Physics 237, Spring 2008
Homework #7
Due in class, Thursday March 27, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that “questions” can be answered briefly; “problems” may be more involved. Each question usually counts 5 points; each problem counts 10 points.

1. E&R, chapter 7, **question** 4.
2. E&R, chapter 7, **question** 5.
3. E&R, chapter 7, **question** 7.
4. E&R, chapter 7, **question** 8.
5. E&R, chapter 7, problem 3.
6. E&R, chapter 7, problem 4.
7. E&R, chapter 7, problem 6.

1) These are very restrictive conditions on wavefunctions. It is single valued and square integrable. Because of these facts the results would not change even in this case, where one chooses different separation variables. We can show this in a very simple example;

Assume instead of $-m_\ell^2$, you choose $+m_\ell^2$. Then

$$\frac{d^2\phi}{d\varphi^2} = m_\ell'^2 \phi \qquad \text{i.e.} \qquad \frac{d^2\Phi}{d\phi^2} = m_\ell'^2 \phi$$

$$\phi(\phi) = Ae^{m_\ell'\phi} + Be^{-m_\ell'\varphi}$$

But from boundedness, m_ℓ' must be pure imaginary

$m_\ell' = i m_\ell$ and since the wavefunction must be single valued, i.e., $\Phi(\phi+2\pi) = \Phi(\varphi)$, m_ℓ must be an integer. We again reached the same result.

(5.) It is a general property of the wavefunction its finite, single valued & well behaved.

$\therefore$ ~~$\Phi(\phi+2\pi)$~~ 0

$\therefore \Phi(\phi+2\pi) = \Phi$ $\qquad \Phi(\varphi) = e^{im_\ell \varphi}$.

$\therefore e^{im_\ell(\varphi+2\pi)} = e^{im_\ell \varphi}$.

or $e^{im_\ell 2\pi} = 1$ $\rightarrow$ This is possible only if m_ℓ is an integer.

(8.) The function $\Theta(\theta)$ & $\Phi(\varphi)$ will remain same provided the potential is a function of the radial co-ordinate only. $\therefore$ $V(r) = \frac{1}{2} m\omega^2 r^2$ or Kr, etc. As long as $V(r)$ is a function of r only and not θ & ϕ the form of $\Theta(\theta)$ & $\Phi(\varphi)$ won't change.

.) The three (3) quantum nos in this problem corresponds to 3 dimensions in which the problem is formulated.

$$\therefore \quad E_n = \frac{-\mu Z^2 e^4}{(4\pi\epsilon_0)^2 \, 2\hbar^2 n^2}$$

$H \rightarrow Z = 1 \qquad \mu = \dfrac{m_p m_e}{m_p + m_e}$

$D \rightarrow Z = 1 \qquad \mu = \dfrac{2m_p m_e}{2m_p + m_e}$

$He \rightarrow Z = 2 \qquad \mu = \dfrac{2m_p m_e}{2m_p + m_e}$

$$E_H : E_D : E_{He} = \frac{m_p m_e}{m_p + m_e} : \frac{2m_p m_e}{2m_p + m_e} : 4 \cdot \frac{2m_p m_e}{2m_p + m_e}$$

If $m_p \gg m_e \Rightarrow \dfrac{m_p}{m_p} : \dfrac{m_p}{m_p} : 4 \, \dfrac{2m_p}{2m_p}$

$= 1 : 1 : 4 \quad = 1 : 1 : 4$

4)

a)

$E_n = \frac{-13.6}{n^2}$ ev ; $E_1 = -13.6$ ev ; $E_2 = -3.4$ ev

$E_3 = -1.5$ ev .

$n = 1, 2, 3.$

b) $E_{12} = (-3.4 + 13.6)$ ev $= 10.2$ ev

$E_{13} = (-1.5 + 13.6)$ ev $= 12.1$ ev

$E_{23} = (+3.4 - 1.5)$ ev $= 1.9$ ev

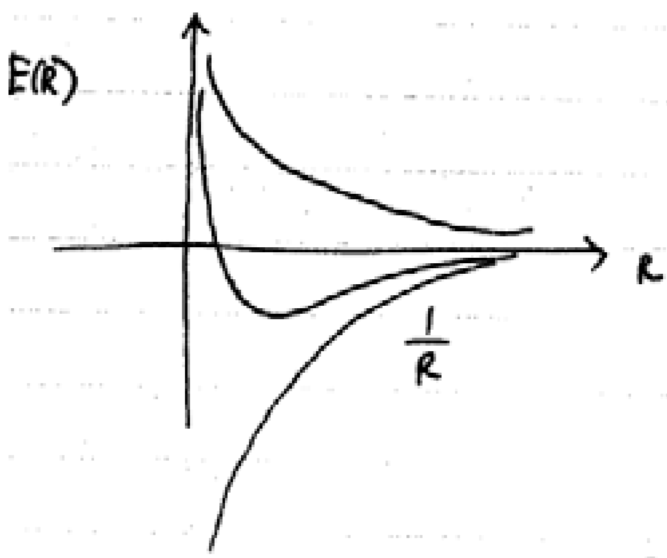
$\cancel{\lambda_{12} = E}$ $\nu_n = \frac{E_n}{h}$; $\lambda_n = \frac{hc}{E_n}$

.6

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 R}$$

$\Delta p = \frac{\hbar}{R}$ $p \simeq \Delta p$; $\Delta x = R$

$E = \frac{\hbar^2}{2mR^2} - \frac{e^2}{4\pi\epsilon_0 R}$ find the minimum of the function $E(R)$



$\frac{dE}{dR} = 0 \Rightarrow -\frac{2\hbar^2}{2mR^3} + \frac{e^2}{4\pi\epsilon_0 R^2} = 0$ i.e. $R = \frac{4\pi\epsilon_0\hbar^2}{me^2} = a_0$

$$E = -\frac{\hbar^2}{2ma_0^2} - \frac{e^2}{4\pi\epsilon_0 a_0} = \frac{\hbar^2}{2m}\frac{m^2e^4}{(4\pi\epsilon_0)^2\hbar^4} - \frac{e^2}{(4\pi\epsilon_0)^2}\frac{me^2}{\hbar^2}$$

$$= -\frac{1}{2}\frac{me^4}{(4\pi\epsilon_0)^2\hbar^2} = 13.6\ \text{eV}$$