Physics 237, Spring 2008
Homework #9
Due in class, Thursday April 10, 2008

Homework is from Eisberg and Resnick (E&R) unless otherwise indicated. Please note that "questions" can be answered briefly; "problems" may be more involved. Each question usually counts 5 points; each problem counts 10 points.

1. E&R, chapter 7, **question** 12.
2. E&R, chapter 7, **question** 16.
3. E&R, chapter 7, problem 13.
4. E&R, chapter 7, problem 16.
5. E&R, chapter 7, problem 18.
6. E&R, chapter 7, problem 19.

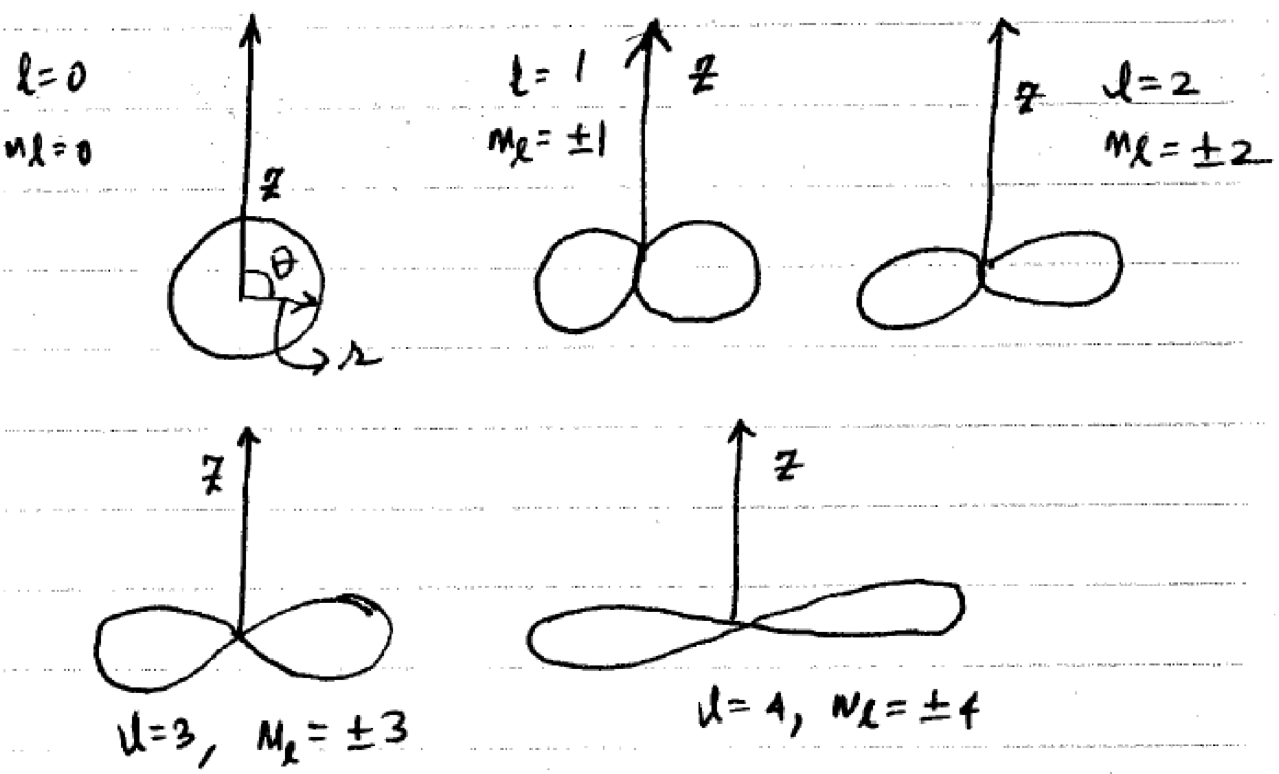
2.) The one electron problem (Hydrogen ~~A~~ atom) is a symmetric (spherical) problem, so the treatment of the problem mathematically is exactly the same whatever the direction of z-axis is taken.

The physical consequence of this is the spherical symmetry of the problem

The application of magnetic or electric field breaks the spherical symmetry of the problem, and the problem is no more solvable in the way that has been formulated so far. It requires what we know as PERTURBATION THEORY.

The three dimensional behavior of $\psi^*_{nlm_l}\psi_{nlm_l}$ is completely specified by the product of the quantity $R^*_{nl}(r)\,R_{nl}(r) = \frac{P_{nl}}{4\pi r^2}$ and $\Theta^*_{lm_l}\,\Theta_{lm}(\theta)$, which plays the role of the directionally dependent modulation factor.

~~Now, today~~ Polar diagrams of the directional depend of the one electron atom probabilities densities for



Only for $l=0$ we have non zero probability at $r=0$ but for all other probabilities there is zero probability at $r=0$.

13.

$$\psi_{432} \approx \psi_{n l m_l} \quad n=4,\ l=3,\ m_l=2$$

(a) $E_n = \frac{-13.6}{n^2}\ eV = -\frac{13.6}{16}\ eV$.

(b) $$\bar{r}_{nl} = \frac{n^2 a_0}{Z}\left\{1+\frac{1}{2}\left[1-\frac{l(l+1)}{n^2}\right]\right\}.$$

for $n=4,\ Z=1,\ l=3$

(c) $L = \sqrt{l(l+1)}\cdot\hbar \quad l=3$

(d) $L_Z = m_l\hbar \quad m_l = 2$

e) $(\Delta L)^2 = \langle L^2 \rangle - (\langle L \rangle)^2$

$= l(l+1)\hbar^2 - l(l+1)\hbar^2 = 0$ $\qquad l = 3$

-) $(\Delta L_z)^2 = \langle L_z^2 \rangle - (\langle L_z \rangle)^2$

$m_l = 2$

$= m_l \hbar^2 - m_l \hbar^2 = 0$

This is so because the hydrogen atom eigenfunction ψ_{432} are eigenfunctions of L^2 & L_z operators.

(a)

$$L_{x\,op} = i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right)$$

$$\psi_{21-1} = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)\sin\theta\, e^{-i\varphi}.$$

$$L_{x\,op}\psi_{21-1} = i\hbar\,\frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0}\left[\sin\varphi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right]\sin\theta\, e^{-i\varphi}$$

$$= \frac{i\hbar}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(\frac{Zr}{a_0}\right)\left[\sin\varphi\cos\theta e^{-i\phi} + \cot\theta\cos\phi\sin\theta(-i)e^{-i\phi}\right]$$

$$L_{ox_{op}} \Psi_{21-1} = \frac{i\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(\frac{Zr}{a_0}\right) \underbrace{e^{-Zr/2a}}_{K} \left[\sin\phi \cos\theta \, e^{-i\phi} + (i)\cos\theta \cos\phi \, e^{-i\phi}\right]$$

$$= \frac{i\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} K \left(\frac{Zr}{a_0}\right) \left[\sin\phi \cos\theta \, e^{-i\phi} - i\cos\theta \cos\phi \, e^{-i\phi}\right]$$

$$= \frac{i\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} K \left(\frac{Zr}{a_0}\right) \left[\cos\theta\right] e^{-i\phi} \left[\sin\phi - i\cos\phi\right]$$

$$= \frac{i\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} K \left(\frac{Zr}{a_0}\right) \left[\cos\theta\right] e^{-i\phi} \left[-i\left(i\sin\phi + \cos\phi\right)\right]$$

$$= \frac{\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} \left(\frac{Zr}{a_0}\right) K \cos\theta \quad e^{-i\phi} \, e^{i\phi}$$

$$= \frac{\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} K \left(\frac{Zr}{a_0}\right) \cos\theta$$

$$= \frac{\hbar}{8\sqrt{\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/2a_0} \left(\frac{Zr}{a_0}\right) \cos\theta$$

$$= \frac{\hbar}{2 \cancel{4}} \frac{\sqrt{2}}{\sqrt{2\pi}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/2a} \left(\frac{Zr}{a_0}\right) \cos\theta$$

$$= \frac{\hbar\sqrt{2}}{2} \Psi_{210} = L_{oxp} \Psi_{21-1}$$

$\therefore$ $L_{oxp} \Psi_{21-1} \propto \Psi_{210}$

$\therefore$ The eigenfunction Ψ_{21-1} is not an eigenfn for L_{oxp}.

(19.)

(a) $L_z = -i\hbar \frac{\partial}{\partial \varphi}$.

(i) $L_z e^{im\varphi} = -i\hbar \frac{\partial}{\partial \phi} e^{im\phi} = m\hbar e^{im\varphi} = m\hbar e^{im\varphi}$

$\therefore e^{im\varphi}$ is an eigenfn.

(ii) $L_z e^{-im\varphi} = -i\hbar \frac{\partial}{\partial \varphi} e^{-im\varphi} = -m\hbar e^{im\varphi} = -m\hbar e^{-im\varphi}$

(iii) $L_z \cos m_\ell \varphi = \cancel{+ m_\ell \hbar \sin m_\ell \varphi}$. $-i m_\ell \hbar \sin m_\ell \varphi$.

(iv) $L_z \sin m_\ell \varphi = +(-i\hbar m_\ell \cos m_\ell \varphi.)$.

(b) $e^{im\varphi}$ & $e^{-im\varphi}$ are eigenfunctions for L_z but

$$\cos m_\ell \varphi = \frac{e^{im_\ell\varphi} + e^{-im_\ell\varphi}}{2} \quad \&$$

$$\sin m_\ell \varphi = \frac{e^{im_\ell\varphi} - e^{-im_\ell\varphi}}{2i}$$

Thus $\cos m_\ell \varphi$ & $\sin m_\ell \varphi$ are a linear combination of different eigenfunctions and are therefore not eigenfunctions of L_{oz}.