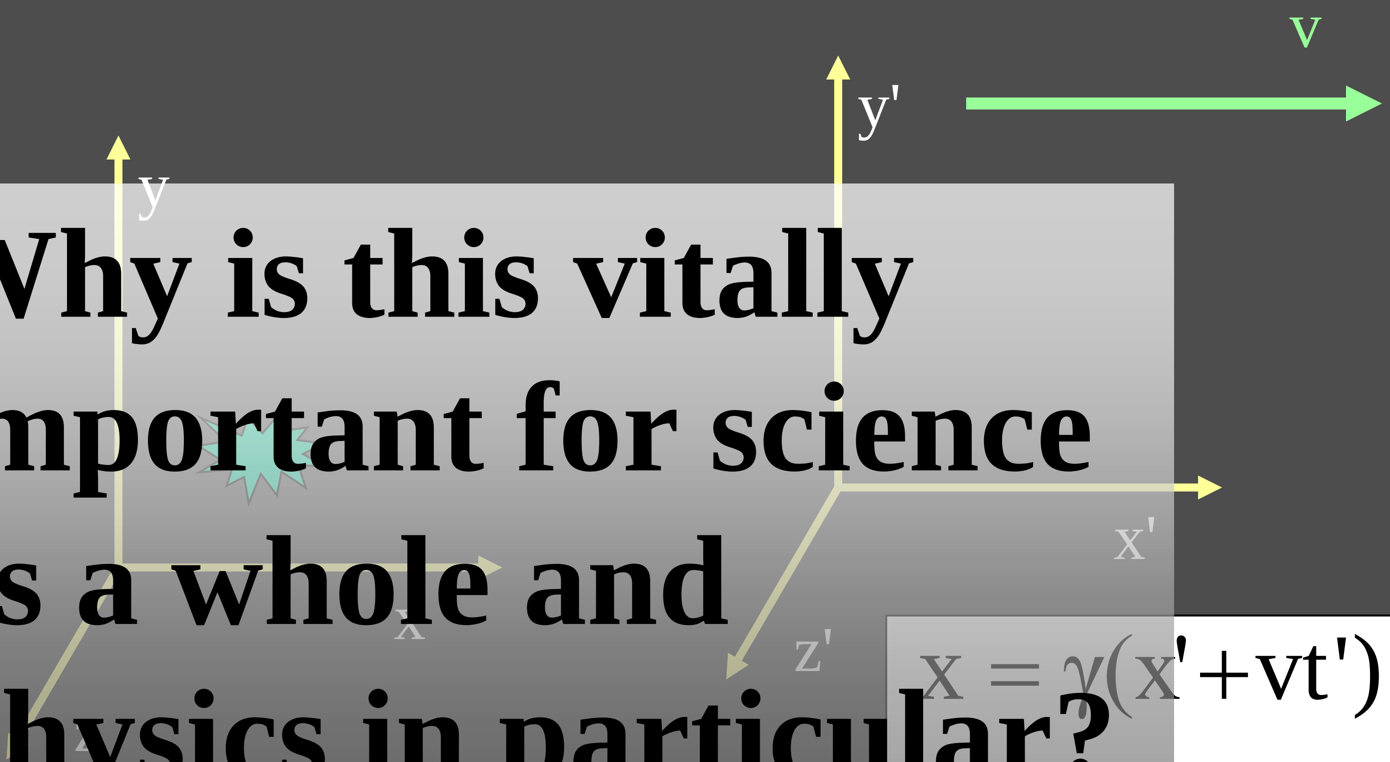
$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + v\frac{x'}{c^2}\right)$$

Lorentz transformations



Why is this vitally important for science as a whole and physics in particular?

How are (x,y,z,t) related to (x',y',z',t')?

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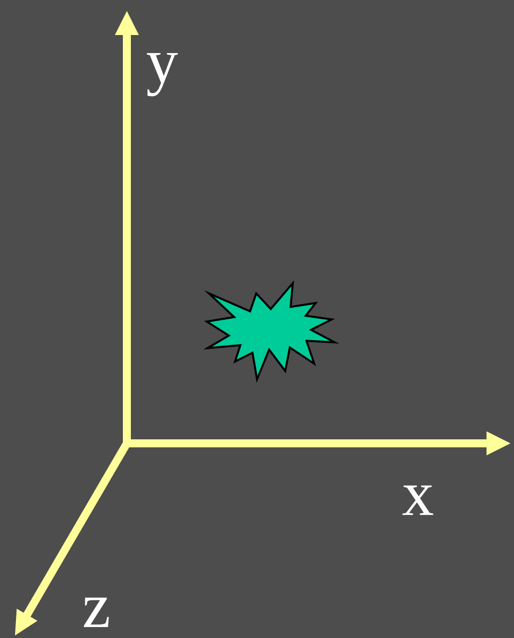
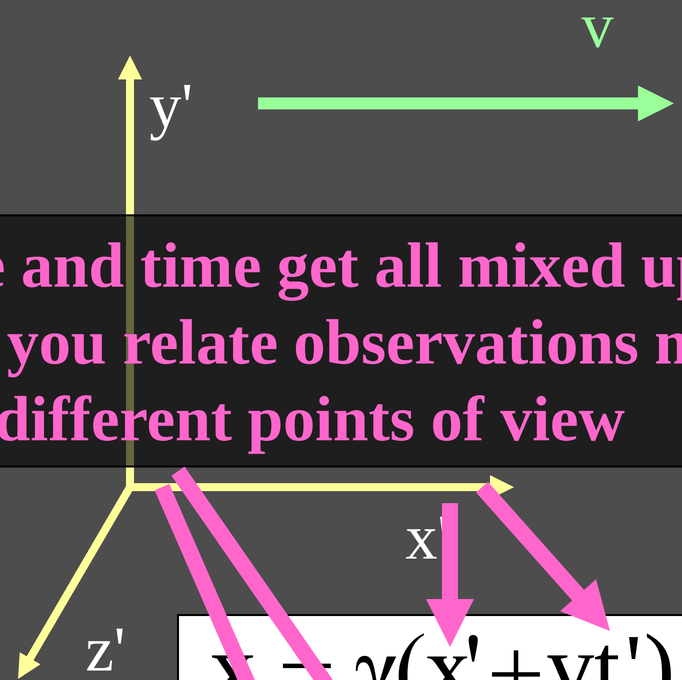
Lorentz transformations





Space and time get all mixed up when you relate observations made from different points of view

$$x = \gamma(x' + vt')$$

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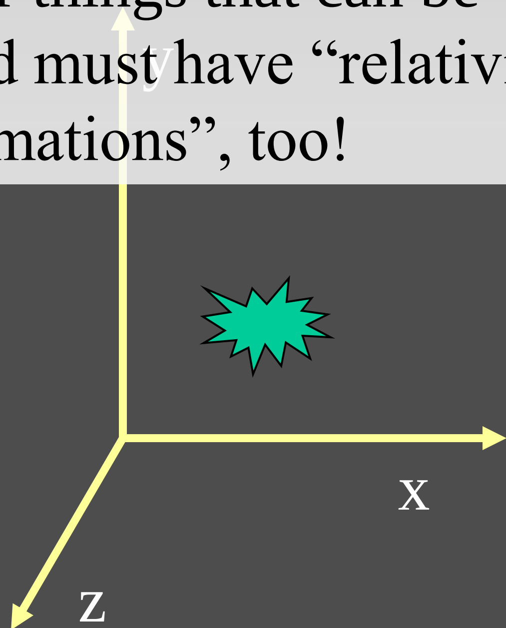
$$z = z'$$

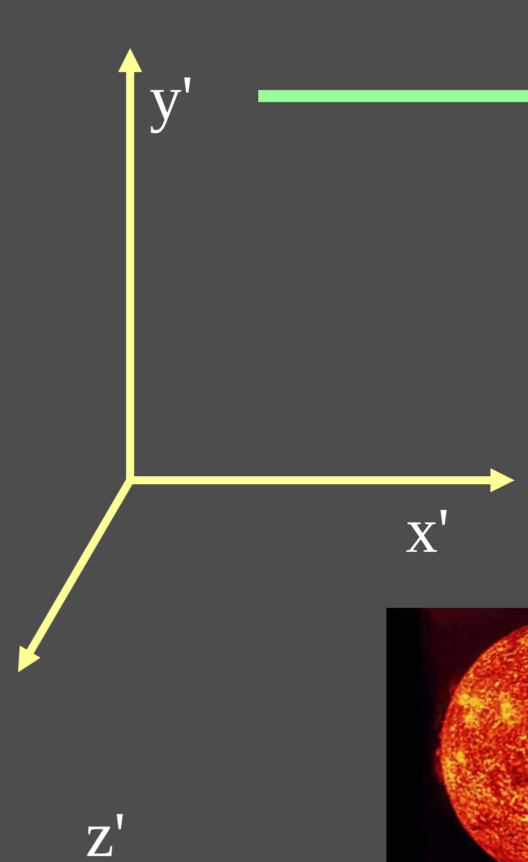
$$t = \gamma\left(t' + v\frac{x'}{c^2}\right)$$

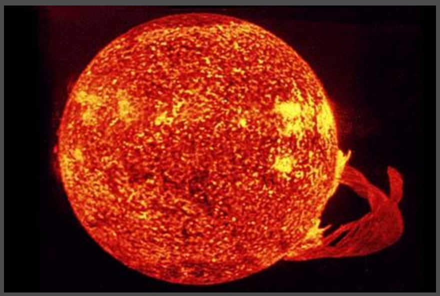
How are (x,y,z,t) related to (x',y',z',t')?

Spacetime

All other things that can be observed must have "relativisitic transformations", too!







$$x = \gamma(x' + vt')$$

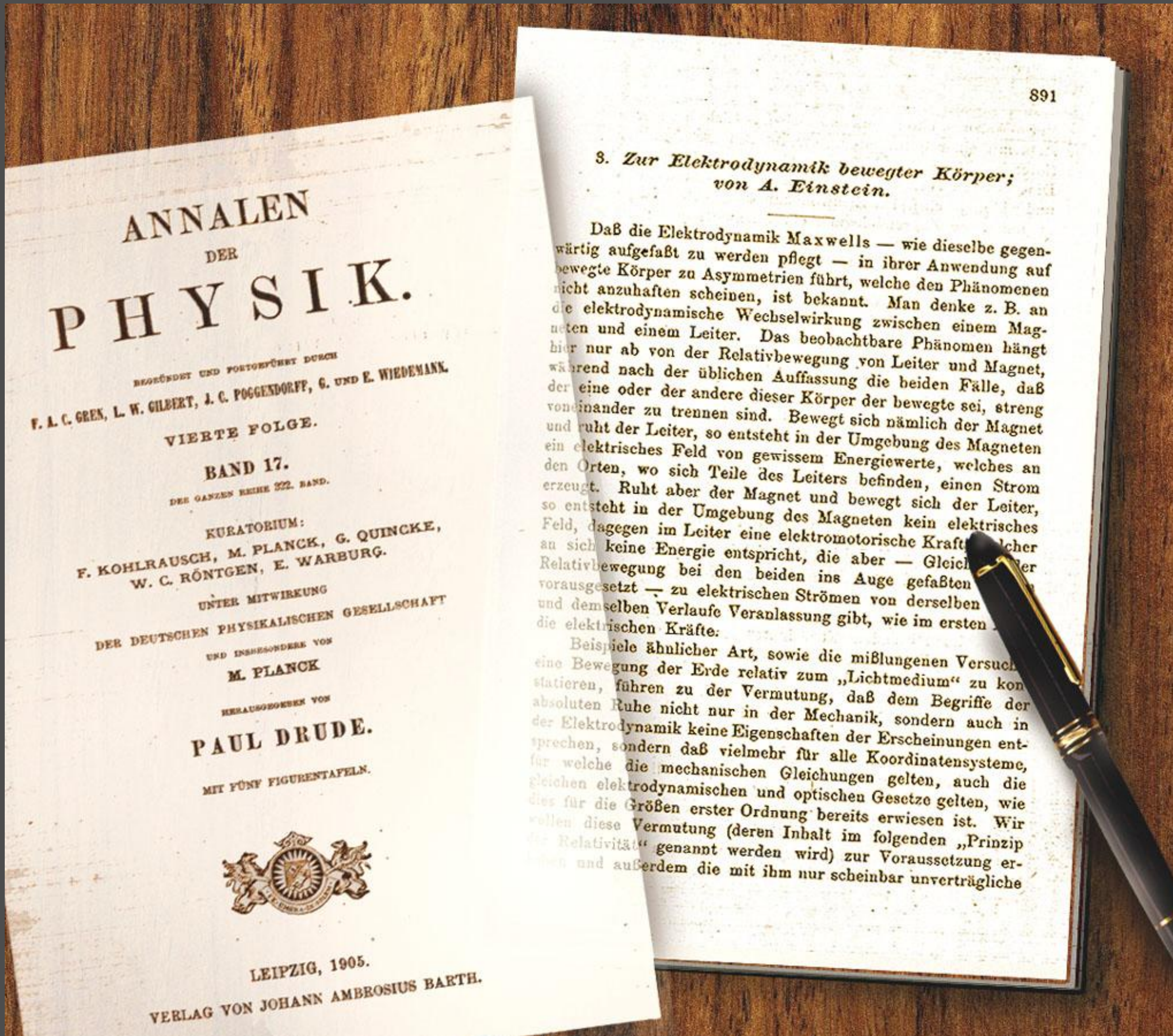
$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + v\frac{x'}{c^2}\right)$$

$$p = mv$$

$$\mathbf{E=mc^2}$$



ANNALEN DER PHYSIK.

BEGRÜNDET UND FORTGEFÜHRT DURCH F. A. C. GREN, L. W. GILBERT, J. C. POGGENDORFF, G. UND E. WIEDEMANN.

VIERTE FOLGE.

BAND 17.

DER GANZEN REIHE 322. BAND.

KURATORIUM: F. KOHLRAUSCH, M. PLANCK, G. QUINCKE, W. C. RÖNTGEN, E. WARBURG.

UNTER MITWIRKUNG DER DEUTSCHEN PHYSIKALISCHEN GESELLSCHAFT UND INSBESONDERE VON M. PLANCK

HERAUSGEGEBEN VON PAUL DRUDE.

MIT FÜNF FIGURENTAFELN.

LEIPZIG, 1905.

VERLAG VON JOHANN AMBROSIUS BARTH.

891

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von A. Einstein.

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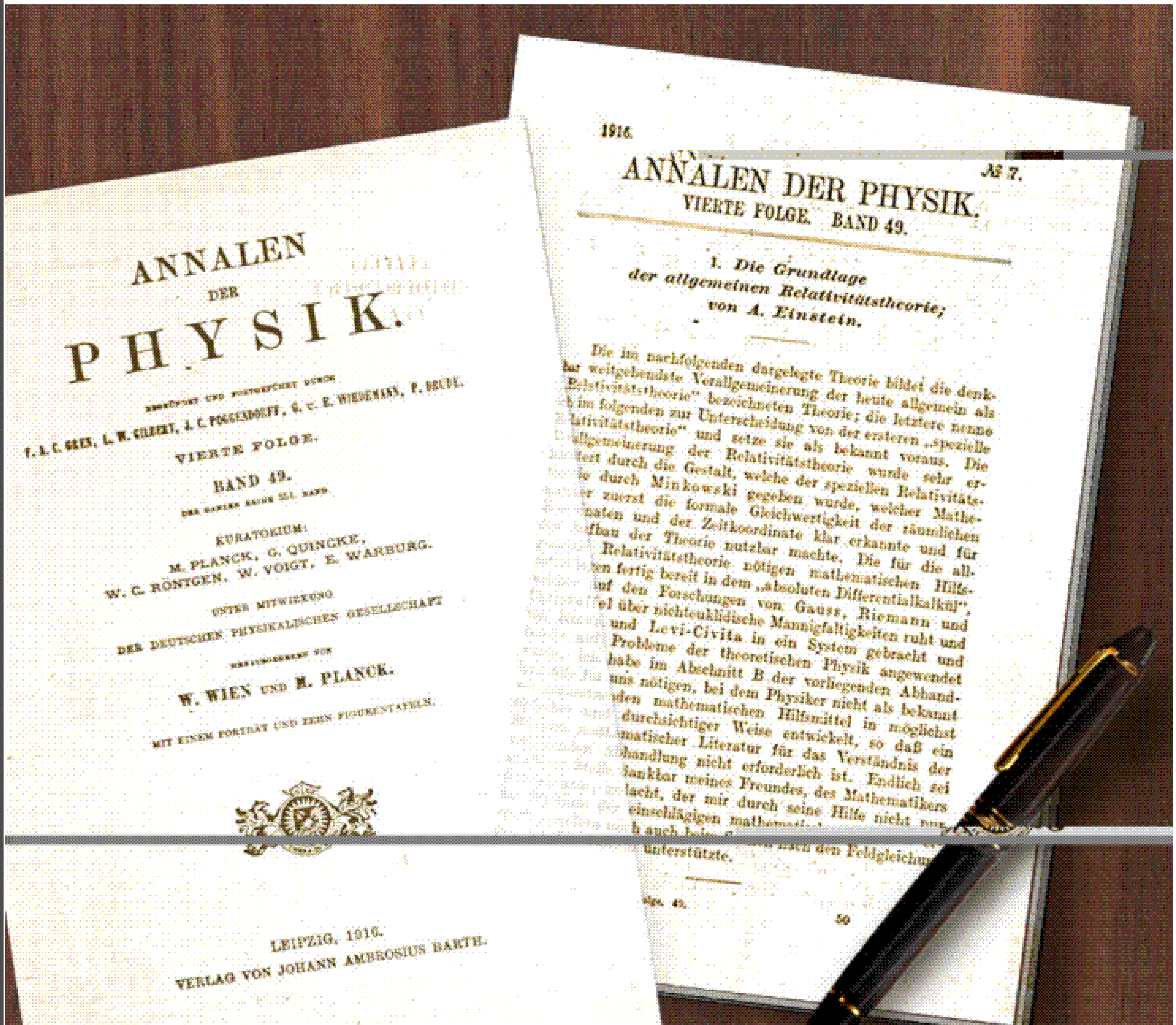
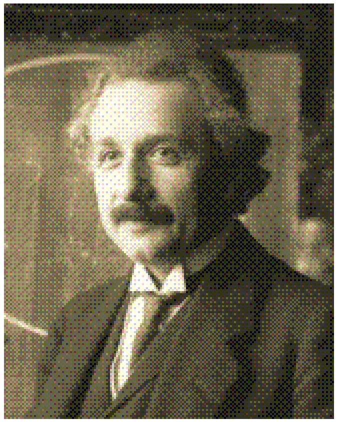
Beispiele ähnlicher Art, sowie die mißlungenen Versuche, eine Bewegung der Erde relativ zum „Lichtmedium" zu konstatieren, führen zu der Vermutung, daß dem Begriffe der absoluten Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften der Erscheinungen entsprechen, sondern daß vielmehr für alle Koordinatensysteme, für welche die mechanischen Gleichungen gelten, auch die gleichen elektrodynamischen und optischen Gesetze gelten, wie dies für die Größen erster Ordnung bereits erwiesen ist. Wir wollen diese Vermutung (deren Inhalt im folgenden „Prinzip der Relativität" genannt werden wird) zur Voraussetzung erheben und außerdem die mit ihm nur scheinbar unverträgliche

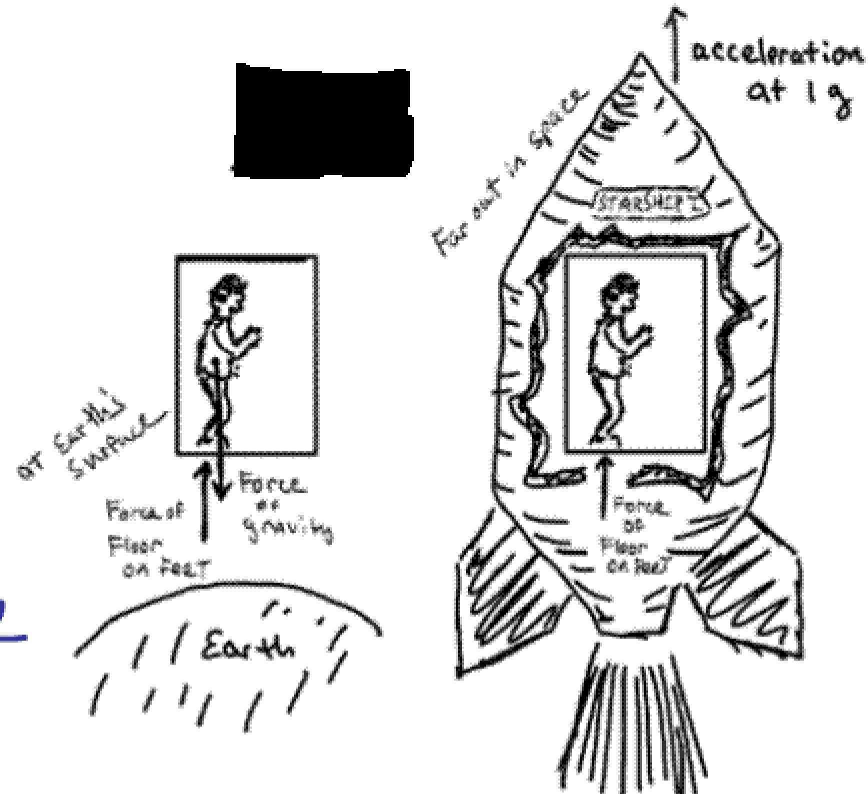
The Theory of General Relativity – Einstein 1916



Gravitation – The general theory of relativity



[redacted]



Equivalence principle

Accelerated reference frame

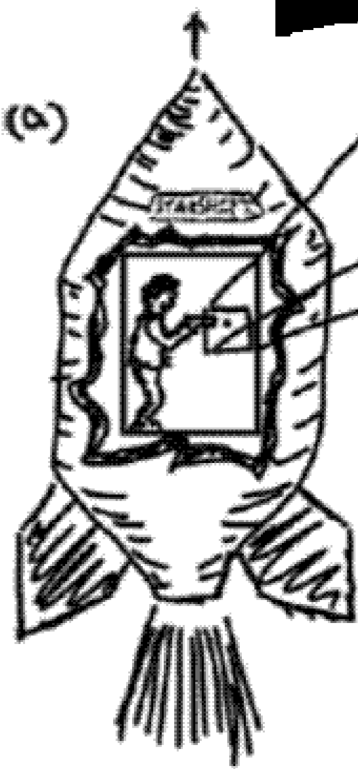
$\equiv$

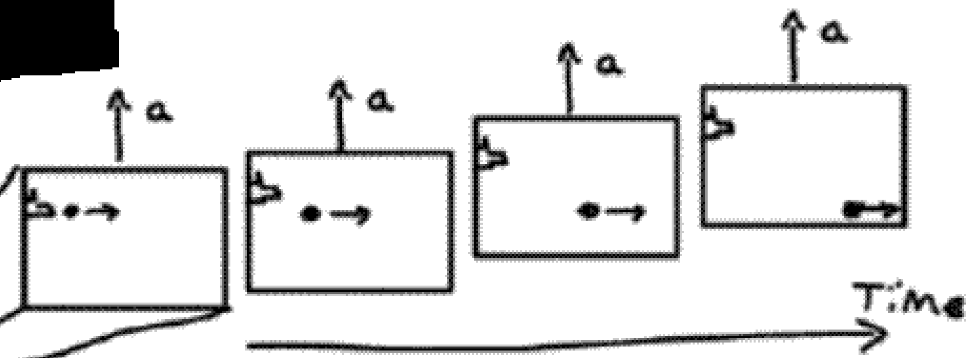
gravitational field

The force of the floor on your feet is the same in both cases. This is what you perceive as your weight.

[redacted]

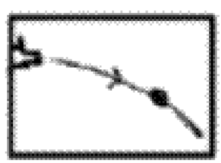
(a)



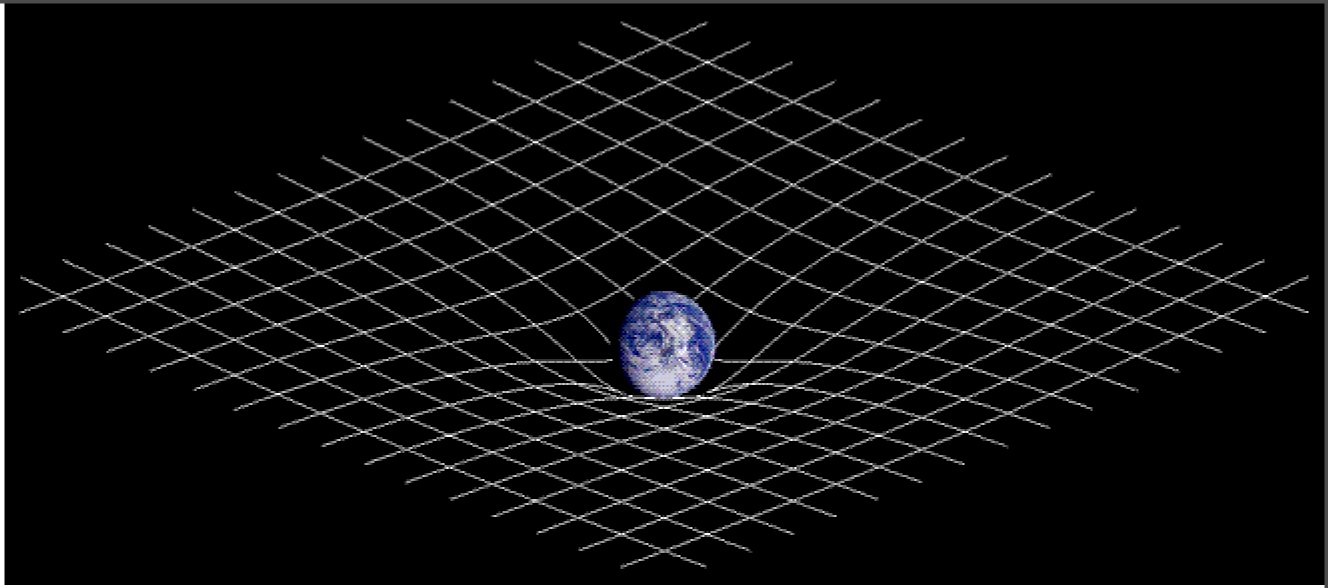


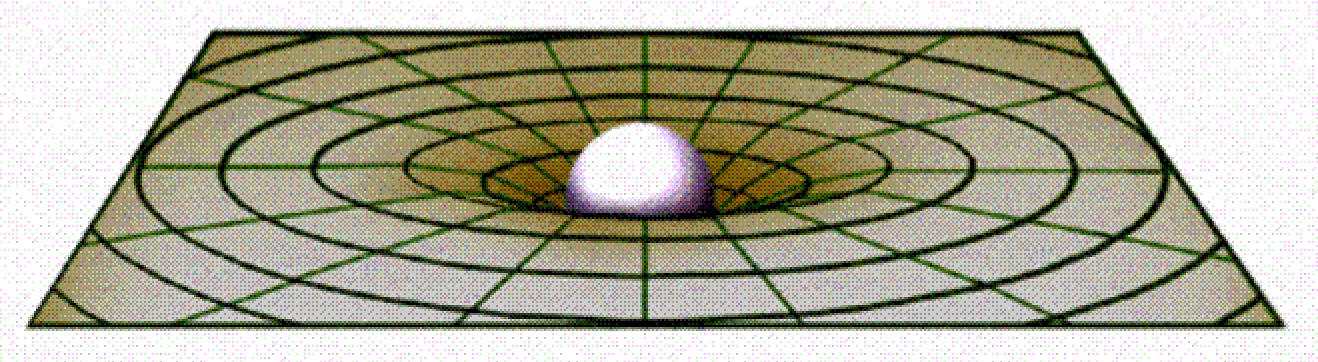
From point of view of observer at rest with respect to rocket ship, the light pulse moves in a straight line while the rocket ship accelerates forward around the pulse.

(b)



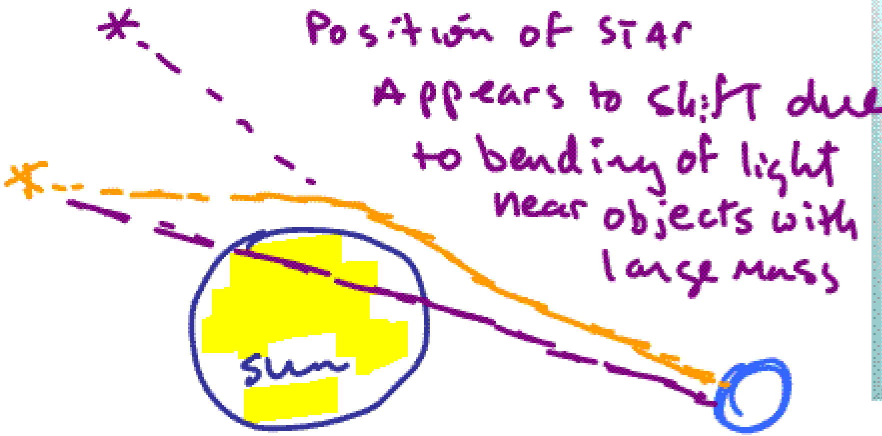
From point of view of observer on the rocket ship, the light pulse seems to travel in a path that curves downward.

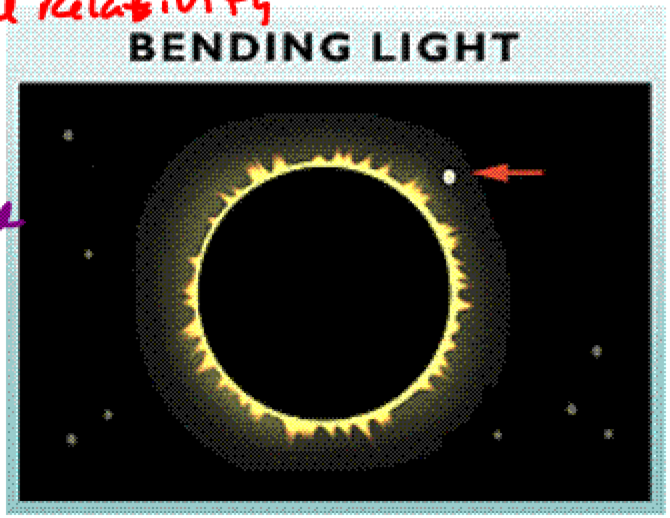




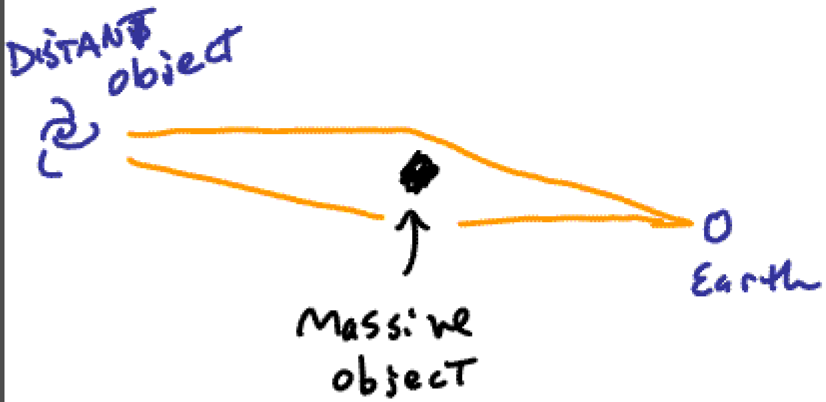
Imagine that mass causes curvature / depression in the fabric of spacetime ... is it true??

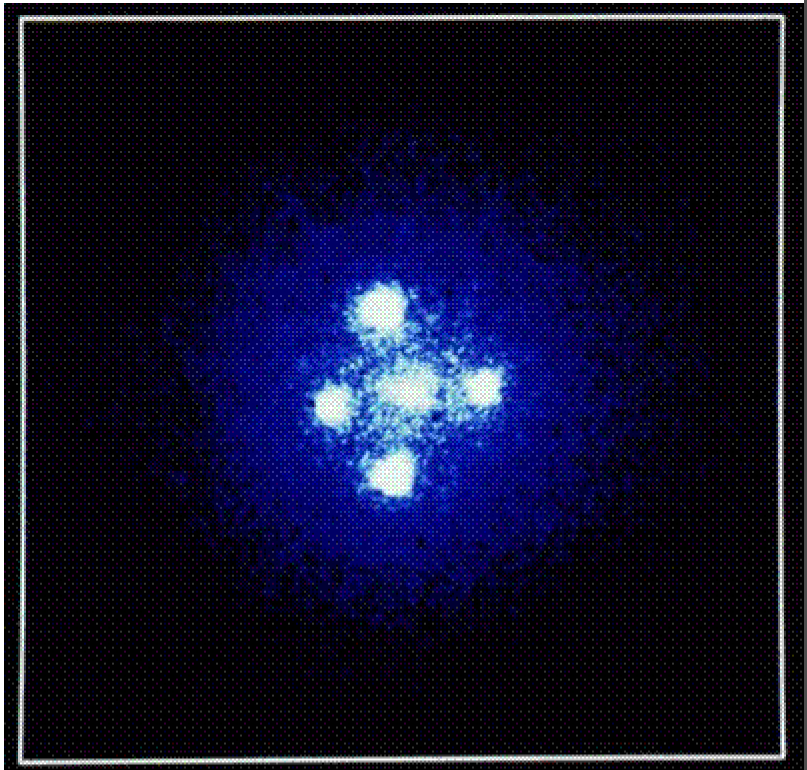
Experimental evidence supporting General Relativity



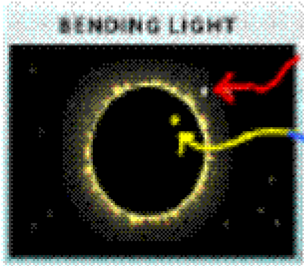


Gravitational Lensing





Gravitational Lens G2237+0305

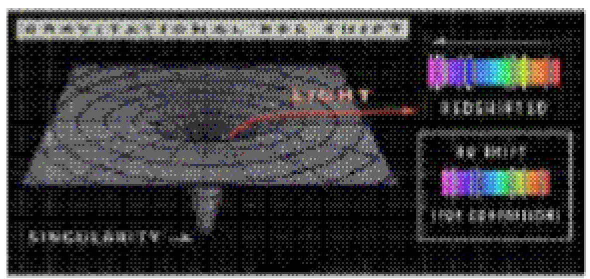


Apparent position

Actual position

- Bending of light by gravitational field ✓

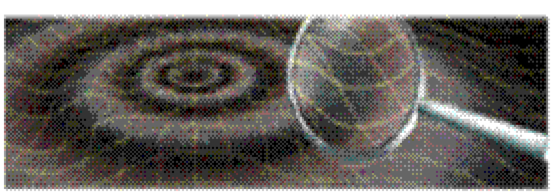
- Gravitational Redshift of light ✓



✓

- Perihelion advance of Mercury

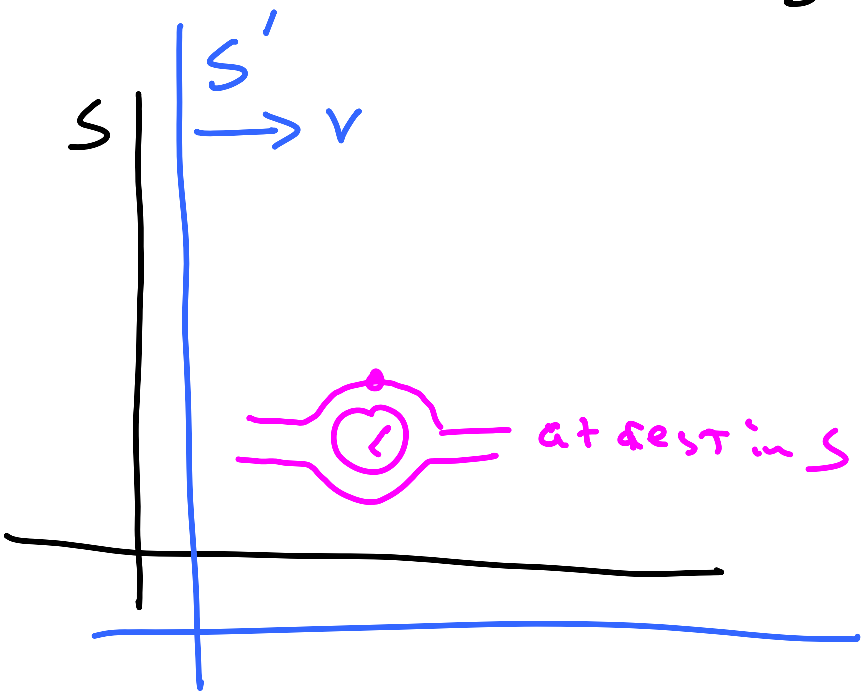
- Gravitational Waves ?



Amplitude $\sim 10^{-16}$ m

LIGO

Special Theory of Relativity



S is proper frame

event at rest

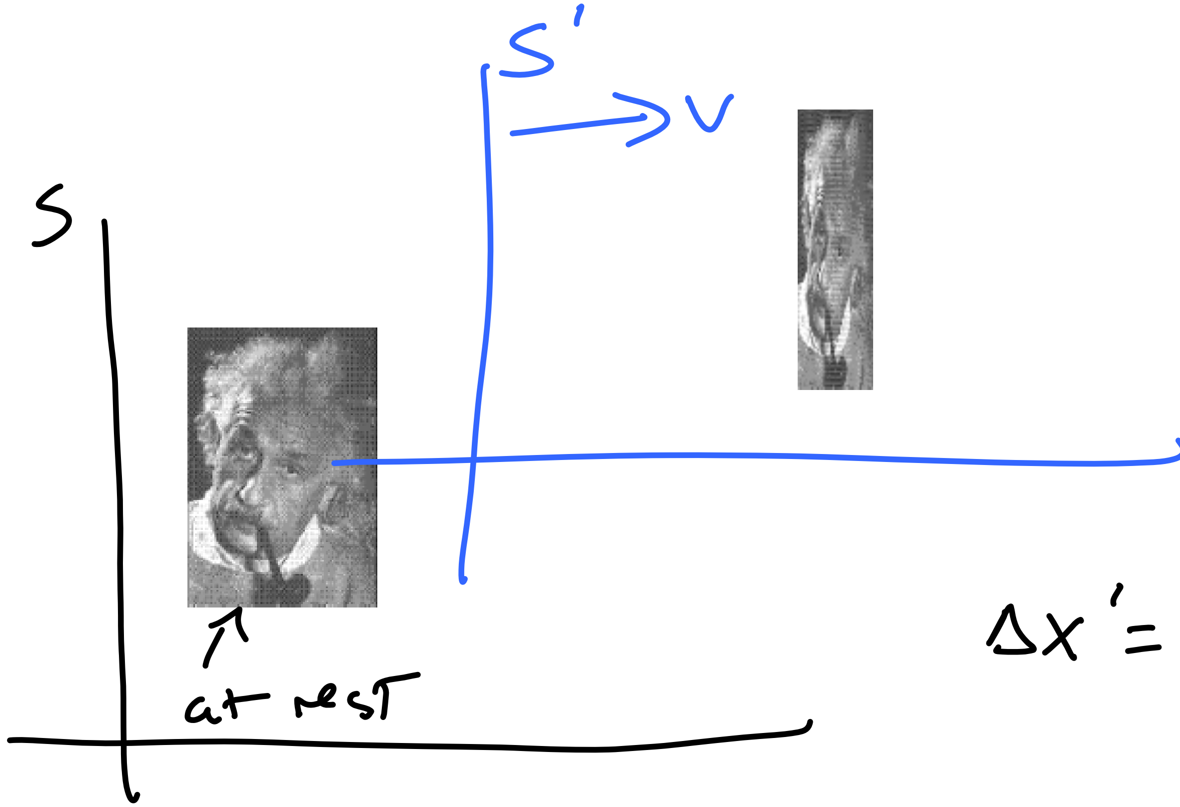
$$\Delta t' = \gamma \Delta t$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

> 1

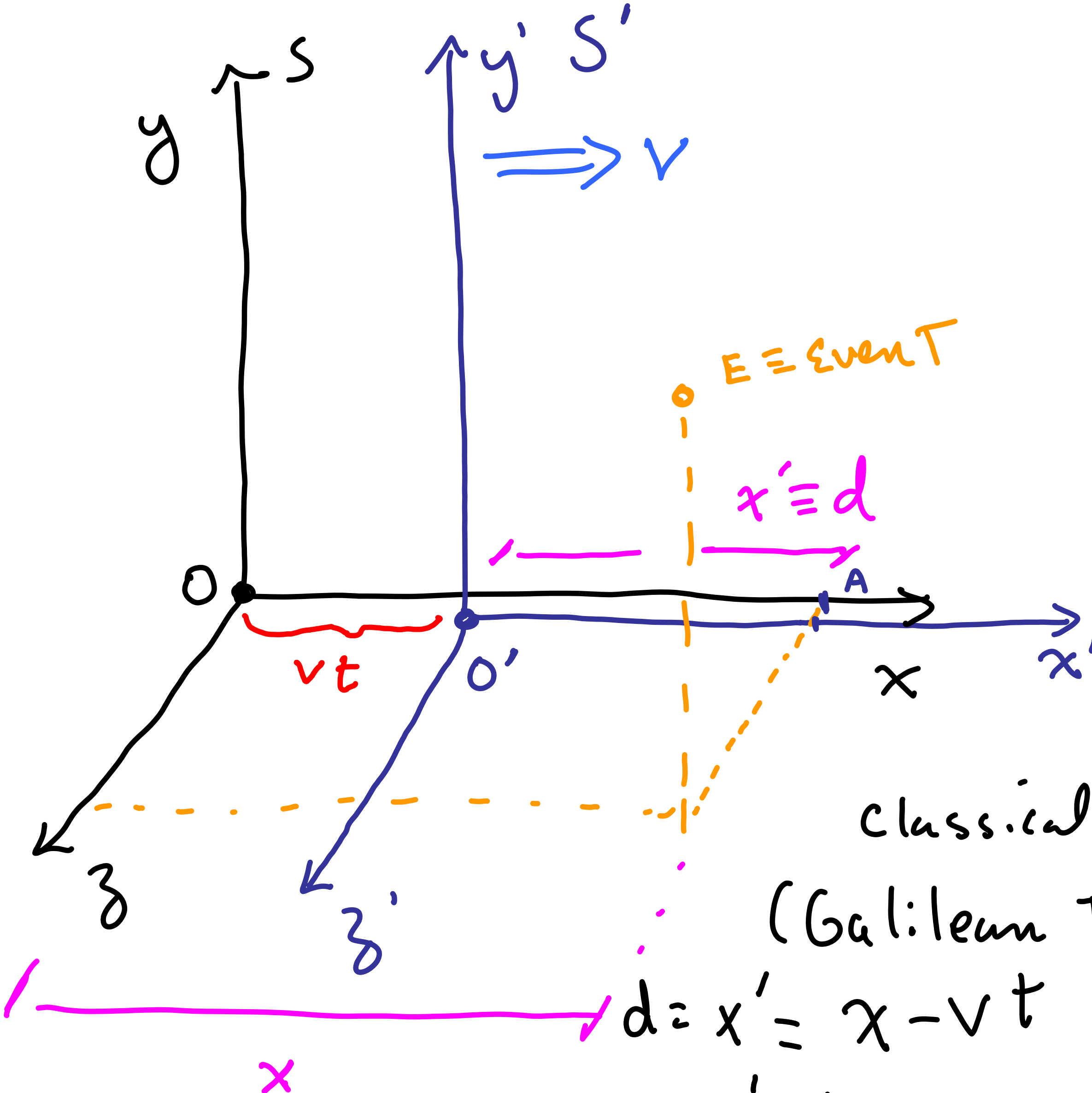
measured time is shortest in <u>proper</u> frame

where event at rest



$$\Delta x' = \frac{\Delta x}{\gamma}$$

Length is greatest in proper frame of reference



at $t=0$, $t'=0$
two systems overlap
$O = O'$

classical physics
(Galilean Transformations)

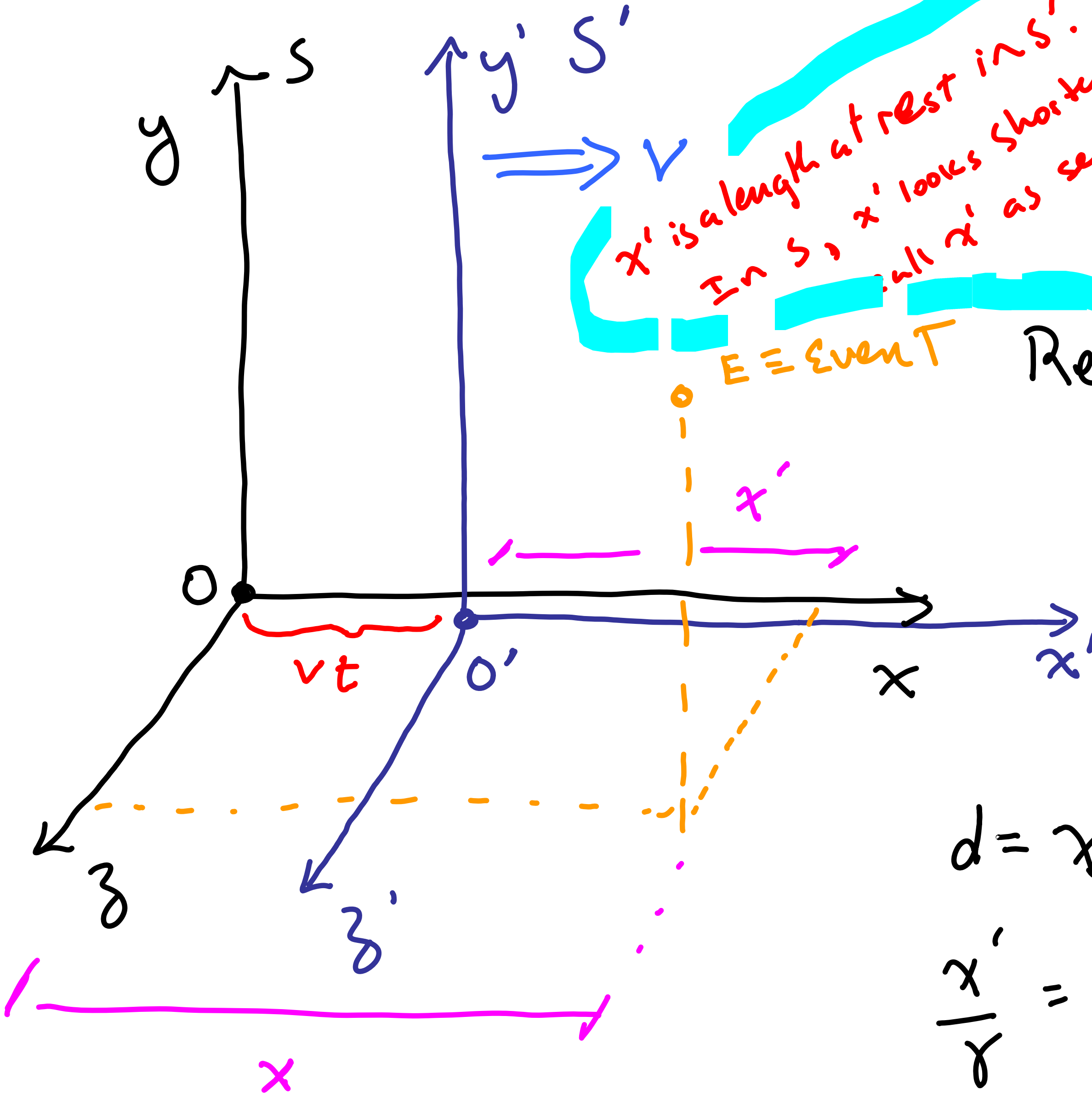
$$d = x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

d is x' as seen in S



x' is a length at rest in S'.
In S, x' looks shorter by 1/γ
call x' as seen in S → d
$d = x'/\gamma$

This is reasoning I didn't recall in class

Relativistic Transformations

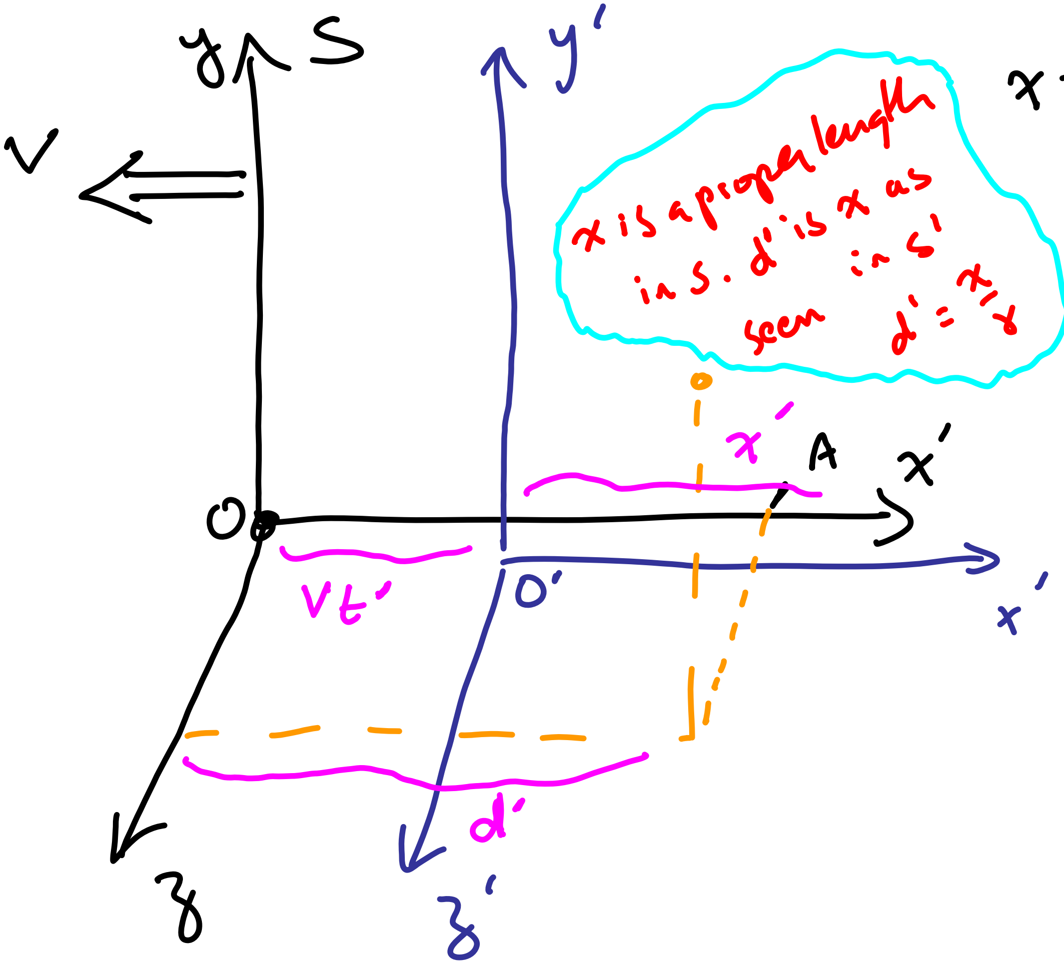
$$d = \frac{x'}{\gamma}$$

$$x' = d\gamma$$

$$d = x - vt$$

$$\frac{x'}{\gamma} = x - vt$$

$$x' = \gamma(x - vt)$$



x = dist from O to A in S

$$d' = \frac{x}{\gamma}$$

$$x' = d' - vt'$$

$$x' = \frac{x}{\gamma} - vt'$$

$$x = \gamma(x' + vt')$$

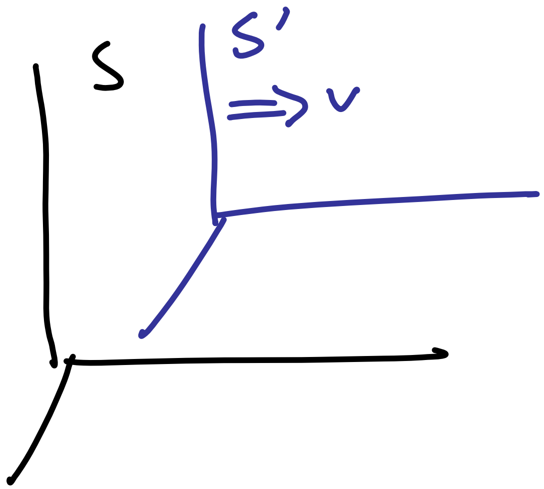
$$x = \gamma(x' + vt') \qquad x' = \gamma(x - vt)$$

substitute in

$$x = \gamma(\gamma(x - vt) + vt']$$

bit of Algebra

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$



Velocity Transformations

$\vec{u} \equiv$ 3 vector velocity in S

$$u_x = dx/dt$$

$$u_y = dy/dt$$

$$u_z = dz/dt$$

$$u_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\gamma\left(\overbrace{\frac{dx}{dt}}^{u_x} - v\right)}{\gamma\left(1 - \frac{v}{c^2}\underbrace{\frac{dx}{dt}}_{u_x}\right)}$$

$$\boxed{u_x' = \frac{\gamma(u_x - v)}{\gamma\left(1 - \frac{v}{c^2}u_x\right)}}$$

$$U_y' = \frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2}dx\right)} = \frac{\frac{dy}{dt}}{\gamma\left(1 - \frac{v}{c^2}\frac{dx}{dt}\right)}$$

U_y

U_x

$$\boxed{U_y' = \frac{U_y}{\gamma\left(1 - U_x v/c^2\right)} \qquad U_z' = \frac{U_z}{\gamma\left(1 - U_x \frac{v}{c^2}\right)}}$$