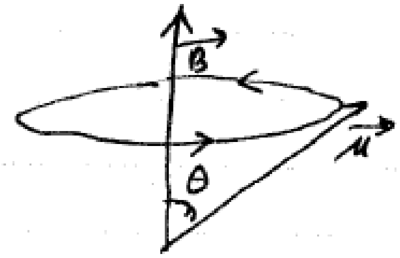
Physics 237, Spring 2008
Homework #10
Due in class, Thursday April 17, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that "questions" can be answered briefly; "problems" may be more involved. Each question usually counts 5 points; each problem counts 10 points.

1. E&R, chapter 8, **question** 2.
2. E&R, chapter 8, **question** 4.
3. E&R, chapter 8, problem 3.
4. E&R, chapter 8, problem 5.
5. E&R, chapter 8, problem 11.

2.)

The Energy of the magnetic dipole in a uniform magnetic field $B = (B_x, B_y, B_z)$ is given by;



$$E = -\vec{\mu} \cdot \vec{B}$$
$$= -(\mu_x B_x + \mu_y B_y + \mu_z B_z$$

Since the dipole moment is a const μ_x, μ_y & μ_z are all constants as well as B_x, B_y & B_z

$$\therefore F = -\nabla E = \mu_x \frac{\partial B_z}{\partial x} + \mu_y \frac{\partial B_y}{\partial y} + \mu_z \frac{\partial B_z}{\partial z}$$
$$= 0 \qquad \because B_x, B_y, B_z \text{ are constants for a uniform magnetic field.}$$

Thus this implies the angle between $\vec{B}$ & $\vec{\mu}$ cannot change.

$$\because \quad E = |\mu||B| \cos\theta$$

$\because$ E is a constant as well as $\vec{\mu}$ & $\vec{B}$ $\cos\theta$ must be constant. The torque which acts on the dipole and is given by;

$\tau = \vec{\mu} \times \vec{B}$ must take account of this constraint that the angle between $\vec{\mu}$ & $\vec{B}$ must remain constant thus leading to precessional motion.

From the experimental observation that the beam of hydrogen atoms is split into two symmetrically deflected components, it is apparent that M_{S_z} can assume just two values, which are equal in magnitude but opposite in sign. If we make the final assumption that the possible values of m_s differ by one and range from $-s$ to $+s$ as is true for quantum numbers m_ℓ & ℓ for orbital angular momentum. Then we can conclude that the two possible values of m_s are

$$m_s = -\frac{1}{2}, \frac{1}{2}$$

$$\therefore -m_s + 1 = m_s$$

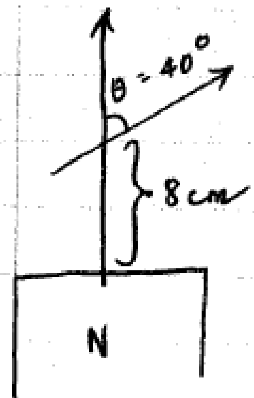
$$\text{or } 2m_s = 1$$

$$m_s = \frac{1}{2} \quad \therefore \quad M_s = \pm\frac{1}{2}$$

$$\therefore S = \frac{1}{2}$$

This is how we conclude that S has to be half integral.

3.



$B(z) = 0.02 + 0.0115 z^2$

$B(z)$ is a non-uniform magnetic field along z axis (z in cm) B in tesla $= \frac{Wb}{m^2}$

(a) $\vec{\tau} = \vec{\mu} \times \vec{B}$ $\quad \vec{\tau} = \vec{\tau}(z)$ since $\vec{B} = \vec{B}(z)$

$|\tau| = \mu B \sin\theta = 1.34 \times 10^{-23} \times (0.756) \times 0.64$

$= 6.51 \times 10^{-24}$ N-m

(b) $|\vec{F}| = \frac{\partial B_z}{\partial z} \mu_z \cos 40^\circ = (0.023 \times 8) \times 1.34 \times 10^{-23}$

$\times \cos 40 / 10^{-2} \left(\frac{\partial}{\partial z} \rightarrow \frac{1}{10^{-2}} \frac{\partial}{\partial z} \right)$

$= 1.88 \times 10^{-22}$ N

(c) $\Delta E = E_f - E_i$. $\quad E = -\vec{\mu} \cdot \vec{B} = \mu_i B_i \cos\theta$

$= -\mu B - (-\mu B \cos 40^\circ$

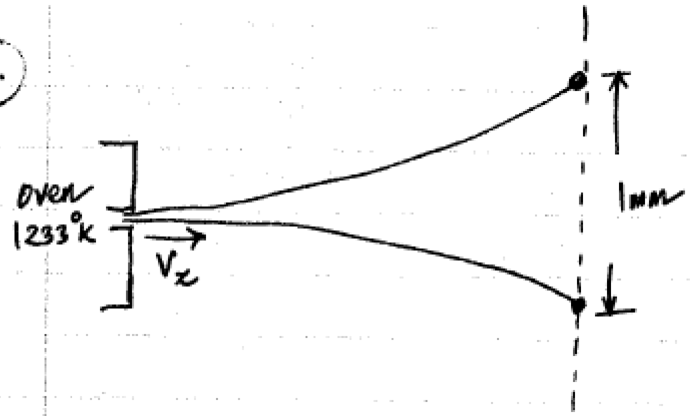
or $\Delta E = \mu B (1 - \cos 40^\circ)$

$= 1.34 \times 10^{-23} \times 0.756 \times 0.23$

$= 0.23 \times 10^{-23}$ J $= \frac{0.23 \times 10^{-23}}{1.6 \times 10^{-19}}$

$= 1.48 \times 10^{-5}$ eV

4.



The mean longitudinal velocity can be found by equi-partition

$$\frac{1}{2} m\overline{v_x^2} = \frac{1}{2} k_B T \sim v_x = \sqrt{\frac{k_B T}{m}}$$

Time to pass the apparatus is then

$$t = \frac{L}{v_x} = L\sqrt{\frac{m}{k_B T}}$$

$$a_z = \frac{F_z}{m} = \frac{1}{m} \pm \left(\frac{\partial B_z}{\partial z}\right) \mu_{e,z}$$

The location of points where the atoms come in

$$z = \pm \frac{1}{2} \left(\cancel{\frac{1}{m}} \frac{\partial B_z}{\partial z} \mu_z\right) \cdot L^2 \frac{\cancel{m}}{k_B T}$$

$$= \pm \frac{1}{2} \frac{\partial B_z}{\partial z} \mu_e \frac{L^2}{k_B T} \quad \sim \quad \Delta z = z_+ - z_-$$

$$= \left(\frac{\partial B_z}{\partial z}\right) \mu_e \frac{L^2}{k_B T}$$

$$\Rightarrow \left(\frac{\partial B_z}{\partial z}\right) = \frac{\Delta z}{\mu_e L^2} k_B T$$

$$= \frac{1\times10^{-3}\,\text{m} \times 1.38\times10^{-23}\,\text{J/K} \times 1233^\circ\text{K}}{0.927\times10^{-23}\,\text{amp-m}^2 \times 0.25\,\text{m}^2}$$

$$= \frac{1.233\times1.38\times4}{0.927}\ \text{Joules/amp-m}^3$$

$$= 7.34\ \text{T/m}.$$