Physics 237, Spring 2008
Homework #5
Due in class, Thursday Feb. 28, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that "questions" can be answered briefly; "problems" may be more involved.

1. E&R, chapter 6, **question** 14.
2. E&R, chapter 6, **question** 15.
3. E&R, chapter 6, problem 6.
4. E&R, chapter 6, problem 9.
5. E&R, chapter 6, problem 16.
6. E&R, chapter 6, problem 20.
7. E&R, chapter 6, problem 21.
8. E&R, chapter 6, problem 23.

14. The fallacy lies in the fact that the particle is eventually detected outside. This could have only taken place only if the process actually happens.

15. The reflection coefficient 4 reflection involves both the potential discontinuities because it depends on the width of the potential barrier as well as the fact the reflection coefficient ~~resonates~~ oscillates, at large energies, because of interference in reflections from its two discontinuities

If the other discontinuity is change to a step, the reflection coefficient is ~~too~~ equal to one for energies less than the hieght of the potential step

$$\therefore T = \frac{4 \times \sqrt{\frac{2mE}{\hbar^2}} \times \sqrt{\frac{2m(E-V_0)}{\hbar^2}}}{\left(\sqrt{\frac{2mE}{\hbar^2}} + \sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2} = \frac{\left(\sqrt{\frac{2m}{\hbar^2}}\right)^2 \times 4\sqrt{1-\frac{V_0}{E}}}{\left(\sqrt{\frac{2m}{\hbar^2}}\right)^2\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}$$

$$\therefore T = \frac{4\sqrt{1-\frac{V_0}{E}}}{\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}$$

5.

$$T = \left\{1 + \frac{\left(e^{k_2 a} - e^{-k_2 a}\right)^2}{16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)}\right\}^{-1} \quad \text{from } 6\text{-}49$$

$$= \left[\frac{16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right) + \left(e^{k_2 a} - e^{-k_2 a}\right)^2}{16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)}\right]^{-1}$$

$$= \left[\frac{16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right) + e^{2k_2 a} + e^{-2k_2 a} + 2}{16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)}\right]^{-1}$$

when $k_2 a$ is large $e^{-2k_2 a} \to 0$ $\quad e^{2k_2 a} \gg 2$ & $16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)$

$$\therefore \quad T = \left[\frac{e^{2k_2 a}}{16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)} \right]^{-1}$$

$$= 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right) e^{-2k_2 a}$$

9.

(a) Since both the proton and the neutron have the same energies the less massive one will have a higher velocity and as a result will hit the walls of the potential well more often and will thus have a higher probability of tunnelling past the barrier. The less massive in this case is the proton and it will have a higher tunnelling probability.

(b) The probability of penetrating the potential barrier can be obtained by calculating the transmission coefficient of the particle

The transmission coefficient is;

$$T = \left[1 + \frac{\sinh^2 ka}{4\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)} \right]^{-1}; \quad k_2 a = \sqrt{\frac{2mV_0 a^2}{\hbar^2}\left(1-\frac{E}{V_0}\right)}$$

For a proton;

$M_p = 1.672 \times 10^{-27}$ kg ; $V_0 = 10$ MeV $= 10 \times 10^6$ ev $\times 1.602 \times 10^{-19}$

$a = 10^{-14}$ m , $\hbar = 1.055 \times 10^{-34}$ $\quad \times 10^{6} \times 10$

$= 1.602 \times 10^{-12}$ J

$$k_2 a = \sqrt{\frac{2 m V_0 a^2}{\hbar^2}\left(1 - \frac{E}{V_0}\right)}$$

$$= \sqrt{\frac{2 \times 1.672 \times 10^{-27} \times 1.602 \times 10^{-12} \times 10^{-28}}{(1.055 \times 10^{-34})^2}\left(1 - \frac{3}{10}\right)}$$

$$= 5.804$$

$$T_p = \left[1 + \frac{\sinh^2 k_2 a}{4\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)}\right]^{-1} = \left[1 + \frac{\sinh^2 5.804}{4 \times \frac{3}{10} \times \left(1 - \frac{3}{10}\right)}\right]$$

$$= 3.05249 \times 10^{-5}$$

For a deuteron

$M_d = 2 m_p = 3.344 \times 10^{-27}$ kg

$V_0 = 1.602 \times 10^{-12}$ J

$a = 10^{-14}$ m

$\hbar = 1.055 \times 10^{-34}$ J

$$k_2 a = \sqrt{\frac{2 \times 3.344 \times 1.602 \times 10^{-12} \times 10^{-28}}{(1.055 \times 10^{-34})^2} \times \left(1 - \frac{3}{10}\right)}$$

$$= \sqrt{2} \times 5.804$$

$$= 8.2087$$

$$T_d = \left[1 + \frac{\operatorname{Sech}^2 8.2087}{4 \times \frac{3}{10} \times \left(1 - \frac{3}{10}\right)} \right]^{-1}$$

$$= 2.49073 \times 10^{-7}$$

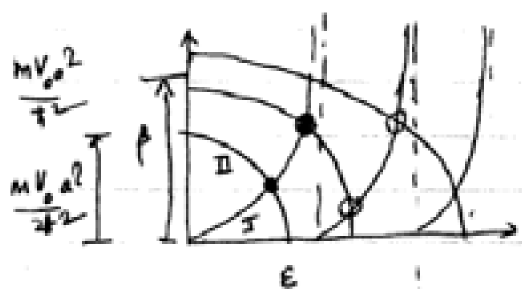
4) 6-16

(6)

For a 1-D square well potential as shown in Q.1) the eigen functions and eigen values depend on the solution of the transcedental equation given by;

$$\underset{\mathrm{I}}{\varepsilon \tan \varepsilon} = \underset{\mathrm{II}}{\sqrt{\frac{mV_0a^2}{2\hbar^2} - \varepsilon}} \cdot = \beta$$

Graphically, these two expressions I & II are plotted on the same graph and the number of possible intersections give the total number of possible eigenstates. Now as seen graphically as V_0a^2 increases, the number of intersections also increases. Thus, the strength determining factor of the well is the product V_0a^2. As already stated in Q.1) the square well has at least one even eigenstate



The dots indicate the possible eigenvalues.

5.)

6-20. (a) The lowest energy state $n=1$, ψ has no nodes. Hence ψ_I must correspond to $n=2$, ψ_{II} to $n=3$. Since $E_n \propto n^2$ hence $E_I = 4\,eV$

$$\frac{E_{II}}{E_I} = \frac{3^2}{2^2} \;;\; E_{II} = 9\,eV$$

(b) By the same analysis;

$$\frac{E_0}{E_I} = \frac{1^2}{2^2} \;;\quad E_0 = 1\,eV.$$

6

6.21) (a) The zero point energy for a neutron in a nucleus, assuming the nucleus to be an infinite square well is;

$$E_n = \frac{\pi^2\hbar^2}{2m a^2} = \frac{(3.141596)^2 \times (1.055 \times 10^{-34})^2 J^2 s^2}{2 \times 1.675_{kg} \times 10^{-27} \times 10^{-28} m^2}$$

$$= 3.279 \times 10^{-13} J$$

$$= 2.05\ MeV$$

(b) This is much less than the zero point energy of the electron, thereby indicating that it is easier to accomodate neutrons inside the nucleus in comparision to the less massive electrons.

7.

6-23 (a)

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

and therefore energy of the adjacent level is;

$$E_{n+1} = \frac{(n+1)^2\pi^2\hbar^2}{2ma^2}$$

So that;

$$E_{n+1} - E_n = \left\{(n+1)^2 - n^2\right\}\frac{\pi^2\hbar^2}{2ma^2}$$

$$= \left\{2n+1\right\}\frac{\pi^2\hbar^2}{2ma^2} = \Delta E_n$$

$$\therefore \frac{\Delta E_n}{E_n} = \frac{(2n+1)\cancel{\frac{\pi^2\hbar^2}{2ma^2}}}{n^2\cancel{\frac{\hbar^2\pi^2}{2ma^2}}} = \frac{2n+1}{n^2}$$

(b) In the classical limit $n \to \infty$; but

$$\lim_{n \to \infty} \frac{\Delta E_n}{E_n} = \lim_{n \to \infty} \frac{2n+1}{n^2} = 0$$

meaning that the energy levels get so close together as to be indistinguishable. Hence, quantum effects are not apparent.