Physics 237, Spring 2008
Homework #3
Due in class, Thursday Feb. 14, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that "questions" can be answered briefly; "problems" may be more involved.

1. E&R, chapter 5, **question** 27.
2. E&R, chapter 5, **question** 32.
3. E&R, chapter 5, problem 11.
4. E&R, chapter 5, problem 12.
5. E&R, chapter 5, problem 13.
6. E&R, chapter 5, problem 14, part a only.
7. E&R, chapter 5, problem 15.
8. E&R, chapter 5, problem 22, part a and b only.

(27) Explain in two or three words how the quantization of energy is related to the well behaved character of acceptable eigenfunctions

In order to be an acceptable solution an eigenfunction $\psi(x)$ and its derivative $\frac{d\psi(x)}{dx}$ are required to have the following properties;

(i) finite (ii) single valued (iii) continuous.

Schrödinger equation like any other differential equation will have a wide variety of solutions but of all the possible solutions only those that obey the above criteria are acceptable. This distinct choice of wavefunctions from a set of many leads to energy quantization.

(32) If a particle is not bound in a potential, its total energy is not quantized. Does this mean potential has no effect on the behavior of the particle? What effect would you expect it to have?

No, this certainly does not mean that the particle is unaffected by the potential. A particle's energy is quantized ~~because~~ for a bound system because;

(i) Region I — $V(x) < E$ — ψ is oscillatory
(ii) Region II — $E < V(x)$ — ψ is exponentially decaying

The continuity of the eigenfunctions in these two regions is possible only for certain solutions of the Schrödinger equation leading to quantization of energy.

Now, when a particle is unbound, $E > V(x)$ always, as a result the wavefunction is oscillatory throughout. The question of quantization does not arise because in this case we do not have the trouble of equating two dissimilar eigenfunctions at the boundaries. Thus, all solutions are allowed and one does not have quantization.

This obviously does not mean that the potential does not affect the particle because the particle may be slowed down as it approaches the potential or may be its speed enhanced as it leaves it.

SOLUTIONS

PHY 237 H·W #3

Calculate the expectation value of p, and the expectation value of p^2, for the particle associated with the wavefunction in Prob 10 (i.e

$$\Psi(x,t) = \sqrt{\frac{2}{a}}\ \sin\frac{2\pi x}{a}\ e^{-iEt/\hbar} \qquad -\frac{a}{2} < x < \frac{a}{2}$$

$$= 0 \qquad \text{otherwise}$$

$$\bar{p} = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar\frac{\partial}{\partial x}\right)\Psi\ dx$$

$$= \int_{-a/2}^{a/2} \sqrt{\frac{2}{a}}\ \sin\frac{2\pi x}{a}\ e^{+iEt/\hbar}\ (-i\hbar)\ \times\sqrt{\frac{2}{a}}\left(\frac{2\pi}{a}\right)\ \cos\frac{2\pi x}{a}\ e^{-iEt/\hbar}\ dx$$

$$= (-i\hbar)\frac{2}{a}\int_{-a/2}^{a/2} \frac{2\pi}{a}\ \cos\frac{2\pi x}{a}\ \sin\frac{2\pi x}{a}\ dx$$

$$= -i\hbar\ \frac{4\pi}{a}\int_{a/2}^{a/2} \cos\frac{2\pi x}{a}\ \sin\frac{2\pi x}{a}\ dx$$

$\cos\frac{2\pi x}{a}$ is an even function, $\sin\frac{2\pi x}{a}$ is an odd func^n

Therefore, the integrand is an odd function and thus the integral vanishes

$$\therefore\ \bar{p} = 0$$

$$= \frac{2}{a}\left[\frac{a^3}{24} - \frac{1}{2}\left\{ + x^2 \sin\frac{4\pi x}{a}\left(\frac{a}{4\pi}\right)\Big|_{-a/2}^{a/2} - 2\frac{a}{4\pi}\left(-x\cos\frac{4\pi x}{a}\left(\frac{a}{4\pi}\right) + \sin\frac{4\pi x}{a}\left(\frac{a}{4\pi}\right)^2\right)\right\}\right.$$

$$= \frac{2}{a}\left[\frac{a^3}{24} - \frac{1}{2}\left\{ x^2 \sin\frac{4\pi x}{a}\left(\frac{a}{4\pi}\right)\Big|_{-a/2}^{a/2} + 2\left(\frac{a}{4\pi}\right)^2 x\cos\frac{4\pi x}{a}\Big|_{-a/2}^{a/2} - 2\left(\frac{a}{4\pi}\right)^3 \sin\frac{4\pi x}{a}\right\}\Big|_{-a/2}^{a/2}\right]$$

$$= \frac{2}{a}\left[\frac{a^3}{24} - \frac{1}{2}\left\{ 2\left(\frac{a^2}{4\pi}\right)^2\left(\frac{a}{2} - \left(-\frac{a}{2}\right)\right)\right\}\right]$$

$$= \frac{2}{a}\left[\frac{a^3}{24} - \frac{1}{2}\left\{ \frac{2(a^2)^2}{16\pi^2} \times a\right\}\right]$$

$$= \frac{2}{a}\left[\frac{a^3}{24} - \frac{2a^3}{32\pi^2}\right] = \frac{1}{a}\left(\frac{a^3}{12} - \frac{a^3}{8\pi^2}\right)$$

$$= \left(\frac{a^3}{12} - \frac{a^3}{8\pi^2}\right)$$

$$\therefore \boxed{\overline{x^2} = \frac{1}{a}\left(\frac{a^3}{12} - \frac{a^3}{8\pi^2}\right)}$$

(11.) Calculate the expectation value of x, and the expectation value of x^2 for

$$\psi(x,t) = \sqrt{\frac{2}{a}}\ \mathrm{Sin}\ \frac{2\pi x}{a}\ e^{-iEt/\hbar} \qquad -\frac{a}{2} < x < \frac{a}{2}$$

$$= 0 \qquad x < -\frac{a}{2} \text{ or } x > \frac{a}{2}$$

$$\bar{x} = \int_{-\infty}^{\infty} \psi^*(x,t)\ x\ \psi(x,t)\, dx$$

$$= \int_{-a/2}^{a/2} \frac{2}{a}\ \mathrm{Sin}^2 \frac{2\pi x}{a}\ x\ dx$$

$\mathrm{Sin}^2 \frac{2\pi x}{a}$ is an even function & $x\ \mathrm{Sin}^2 \frac{2\pi x}{a}$ is an odd function therefore the integral is zero.

$$\therefore \bar{x} = 0$$

$$\overline{x^2} = \int_{-a/2}^{a/2} \frac{2}{a}\ \mathrm{Sin}^2 \frac{2\pi x}{a}\ x^2\, dx$$

$$= \frac{2}{a} \int_{-a/2}^{a/2} x^2\ \mathrm{Sin}^2 \frac{2\pi x}{a}\, dx$$

$$= \frac{2}{a} \int_{-a/2}^{a/2} \frac{x^2}{2}\left[1 - \mathrm{Cos}\ \frac{4\pi x}{a}\right] dx$$

$$= \frac{2}{a}\left[\frac{x^3}{6}\Bigg|_{-a/2}^{a/2} - \frac{1}{2}\int_{-a/2}^{a/2} x^2\ \mathrm{Cos}\ \frac{4\pi x}{a}\, dx \right]$$

$$= \frac{2}{a}\left[\frac{1}{6}\ \frac{a^3}{8} \times 2 \ -\frac{1}{2}\left\{ +x^2\ \mathrm{Sin}\ \frac{4\pi x}{a}\left(\frac{a}{4\pi}\right)\Bigg|_{-a/2}^{a/2} - \frac{2a}{4\pi}\int x\ \mathrm{Sin}\ \frac{4\pi x}{a}\, dx \right\}\right.$$

$$\overline{p^2} = \int_{-\infty}^{\infty} \psi^* (-i\hbar)^2 \frac{\partial^2}{\partial x^2} \psi \, dx$$

$$= \cancel{\int_{-a/2}^{a/2} \sqrt{\frac{2}{a}} (-i\hbar)^2 \frac{\partial^2}{\partial x^2}}$$

$$= \int_{-a/2}^{a/2} \sqrt{\frac{2}{a}} \sin\frac{2\pi x}{a} \, e^{iEt/\hbar} \, (-i\hbar)^2 \frac{\partial^2}{\partial x^2} \sqrt{\frac{2}{a}} \cos\frac{2\pi x}{a} \, e^{-iEt/\hbar} \, dx$$

$$= \frac{2}{a}\int_{-a/2}^{a/2} \sin\frac{2\pi x}{a} \times (-\hbar^2) \times \left(-\left(\frac{2\pi}{a}\right)\right)^2 \times \sin\frac{2\pi x}{a} \, dx$$

$$= \frac{+8\pi^2}{a^3} \times \hbar^2 \int_{-a/2}^{a/2} \sin^2\frac{2\pi x}{a} \, dx$$

$$= \frac{8\pi^2 \hbar^2}{a^3} \int_{-a/2}^{a/2} \frac{\left(1 - \cos\frac{4\pi x}{a}\right)}{2} \, dx$$

$$= \frac{8\pi^2 \hbar^2}{a^3} \int_{-a/2}^{a/2} \frac{dx}{2} - \cancel{\int_{-a/2}^{a/2} \frac{\cos 4\pi x \, dx}{2}} \rightarrow 0$$

$$= \frac{8\pi^2 \hbar^2}{a^3} \times \frac{a}{2} = \frac{4\pi^2 \hbar^2}{a^2}$$

$$\therefore \quad \overline{p^2} = \frac{4\pi^2 \hbar^2}{a^2}$$

(13.)

(a) Use quantities calculated in the preceding two problems to calculate the product of uncertainties in position and momentum of a particle in the first excited state of the system being considered

(b) Compare the uncertainty product when the particle is in the lowest energy state of the system. Explain why the uncertainty products differ.

(a) $(\overline{\Delta x})^2 = \overline{x^2} - (\bar{x})^2$

$(\overline{\Delta p})^2 = \overline{p^2} - (\bar{p})^2$

$\therefore (\overline{\Delta x})^2 = \overline{x^2}$ & $(\overline{\Delta p})^2 = \overline{p^2}$; $\overline{\Delta x} \cdot \overline{\Delta p} = \sqrt{\overline{x^2} \cdot \overline{p^2}}$

or $\overline{\Delta x}\ \overline{\Delta p} = \sqrt{\overline{x^2} \cdot \overline{p^2}} = \sqrt{\frac{A^3}{\cancel{a}}\left(\frac{1}{12} - \frac{1}{8\pi^2}\right) \times \frac{4\pi^2\hbar^2}{\cancel{A^2}}}$

$= 1.67\hbar$

or $\overline{\Delta x} \cdot \overline{\Delta p} = 1.67\hbar.$

(b)

$E = \frac{p^2}{2m}$; Since this wavefunction corresponds to the excited state (1st) of the system, thus it is higher than the ground state energetically, increase in E leads to an increase in $\overline{p^2}$ which is equal to $(\Delta p)^2$

thus increasing the uncertainty p. It is to be noted that the wavefunction for the 1st excited state has more wiggles (oscillations) than that for the ground state. Greater the no: of oscillations more is the uncertainty of finding the particle in a certain position thus leading to an increase in the uncertainty Δx. Thus, the uncertainty product is higher in this case as compared to the uncertainty product in the ground state which was $0.57\hbar$.

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(a) Calculate the expectation value of the K.E & the P.E for a particle in the lowest energy state of a Simple Harmonic Oscillator, using the wavefunction

$$\Psi = \frac{(Cm)^{1/8}}{(\pi\hbar)^{1/4}} e^{-\left(\frac{\sqrt{Cm}}{2\hbar}\right)x^2} e^{-\frac{i}{2}\sqrt{\frac{C}{m}}t}$$

$$E = \frac{p^2}{2m} + \frac{1}{2}Cx^2$$

$$K\cdot E = \frac{p^2}{2m} \qquad \therefore \quad \overline{K\cdot E} = \frac{\overline{p^2}}{2m} \quad \& \quad \overline{P\cdot E} = \frac{1}{2}C\overline{x^2}$$

$$\Psi = Ae^{-\alpha x^2}e^{-i\beta t} \qquad A = \frac{(Cm)^{1/8}}{(\pi\hbar)^{1/4}} \; ; \; \alpha = \frac{\sqrt{Cm}}{2\hbar} \; ; \; \beta = +\frac{1}{2}\sqrt{\frac{C}{m}}$$

$$\overline{p^2} = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar\frac{\partial}{\partial x}\right)^2 \Psi \, dx$$

$$= \int_{-\infty}^{\infty} A e^{-\alpha x^2} e^{+i\beta t} (-i\hbar)^2 \frac{\partial^2}{\partial x^2} A e^{-\alpha x^2} e^{-i\beta t} \, dx$$

$$= \int_{-\infty}^{\infty} A^2 e^{-\alpha x^2} \times \left\{ \frac{\partial}{\partial x} \left[(-i\hbar)^2 (-2x\alpha) e^{-\alpha x^2} \right] \right\} dx$$

$$= \int_{-\infty}^{\infty} A^2 e^{-\alpha x^2} \left[(-i\hbar)^2 (-2\alpha) e^{-\alpha x^2} + (-i\hbar)^2 (-2\alpha x)(-2\alpha x) e^{-\alpha x^2} \right] dx$$

$$\overline{p^2} = \int_{-\infty}^{\infty} A^2 e^{-\alpha x^2}\left[2\alpha\hbar^2 e^{-\alpha x^2} - \hbar^2 (2\alpha)^2 x^2 e^{-\alpha x^2}\right] dx$$

$$= \int_{-\infty}^{\infty} A^2 (2\alpha\hbar^2)\, e^{-2\alpha x^2} dx - \int_{-\infty}^{\infty} A^2 (2\alpha\hbar)^2 x^2 e^{-2\alpha x^2} dx$$

$$= 2\alpha\hbar^2 \underbrace{\int_{-\infty}^{\infty} A^2 e^{-2\alpha x^2} dx}_{\mathrm{I}} - (2\alpha\hbar)^2 \underbrace{\int_{-\infty}^{\infty} A^2 x^2 e^{-2\alpha x^2} dx}_{\mathrm{II}} \quad \dots\dots (a)$$

Term I

$$\int_{-\infty}^{\infty} A^2 e^{-2\alpha x^2} dx = \int_{-\infty}^{\infty} \psi^*(x)\,\psi(x)\, dx = \int_{-\infty}^{\infty} |\psi|^2 dx = 1 \quad \dots\dots (1)$$

Term II

$$\int_{-\infty}^{\infty} A^2 x^2 e^{-2\alpha x^2} dx$$

~~Now;~~ $\cancel{\int_{-\infty}^{\infty} \frac{\partial}{\partial\alpha}\left(A^2 x^2 e^{-2\alpha x^2}\right) = 2A\frac{\partial A}{\partial \alpha}}$

Now; $$\frac{\partial}{\partial\alpha}\left(A^2 e^{-2\alpha x^2}\right) = 2A\frac{\partial A}{\partial\alpha} e^{-2\alpha x^2} + A^2(-2x^2)\, e^{-2\alpha x^2}$$

$$\therefore \quad 2A\frac{\partial A}{\partial\alpha} e^{-2\alpha x^2} - \frac{\partial}{\partial\alpha}\left(A^2 e^{-2\alpha x^2}\right) = 2A^2 x^2 e^{-2\alpha x^2}$$

or $$A^2 x^2 e^{-2\alpha x^2} = \frac{1}{2}\left[2A\frac{\partial A}{\partial\alpha} e^{-2\alpha x^2} - \frac{\partial}{\partial\alpha}\left(A^2 e^{-2\alpha x^2}\right)\right]$$

$$\int_{-\infty}^{\infty} A^2 x^2 e^{-2\alpha x^2}\,dx = \frac{1}{2}\; 2A\,\frac{\partial A}{\partial \alpha}\int_{-\infty}^{\infty} e^{-2\alpha x^2}\,dx$$

$$-\;\frac{1}{2}\cancelto{0}{\frac{\partial}{\partial\alpha}\int_{-\infty}^{\infty} A^2 e^{-2\alpha x^2}\,dx}$$

Now $\displaystyle\int_{-\infty}^{\infty} A^2 e^{-2\alpha x^2}\,dx = \int_{-\infty}^{\infty} |\psi|^2\,dx = 1$

$$\therefore\quad -\frac{1}{2}\frac{\partial}{\partial\alpha}(1) = 0$$

$$\frac{1}{2}\;2A\,\frac{\partial A}{\partial\alpha}\int_{-\infty}^{\infty} e^{-2\alpha x^2}\,dx = \frac{1}{A}\frac{\partial A}{\partial\alpha}\int_{-\infty}^{\infty} A^2 e^{-2\alpha x^2}\,dx$$

$$= \frac{1}{A}\frac{\partial A}{\partial\alpha}$$ (for the same reason again)

$$\therefore\quad \int_{-\infty}^{\infty} A^2 x^2 e^{-2\alpha x^2} = \frac{\partial \ln A}{\partial\alpha}$$

$$A = \frac{(Cm)^{1/8}}{(\pi\hbar)^{1/4}} = \frac{(Cm)^{1/8}}{\pi^{1/4}}\,\frac{2^{1/4}}{(2\hbar)^{1/4}}$$

$$= \left(\frac{2}{\pi}\right)^{1/4}\left(\frac{\sqrt{Cm}}{2\hbar}\right)^{1/4}$$

But $\displaystyle\frac{\sqrt{Cm}}{2\hbar} = \alpha \quad\therefore\quad A = \left(\frac{2}{\pi}\right)^{1/4}\alpha^{1/4}$

$$\frac{\partial \ln A}{\partial\alpha} = \frac{1}{4\alpha}$$

$$\therefore\quad \int_{-\infty}^{\infty} A^2 x^2 e^{-2\alpha x^2} = \frac{1}{4\alpha}\quad \cdots\cdots \quad (2)$$

Substituting (1) & (2) in (a)

one has;

$$\overline{p^2} = 2\alpha\hbar^2 - (2\alpha\hbar)^2 \times \frac{1}{4\alpha}$$

$$= 2\alpha\hbar^2 - \alpha\hbar^2 = \alpha\hbar^2$$

$$\therefore \overline{K \cdot E} = \frac{\overline{p^2}}{2m} = \frac{\alpha\hbar^2}{2m} = \frac{\hbar^2}{2m}\frac{\sqrt{cm}}{2\hbar}$$

$$= \frac{\hbar}{4}\sqrt{\frac{c}{m}}$$

$$\therefore \overline{K \cdot E} = \frac{\hbar}{4}\sqrt{\frac{c}{m}}$$

$$\overline{P \cdot E} = \frac{1}{2} c \overline{x^2}$$

$$\overline{x^2} = \int_{-\infty}^{\infty} \psi^* x^2 \psi \, dx$$

$$= \int_{-\infty}^{\infty} A^2 e^{-\alpha x^2} e^{i\beta t} x^2 e^{-\alpha x^2} e^{i\beta t} dx$$

$$= \int_{-\infty}^{\infty} A^2 e^{-2\alpha x^2} x^2 \, dx$$

As calculated before in (2) this integral gives $\frac{1}{4\alpha}$

$$\therefore \overline{P.E} = \frac{1}{2}\frac{c}{4\alpha} = \frac{1}{8}\frac{c}{\sqrt{cm}} \times 2\hbar = \frac{\hbar}{4}\sqrt{\frac{c}{m}}$$

$$\therefore \overline{P.E} = \frac{\hbar}{4}\sqrt{\frac{c}{m}}.$$

15.) In calculating the expectation value of the product of the position times momentum, an ambiguity arises because it is not apparent which of the two expressions.

$$\overline{xp} = \int_{-\infty}^{\infty} \Psi^* x \left(-i\hbar \frac{\partial}{\partial x}\right) \Psi \, dx$$

$$\overline{px} = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) x \Psi \, dx$$

should be used

(a) Show that neither is acceptable because both violate the obvious requirement that $\overline{xp}$ should be real since it is measurable (b) Show that the expression;

$$\overline{xp} = \int_{-\infty}^{\infty} \Psi^* \frac{\left[x\left(-i\hbar \frac{\partial}{\partial x}\right) + \left(-i\hbar \frac{\partial}{\partial x}\right) x\right]}{2} \Psi \, dx$$

is acceptable because it does satisfy the requirement.

(a) $\overline{xp} = \int_{-\infty}^{\infty} \psi^* x \left(-i\hbar \frac{\partial}{\partial x}\right) \psi \, dx = -i\hbar \int_{-\infty}^{\infty} \psi^* x \frac{\partial \psi}{\partial x} \, dx$

Take complex conjugate of $\overline{xp}$

$$\overline{xp}^* = i\hbar \int_{-\infty}^{\infty} \psi x \frac{\partial \psi^*}{\partial x} \, dx.$$

Evaluate this integral; $I = \int_{-\infty}^{\infty} \psi^* x \frac{\partial \psi}{dx} \, dx$ using the condition that $\psi(x)$ is a square integrable function. [i.e $\int_{-\infty}^{\infty} |\psi|^2 dx < \infty$ ie as $|x| \to \infty$, $\psi(x)$ must vanish faster than $\frac{1}{\sqrt{x}}$]

$$I = \int_{-\infty}^{\infty} \underbrace{\psi^* x}_{u} \underbrace{\frac{\partial \psi}{\partial x}}_{v} \, dx = \psi^* x \psi \Big|_{-\infty}^{\infty} - \int \psi \frac{\partial}{\partial x}(\psi^* x) \, dx$$

The first term vanishes since $\psi(x)$ as $x \to \infty$ has at least $\frac{1}{x^{1/2+\varepsilon}}$ form. So $\psi^* x \psi \sim \frac{1}{x^{1+2\varepsilon}} x \sim \frac{1}{x^{2\varepsilon}} \to 0$ as $x \to \infty$

$$I = -\int \psi \frac{\partial}{\partial x}(\psi^* x) \, dx = -\int \psi \left(\psi^* + x \frac{\partial \psi^*}{\partial x}\right) dx$$

$$= -\left[\int_{-\infty}^{\infty} \psi \psi^* dx + \int_{-\infty}^{\infty} \psi x \frac{\partial \psi^*}{\partial x}\right]$$

$$\overline{xp} = i\hbar + i\hbar \underbrace{\int_{-\infty}^{\infty} \psi \frac{x \, \partial \psi^*}{\partial x}}_{\longrightarrow \; \overline{xp}^*}$$

$\therefore \quad \overline{xp} = i\hbar + \overline{xp}^* \qquad$ i.e $\qquad \boxed{\overline{xp} \ne \overline{xp}^*}$

Using the same arguments; let us find $\overline{px}$

$$\overline{px} = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar\frac{\partial}{\partial x}\right) x \psi \, dx = -i\hbar \int_{-\infty}^{\infty} \psi^* \frac{\partial}{\partial x}(x\psi) dx$$

$$= -i\hbar \left[\cancel{\psi^* x \psi \Big|_{-\infty}^{\infty}}^{\,\to 0} - \int \psi x \frac{\partial \psi^*}{\partial x} dx \right]$$

$$= i\hbar \int \psi x \frac{\partial \psi^*}{\partial x} dx$$

$$\overline{px} = -\overline{xp}$$

So; $\overline{px}^* = -\overline{xp}^* = -(\overline{xp} - i\hbar) = i\hbar - \overline{xp} = i\hbar + \overline{px}$;

Hence $\qquad \boxed{\overline{px} \ne \overline{px}^*}$

So, both constructions, $\overline{xp}$ or $\overline{px}$ are not real, But the quantity that we are searching is real.

(b) Let us try the symmetric combination

$$(\overline{xp})_s = \frac{1}{2}(\overline{xp} + \overline{px})$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \psi^* \left(x \left(-i\hbar\frac{\partial}{\partial x}\right) + \left(-i\hbar\frac{\partial}{\partial x} x\right) \right] \psi(x) \, dx$$

$$\overline{xp}^* = \frac{1}{2} \left\{ \overline{xp}^* + \overline{px}^* \right.$$

$$= \frac{1}{2} \left\{ \overline{xp} - i\hbar + i\hbar + \overline{px} \right\}$$

$$= \frac{1}{2}\left(\overline{xp} + \overline{px}\right)$$

Hence $(\overline{xp})_s = (\overline{xp})_s^*$

The expectation value of the symmetric combination is a real quantity.

(a) The curvature of ψ is proportional to $|V-E|$: where $|V-E|$ is large the function oscillates rapidly in x, and where $|V-E|$ is small it oscillates less rapidly (hence nodes are closer in the former case and further apart in latter case). In the first case, $|V-E|$ is just large enough to turn ψ over : no nodes. The 10 th state will have $10-1=9$ nodes leading to an odd function since V is symmetrical about the origin. The wavefunction decays exponentially where $V > E$, the classically forbidden region.