Physics 237, Spring 2008
Homework #6
Due in class, Thursday March 6, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that "questions" can be answered briefly; "problems" may be more involved. Each question usually counts 5 points; each problem counts 10 points.

1. E&R, chapter 6, **question** 30. (10 points)
2. E&R, chapter 6, problem 30.
3. E&R, chapter 6, problem 31.
4. E&R, chapter 6, problem 33.
5. E&R, chapter 6, problem 34.
6. E&R, Appendix I, problem 1.
7. E&R, Appendix I, problem 2.


diatomic linear molecule.

The energy expression for the diatomic molecule is;

$$E = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2} c (x_1 - x_2)^2$$

Use centre of mass and relative co-ordinates to extract vibrational energy.

$$x = x_{real} = x_1 - x_2$$
$$X = X_{cm} = \frac{x_1 + x_2}{2}$$

In this co-ord.

$$E = \frac{P^2}{2(2m)} + \frac{p^2}{2\left(\frac{m}{2}\right)} + \frac{1}{2} c x^2$$

Total mass $M = 2m$ , reduced mass is $\mu = \frac{m}{2}$

$\frac{P^2}{2(2m)} = \frac{P^2}{4M}$ describes C.M. motion

$$E = E_{cm} + E_{vibrational}$$

$$E_{vib} = \frac{p^2}{2\mu} + \frac{1}{2} c x^2$$

The energy levels of this system is

$$E_{vib} = \hbar\omega(n+\frac{1}{2}) \quad \text{where} \quad \omega = \sqrt{\frac{C}{\mu}}.$$

The zero point energy is

$$E_0 = \frac{\hbar\omega}{2} \quad \text{where} \quad \omega = \sqrt{\frac{C}{\mu}}.$$

$$\mu = \frac{m}{2} = \frac{M}{4} \rightarrow \text{Mass of molecule}$$

$$\omega = 2\sqrt{\frac{C}{M}} \quad \therefore \quad E_0 = \frac{\hbar\omega}{2}$$

$$\omega = 2\sqrt{\frac{10^3}{4.1\times10^{-26}}} = 3.1\times10^{14}\ \text{sec}^{-1}$$

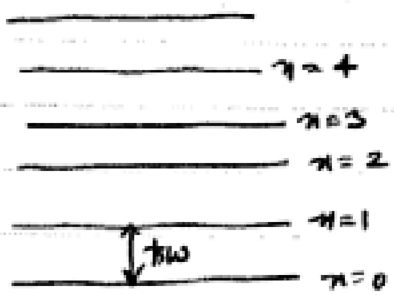
$$E_0 = \frac{1.055\times10^{-34} \times 3.1\times10^{14}}{2} = 1.6\times10^{-20}\ \text{Joule}$$

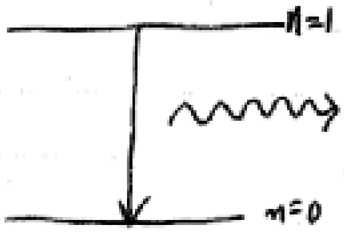
$$E_0 = 1.6\times10^{-20}\ \text{Joule} \times \frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 1\times10^{-1}\ \text{eV} = 0.1\ \text{eV}$$

3)

a) $E_n = \hbar\omega(n+\frac{1}{2})$ $\qquad \omega = 2\sqrt{\frac{C}{M}}$.

$E_2 - E_1 = \hbar\omega = 0.2\,eV$




b) 


The energy of emitted γ quantum is $E_1 - E_0 = \hbar\omega = 1.28\,eV$

$$\Psi(x,t) = N\left(\Psi_0(x)\, e^{-iE_0 t/\hbar} + \Psi_1(x) e^{-iE_1 t/\hbar}\right)$$

$$|\Psi(x,t)|^2 = |N|^2\left(|\Psi_0(x)|^2 + |\Psi_1(x)|^2 + 2\Psi_0(x)\Psi_1(x)\cos\frac{E_1-E_0}{\hbar}t\right)$$

c) Frequency of the photon is $\nu = \frac{\omega}{2\pi} = \frac{(E_1-E_0)}{\hbar\, 2\pi} = \frac{3.1\times10^{14}\,sec^{-1}}{2\pi}$

$\nu = 0.5\times10^{14}\,sec^{-1}$

Classical Oscillation frequency is $\nu = \frac{\omega}{2\pi} = 2\sqrt{\frac{C}{m}} = 5\times10^{15}\,sec^{-1}$

) $\lambda = \frac{3\times10^8}{0.5\times10^{14}} = 6\times10^{-6}\,m = 6\times10^4\,A^\circ$ (Infrared)

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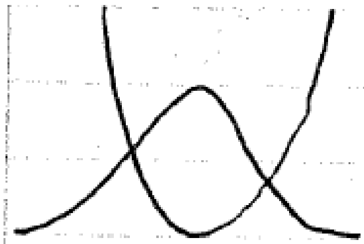

$$V(x) = \frac{1}{2}kx^2 + \frac{1}{2}m\omega^2x^2$$

Consider classically forbidden region

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0 \quad ; \quad k^2(x) = -\frac{2m}{\hbar^2}\left(E - V(x)\right)$$

$$\frac{d^2\psi}{dx^2} - k^2(x)\,\psi(x) = 0$$

$V(x) > E$ in that reg

The solution behaves as $\psi(x) \sim e^{-k(x)x}$ in a regi where $k(x)$ changes very little.

$$\psi(x) \sim e^{-k(x)x}$$

Since E is constant for any level.

$$k(x) = \sqrt{\frac{2m}{\hbar^2}\left(E - V(x)\right)} \approx \sqrt{\frac{2mV(x)}{\hbar^2}}\left(1 - O\left(\frac{E}{V}\right)\right)^{1/2}$$

$$\approx \sqrt{\frac{2mV(x)}{\hbar^2}} = \sqrt{\frac{2m^2\omega^2x^2}{2\hbar^2}} = \frac{m\omega x}{\hbar}$$

Hence $\psi(x) \sim e^{-\frac{m\omega}{\hbar}x\cdot x} = e^{-\frac{m\omega}{\hbar}x^2}$

·) The recursion relation in Appendix I is;

$$a_{l+2} = \frac{-\left(\frac{\beta}{\alpha} - 1 - 2l\right)}{(l+1)(l+2)} a_l \cdot$$

where the actual differential equation is;

$$\frac{d^2\psi}{dx^2} + (\beta - \alpha^2 x^2)\psi = 0$$

& after assuming a solution of the form

$$\psi(u) = A e^{-u^2/2} H(u)$$

$$\frac{d^2 H}{du^2} - 2u \frac{dH}{du} + \left(\frac{\beta}{\alpha} - 1\right) H = 0$$

The recursion relation is arrived at ~~where~~ from this equation.

$$\psi_0(u) = A e^{-u^2/2} a_0$$

$$\psi_1(u) = A e^{-u^2/2} \left[a_0 + a_1 u\right]$$

$$\psi_2(u) = A e^{-u^2/2} \left[a_0 + a_1 u + \frac{a_2}{a_0} a_0 u^2\right]$$

$$\psi_3(u) = A e^{-u^2/2} \left[a_0 + a_1 u + a_2 u^2 + a_3 u^3\right]$$

$$\psi_4 = A e^{-u^2/2}\left[a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4\right]$$

$$\psi_5 = A e^{-u^2/2}\left[a_0 + a_1 u + a_2 u^2 + a_3 u^3 + a_4 u^4 + a_5 u^5\right]$$

$a_1$ & $a_0$ are arbitrary constants

$$a_2 = -\frac{\left(\frac{\beta}{\alpha} - 1\right)}{2} a_0 \quad ; \quad a_4 = -\frac{\left(\frac{\beta}{\alpha} - 5\right)}{12} a_2$$

$$= \frac{\left(\frac{\beta}{\alpha} - 5\right)\left(\frac{\beta}{\alpha} - 1\right)}{2 \cdot 12} a_0$$

$$a_3 = -\frac{\left(+\frac{\beta}{\alpha} - 3\right)}{6} a_1 \quad ; \quad a_5 = -\frac{\left(\frac{\beta}{\alpha} - 7\right)}{20} a_3$$

$$= \frac{\left(\frac{\beta}{\alpha} - 7\right)\left(\frac{\beta}{\alpha} - 3\right)}{6 \cdot 20} a_1$$

substituting the values of the coefficients one gets
the solutions in terms of $a_1$ & $a_0$

$$\frac{d^2\psi}{du^2} + \left(\frac{\beta}{\alpha} - u^2\right)\psi = 0$$

$$\psi = \sum_{n=0}^{\infty} a_n u^n$$

$$\psi'' = \sum_{n=2}^{\infty} a_n\, n(n-1)\, u^{n-2} \quad ; \quad \psi' = \sum_{n=1}^{\infty} a_n (n-1)\, u^{n-1}$$

∴ Substituting in the equation one has;

$$\sum_{n=2}^{\infty} a_n\, n(n-1)\, u^{n-2} + \left(\frac{\beta}{\alpha} - u^2\right) \sum_{n=0}^{\infty} a_n u^n$$

$$\sum_{n=2}^{\infty} a_n\, n(n-1)\, u^{n-2} + \frac{\beta}{\alpha} \sum_{n=0}^{\infty} a_n u^n - \sum_{n=0}^{\infty} a_n u^{n+2}$$

$n-2 = j$ one has;

$$\sum_{j=0}^{\infty} a_{j+2}\, (j+2)(j+1)\, u^j + \frac{\beta}{\alpha} \sum_{j=-2} a_j u^j - \sum_{j=-2} a_j u^{j+4}$$

$a_j = 0$ for $j < 0$

$$\therefore \sum_{j=0}^{\infty} a_{j+2}\, (j+2)(j+1)\, u^j + \frac{\beta}{\alpha} \sum_{j=-2}^{\infty} a_{j+2} u^{j+2} - \sum_{j=-2}^{\infty} a_{j+2} u^{j+4}.$$

~~[illegible]~~ If we now take the co-eff of $u^j$

we have $a_{j+2}\,(j+2)(j+1) + \frac{\beta}{\alpha} a_j - a_{j-2}$

$\therefore\ a_{j+2}\,(j+2)\,(j+1) + \frac{\beta}{\alpha} a_j - a_{j-2} = 0$

The recursion relation is ~~to~~ ~~now~~ now has three terms unlike the two term recursion relations one normally has and it can be shown that solution for this does not work out in the ordinary way. This is an indication that the solution must be attempted by the asymptotic method.