Physics 237, Spring 2008
Homework #4
Due in class, Thursday Feb. 21, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that "questions" can be answered briefly; "problems" may be more involved.

1. E&R, chapter 5, problem 23.
2. E&R, chapter 5, problem 27.
3. E&R, chapter 5, problem 28.
4. E&R, chapter 6, **question** 5.
5. E&R, chapter 6, **question** 11.
6. E&R, chapter 6, problem 2.
7. E&R, chapter 6, problem 3.
8. E&R, chapter 6, problem 4.

23')

(a) $E < V_0$: no allowed energy values

(b) $V_0 < E < V_1$: discretely separated

(c) $V_1 < E < V_2$: " "

(d) $V_2 < E < V_3$: continuously distributed

(e) $V_3 < E$: " "

(27.) The Schrödinger equation is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

In the region in question, $V = V_0 =$ constant. $E < V_0$ so that

$$q^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0.$$

Hence;

$$\psi = Ae^{-qx} + Be^{qx}$$

is the general solution. However, $\psi(x=\infty) = 0$, requiring $B = 0$, leaving

$$\psi = Ae^{-qx}$$

as the wavefunction.

$$q = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

(28.)

$$|\Psi|^2_{x=a/2} = f \quad ; \quad |\Psi|^2_{x=\frac{a}{2}+D} = g$$

$$\frac{f}{g} = e \Rightarrow \text{or} \quad \frac{\cancel{A^2}\, e^{-2\sqrt{\frac{2m(V_0-E}{\hbar}}\,\frac{a}{2}}}{\cancel{A^2}\, e^{-2\sqrt{\frac{2m(V_0-E}{\hbar^2}}\left(\frac{a}{2}+D\right)}} = \left(\frac{1}{e}\right)^{-1}$$

$$\text{or} \quad e^{+2\sqrt{\frac{2m(V_0-E)}{\hbar}}\,D} = e$$

$$\text{or} \quad D\left(2\sqrt{\frac{2m(V_0-E)}{\hbar}}\right) = 1$$

$$\therefore \boxed{D = \frac{\hbar}{2\sqrt{2m(V_0-E}}}$$

SOLUTIONS H.W # 6

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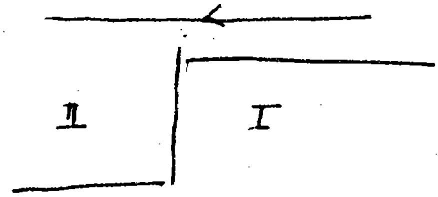
(5) A discontinuous potential function is a reasonable approximation to an actual system whenever the potential function changes substantially within a short distance. A good example is the potential energy function for a charged particle moving along the axis of a system of 2 electrodes, separated by a very narrow gap, which are held at different voltages.

(1P) The statement that the reflection coefficient is one for a particle incident on a potential step with total energy less than step hieght implies that a particle will never get transferred across a step potential function. This in agreement with the predictions of classical mechanics

Since ; $R+T=1$, this implies that we have a finite value for T as there is a finite probability for a particle to get reflected.

$T<0$ implies negative ~~partic~~ probability or for $R>1$ more number of particles are reflected than those incident. Extra particles cannot be created out of nowhere.

The Schrödinger equation is given by ;



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$\therefore$ for a particular E

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi \quad \text{for } x < 0$$

&

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi \quad \text{for } x > 0.$$

$\therefore$ For $x < 0$

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E-V_0)\psi \qquad \because E > V_0 \quad \frac{2m(E-V_0)}{\hbar^2} = k_1^2 > 0$$

$$\Rightarrow \frac{d^2\psi}{dx^2} = -k_1^2\psi$$

$$\therefore \psi_I = Ae^{ik_1x} + Be^{-ik_1x}$$

Similarly for $x > 0$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2} \quad \because E > 0 \quad \therefore k_2^2 = \frac{2mE}{\hbar^2}$$

$$\psi_{II} = Ce^{ik_2x} + De^{-ik_2x}$$

Now for a wave incident from the right in the second region $x>0$ there won't be any wave travelling to the right $\therefore C=0$ in $\underline{\Psi_{II}}$

$\therefore$ Appling the condition of continuity for the wavefunction & its derivatives we has;

at $x=0$

$$A+B = D \quad \rightarrow \quad A+B = D \qquad \cdots \quad (a)$$

$$ik_1(A-B) = -ik_2 D \qquad B-A = \frac{k_2}{k_1} D \qquad \cdots \quad (b)$$

$(a)+(b)$ yields

$$2B = \frac{k_2+k_1}{k_1} D \qquad \text{or} \qquad \frac{2k_1}{k_1+k_2} B = D$$

Similarly;

$$-(a) + \frac{k_1}{k_2}(b)$$

$$\cancel{-(k_2-k_1)A + B(k_1+k_2) = 0}$$

$$-A - B + \frac{k_1 B}{k_2} - \frac{k_1 A}{k_2} = 0$$

$$\text{or} \quad -A(k_1+k_2) + B(k_1-k_2) = 0$$

$$\text{or} \quad \frac{B(k_1-k_2)}{k_1+k_2} = A.$$

$$\therefore \quad A = \frac{B(k_1 - k_2)}{k_1 + k_2} \quad ; \quad D = \frac{2k_1}{k_1 + k_2} B$$

$$R = \frac{|\vec{J}_{refl}|}{|\vec{J}_{inc}|} = \frac{v_1 A^* A}{v_1 B^* B} = \frac{A^* A}{B^* B} = \frac{(k_1 - k_2)^*}{(k_1 + k_2)^*} \frac{(k_1 - k_2)}{(k_1 + k_2)}$$

$$= \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$$

$$T = \frac{|\vec{J}_{trans}|}{|\vec{J}_{inc}|} = \frac{v_2 D^* D}{v_1 B^* B} = \frac{v_2}{v_1} \left(\frac{2k_1}{k_1 + k_2} \right)^2$$

Since $v_1 = \frac{p_1}{m} = \frac{\hbar k_1}{m}$, $v_2 = \frac{p_2}{m} = \frac{\hbar k_2}{m}$

$$T = \frac{k_2}{k_1} \left(\frac{2k_1}{k_1 + k_2} \right)^2 = \frac{4 k_1 k_2}{(k_1 + k_2)^2}$$

The transmission & reflection coefficients are thus same as those obtained from sec 6.4.

3. $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$; $T = \frac{4 k_1 k_2}{(k_1 + k_2)^2}$

$$R + T = \frac{(k_1 - k_2)^2 + 4 k_1 k_2}{(k_1 + k_2)^2} = \frac{\cancel{(k_1 + k_2)^2}}{\cancel{(k_1 + k_2)^2}} = 1$$

4. $\left(\frac{k_1-k_2}{k_1+k_2}\right)^2 = R$ & $T = \frac{4k_1k_2}{(k_1+k_2)^2}$

where $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$ & $k_2 = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

$$R = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2 = \left(\frac{\sqrt{\frac{2mE}{\hbar^2}} - \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}{\sqrt{\frac{2mE}{\hbar^2}} + \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}\right)^2$$

$$= \left(\frac{\left(\sqrt{2mE} - \sqrt{2m(E-V_0)}\right)\left(\sqrt{2mE}\right)}{\left(\sqrt{2mE} + \sqrt{2m(E-V_0)}\right)\left(\sqrt{2mE}\right.}\right)^2$$

$$= \left(\frac{2mE - 2mE\sqrt{1-\frac{V_0}{E}}}{2mE + 2mE\sqrt{1-\frac{V_0}{E}}}\right)^2$$

$$\therefore R = \left(\frac{1 - \sqrt{1-\frac{V_0}{E}}}{1 + \sqrt{1-\frac{V_0}{E}}}\right)^2$$

$$T = \frac{4k_1k_2}{(k_1+k_2)^2}$$

$$\therefore T = \frac{4 \times \sqrt{\frac{2mE}{\hbar^2}} \times \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}{\left(\sqrt{\frac{2mE}{\hbar^2}} + \sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2} = \frac{\left(\sqrt{\frac{2m}{\hbar^2}}\right)^2 \times 4\sqrt{1-\frac{V_0}{E}}}{\left(\sqrt{\frac{2m}{\hbar^2}}\right)^2\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}$$

$$\therefore T = \frac{4\sqrt{1-\frac{V_0}{E}}}{\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}$$