SOLUTIONS H·W #1

PHY-237

Chap-I

Q.16) Show that Planck's constant has the dimension of angular momentum. Does this necessarily suggest that angular momentum is a quantized quantity?

Units for Planck's constant $h \rightarrow Js = kg \times \frac{m}{s^2} \times m \times s$

$$= kg\,m^2 s^{-1} = ML^2T^{-1}$$

Units for Angular momentum $\rightarrow mvr = kg \times \frac{m}{s} \times m = kg\,m^2 s^{-1}$

$$= ML^2T^{-1}$$

$\therefore$ The dimensions of angular momentum & the Planck's constant are similar

This does not necessarily imply that Angular momentum is quantized. For any quantity to be quantized, there are other important considerations to be taken care of before such a statement can be made.

The relation between spectral radiancy $R_T(\nu)$ and energy density $\rho_T(\nu)$ is ;

$$R_T(\nu)\, d\nu = \frac{c}{4}\, \rho_T(\nu)\, d\nu.$$

$$\rho_T(\nu) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$R_T = \int_0^\infty \frac{2\pi h}{c^2} \frac{\nu^3\, d\nu}{e^{h\nu/kT} - 1}$$

Let $\quad q = \frac{h\nu}{kT} \quad ; \quad dq = \frac{h\, d\nu}{kT}$

$$R_T = \int_0^\infty \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4 \frac{q^3\, dq}{e^{q}-1} = \frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{q^3\, dq}{e^{q}-1}$$

$$= \frac{2\pi h}{c^2} \times \left(\frac{kT}{h}\right)^4 \times \frac{\pi^4}{15} = \sigma T^4$$

where $\sigma = \dfrac{2\pi^5 k^4}{15 c^2 h^3}$

Problem 8.) At what wavelength does the human body emit its maximum temperature radiation? List assumptions you make at arriving at the answer.

Assumptions

1.) The human body is the source of thermal radiation. The energy of radiation is proportional to its frequency, high frequency radiation corresponds to high energy radiation, which is in turn proportional to its temperature. Thus, WIEN'S LAW is valid and applicable in this case

2.) The Normal human body temp which is fairly constant and may be assumed as the maximum temperature is 310°K.

$$\nu_{max} \propto T ,$$

$$\frac{c}{\lambda_{max}} \propto T \quad \text{or} \quad \lambda_{max} T = \text{Const}$$
$$= 2.898 \times 10^{-3} \text{ m °K}$$

$$\lambda_{max} = \frac{2.898 \times 10^{-3} \text{ m°K}}{310\text{°K}} = 9.348 \times 10^{-6} \text{ m}$$

Chap-II
Q.9 Does a photon of energy E have mass? If so evaluate it

The photon ~~of energy~~ has a zero rest mass. One may argue that it has a inertial mass by the virtue of its motion.

$$M = \frac{E}{c^2} = \frac{h\nu}{c^2}$$

But, this is not correct then photon with different frequencies will seem to have different masses, which physically does not make much sense.

Prob: 13 What are the frequency, wavelength and momentum of a photon whose energy equals the rest mass energy of an electron?

Rest mass energy of an electron : $E = m_e c^2$

m_e = Electron mass

$m_e = 9.1091 \times 10^{-31}$ kg $\quad c = 3 \times 10^8$ m/s

$E = 9.1091 \times 10^{-31} \times 9 \times 10^{16} = 8.1982 \times 10^{-14}$ J

$h\nu = E$ or $\nu = \frac{E}{h} = \frac{8.1982 \times 10^{-14}}{6.6256 \times 10^{-34}} = 1.2363 \times 10^{20}\ s^{-1}$

$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{1.2363 \times 10^{20}} = 2.4265 \times 10^{-12}$ m $= 2.4265 \times 10^{-2}$ A°

$p = \frac{E}{c} \quad \because E = pc = \cancel{2.732} \quad \frac{8.1982 \times 10^{-14}}{3 \times 10^8}$

$= 2.7327 \times 10^{-22}$ kg m/s.

Chap III

Q.1) Why is the wave nature of matter not more apparent to us in our daily observations?

According to De-Broglie's hypothesis;

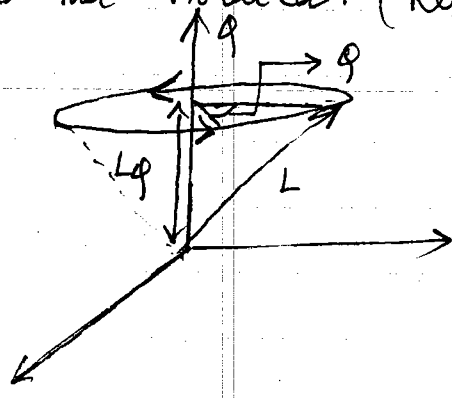
$\lambda = \frac{h}{p}$. p for a normal macroscopic body is much larger than the magnitude of Planck's constant.

Thus, the wavelength associated with the object is really very small to be actually observed or make any sense at all.

~~Q.11)~~ Q.17) The Uncertainty principle is some times stated in terms of angular quantities as $\Delta L_\varphi \Delta \varphi \geq \frac{\hbar}{2}$ where ΔL_φ is the uncertainty in a component of angular momentum and $\Delta\varphi$ is the uncertainty in the corresponding angular position. In some quantum mechanical system the angular momentum is measured to have a definite quantized magnitude. Does this contradict the statement of Uncertainty Principle?

$\Delta L_\varphi \Delta \varphi \geq \frac{\hbar}{2}$

$\because L_\varphi$ has definite quantized values $\therefore \Delta L_\varphi = 0$
According to the uncertainty principle this implies that $\Delta\varphi \geq \infty$. Thus, there is no certainty in the value of φ. Thus at any instant for a particular value of L_φ, φ can take on any value. This thing is realised in nature as the precession of the angular momentum vector along the φ direction keeping L_φ constant. Thus, Uncertainty Principle is not violated. (Ref: Vector Atom Model)



Prob 32

The energy of a linear harmonic oscillator is

$$E = \frac{p_c^2}{2m} + \frac{Cx^2}{2}$$

(a) Show using Uncertainty relation, that this can be written as $E = \dfrac{h^2}{32\pi^2 m x^2} + \dfrac{1}{2} C x^2$

(b) Then show that the minimum energy of the oscillator is $\dfrac{h\nu}{2}$

The energy of a Simple Harmonic Oscillator is;

$$E = \frac{p_x^2}{2m} + \frac{1}{2} C x^2$$

From the uncertainty relation, $\Delta p_x \, \Delta x \geq \frac{\hbar}{2} = \frac{h}{4\pi}$.

At the lower limit, $\Delta p_x \, \Delta x = \frac{h}{4\pi}$.

$$(\Delta p_x)^2 = \overline{p_x^2} - (\bar{p}_x)^2$$
$$(\Delta x)^2 = \overline{x^2} - (\bar{x})^2$$

$\bar{x}$ and $\bar{p}$ denotes the mean value of position and momentum for a harmonic oscillator. From symmetry, it is easy to see that $\bar{x} = 0$ and $\bar{p} = 0$
Hence

$$(\Delta p_x)^2 (\Delta x)^2 = \frac{h^2}{16\pi^2} = \overline{p_x^2} \, \overline{x^2}$$

$$p_x^2 = \frac{h^2}{16\pi^2} \frac{1}{x^2}$$ (Omit bars for notational purposes)

$$E = \frac{h^2}{32\pi^2 x^2 m} + \frac{1}{2} C x^2$$ or similarly

$$E = \frac{p_x^2}{2m} + \frac{1}{2} C \frac{h^2}{16\pi^2 p_x^2}$$

(b) Now, Energy is a function of X only; $E(x) = \frac{h^2}{32\pi^2 x^2 m} + \frac{1}{2}cx^2$

$$\frac{dE(x)}{dx} = \frac{-2h^2}{32\pi^2 x^3 m} + Cx$$

For minimum

$$\frac{dE(x)}{dx} = 0 \quad \& \quad \frac{d^2E(x)}{dx^2} > 0$$

$\therefore$ for $\frac{dE(x)}{dx} = 0$ $\qquad x^4 = \frac{h^2}{16\pi^2 cm}$

$$x^2 = \frac{h}{4\pi\sqrt{cm}}$$

$$\frac{d^2E}{dx^2} = \frac{3\times 2h^2}{32\pi^2 x^4} + C$$

$\because c > 0 \ \& \ x^4 > 0$

$$\therefore \frac{d^2E}{dx^2} > 0$$

$$\therefore E(x)_{min} = \frac{h^2}{32\pi m} \times \frac{4\pi\sqrt{cm}}{h} + \frac{1}{2}c \times \frac{h}{4\pi\sqrt{cm}}$$

$$= \frac{h}{8\pi}\sqrt{\frac{c}{m}} + \frac{h}{8\pi}\sqrt{\frac{c}{m}}$$

$$= \frac{h}{2}\,\frac{1}{2\pi}\sqrt{\frac{c}{m}}$$

$$= \frac{h\nu}{2}$$

Chap 4
Q: 12

Can hydrogen atom absorb a photon whose energy exceeds its binding energy, 13.6 ev?

Yes, the hydrogen atom can absorb a photon whose energy exceeds its binding energy and get ionized in the process. The ejected electron carries the excess energy as its Kinetic Energy. This is so because ~~Energy quantization does~~ outside the atom, the electron is free and can take on any energy value, thus any photon of energy exceeding 13.6 ev may be absorbed. Same cannot be said for energies below 13.6 ev, then the photon absorbed depends on the difference between the energy values of the stationary energy levels of the hydrogen atom.

Prob 14

Compare the gravitational attraction of an electron and proton in the ground state of a hydrogen atom to the Coulomb attraction. Are we justified in ignoring the gravitational force?

$$F_G = \frac{G M_e M_p}{R^2} \qquad F_c = \frac{e^2}{4\pi\epsilon_0 R^2}$$

$$\therefore \frac{F_G}{F_c} = \frac{G M_e M_p}{e^2} \times 4\pi\epsilon_0$$

$G = 6.67\times10^{-11}\ Nm^2/kg^2$

$m_e = 9.109 \times 10^{-31}$ kg

$m_p = 1.672 \times 10^{-27}$ kg

$4\pi\epsilon_0 = \frac{1}{9\times10^9}$

$e = 1.6\times10^{-19}$ Coulombs

$$= \frac{6.67\times10^{-11}\times9.109\times1.672\times\cancel{10^{-58}}\ 10^{-20}}{2.56\times\cancel{10^{-38}}\times9\times10^9}$$

$$= \frac{6.67\times9.109\times1.672}{2.56\times9}\times10^{-40}$$

$$= 4.4\times10^{-40}$$

$\therefore F_c \gg F_G.$

Thus, ignoring Gravitational attraction is justified.