Intro/Review: Why Quantum Mechanics?

In the late 1800's, many people thought physics was done. Newton's classical mechanics was long since understood, Maxwell had successfully unified Electricity + Magnetism + explained light into the bargain, and the Kinetic theory took care of thermodynamics.

As of the very late 1800's and early 1900's, though, some phenomena came up that classical physics failed spectacularly to explain. This led to (among other things) the development of quantum mechanics.

You are familiar with this story from P143 or P123. Here I'll just review a few of the main developments that led to the quantum theory, focusing on where the classical theory went wrong and how quantum ideas got it right. We'll discuss

- Blackbody radiation (Planck, 1900)
- Photoelectric effect (Einstein, 1905)
- Particles as Waves (de Broglie, 1923)
- Atomic spectra + stability (Bohr, 1913)

Thermal Radiation from Black Bodies

All bodies emit thermal radiation that depends on their temperature and other things. Remember that heat is just motion, and the particles in bodies jiggling around radiate electromagnetic waves.

Blackbody is a special case: it absorbs all of the thermal radiation that's incident on it, without reflection. The lack of reflection is why it looks black, unless it's very hot and its *own* thermal radiation makes it visible.

Blackbodies are special because the properties of their radiation depend *only* on their temperature, and nothing else. These properties are usually identified in terms of the spectrum.

[*Aside*: A *spectrum* is a distribution (in this case, of radiated energy) as a function of frequency or wavelength. (A spectrum can also be expressed as a function of energy, or even mass. So, for example when people talk about particle spectroscopy, they mean the distribution of masses and/or excitation energies).

For blackbody radiation we'll consider the *frequency spectrum*]

Aside #2. Recall the relation between wavelength λ + frequency ν of a wave.

λ = wavelength = dist. fr/ peak to peak

ν = frequency = cycles of the wave / time
= $1/T$ where T = period

$\boxed{\lambda\nu = c}$ where c is the speed of the wave.

So increasing $\lambda \Leftrightarrow$ decreasing ν, + vice versa.

Also, for future reference angular freq. $\omega = 2\pi\nu$ = radians/sec.
(One complete cycle = 2π radians)

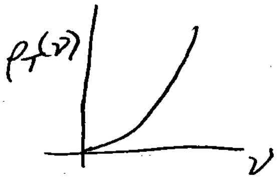
Classical calc. of blackbody spectrum of a cavity

Rayleigh + Jeans calculated the frequency spectrum by
- considering EM waves in a cavity, + counting the standing waves
- using classical kinetic theory to find the avg. total energy (in thermal equilibrium)

$\Rightarrow \rho_T(\nu)\,d\nu$ = energy/vol in cavity for frequencies btwn ν + $\nu + d\nu$ at temp T

$$= \frac{8\pi\nu^2 kT}{c^3}\,d\nu$$ Classical result

It increases like the square of the freq ν + is proportional to temp.



There's a <u>problem</u> here, even before you start to compare with experiment.

- <u>Q</u>. What is the <u>total</u> <u>energy</u> for all frequencies?

  <u>A</u>. It's <u>infinite</u>! To get it, integrate over ν:

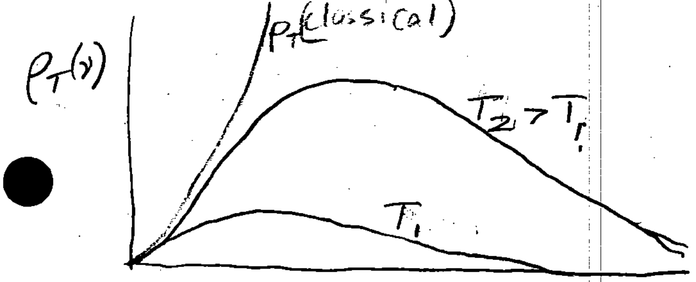
$$R_{tot} = \int^{\infty} \rho_T(\nu)\, d\nu \quad \propto \nu^3 \rightarrow \infty !$$

This cannot be right, and it's called the <u>ultraviolet</u> <u>catastrophe</u>

↳ ultraviolet because it happens at high frequencies (= high energies), and UV is the next higher freq. range after the visible. Ultraviolet is used generically for high freq. + high energy. Similarly, infrared is used for low freq. or low en.

So the classical <u>theory</u> itself is already <u>sick</u>, indep. of specific experimental results. Of course it contradicts those, too:

<u>Exp'tal results</u> for black body radiation: The spectrum doesn't rise indefinitely; it looks like



- $R_{tot} = \underset{\uparrow\ \text{Stefan-Boltzmann const}}{\sigma} T^4$, not ∞

- Shifts to higher ν w/ higher en: max energy at freq

  $\nu_{max} \propto T$, not ∞ as in classical result

Solution:
Planck comes to the rescue with a quantum hypothesis.

In summing up the energy, don't assume it's continuous; assume it comes in discrete packets:

$$\boxed{\Delta E = h\nu} \text{ and multiples there of}$$

Planck's const; $h = 6.63 \times 10^{-34}$ Joule-sec

Note: Recall that the thermal radiation comes from oscillating electrons in the cavity wall. Planck's assumption is that the energy of oscillation is quantized.

It works! Now you get

$$\rho_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1} \quad \text{quantum result.}$$

and it fits the data beautifully, and has all the right features. One of these is that it reduces to the classical result at low freq., which it should because the class. result works fine there:

$$\rho_T^{quant} \xrightarrow[\text{small } \nu]{} \frac{8\pi\nu^3}{c^3}\frac{h\nu}{1+\frac{h\nu}{kT}-1} = \frac{h\pi\nu^2}{c^3}\frac{h\nu}{h\nu/kT}$$

$$= \frac{h\pi\nu^2}{c^3}kT = \text{classical result}$$

Comment: Planck had no good reason to make his hypothesis - he was desperate to find something that worked, and this did.

Photoelectric Effect 1905

It was known at the turn of the century that if you shine ultraviolet light on a metal electrode, electrons come off. This is called the photoelectric effect + the electrons are called photoelectrons.

Classically the picture is that the waves are incident on the metal + the electrons soak up energy until they escape from the metal.

Classical predictions:

1. Higher intensity increases KE of electrons.
   Recall intensity $\propto |\vec{E}|^2 \propto$ energy density of field.
   Higher intensity $\Rightarrow$ more energy
2. The effect happens for any frequency.
3. The KE is independent of frequency.

What happens in real life:

1. Higher intensity doesn't change KE; it only gives more electrons.
2. There is a cutoff frequency ν_0 below which the effect doesn't happen.
3. KE increases with increasing frequency.

Einstein provided the explanation in 1905 (+ later won the Nobel Prize for it). He extended Planck's views to postulate that

> the absorbed energy is quantized in bundles, $E = h\nu$; the electron absorbs all or nothing.

These bundles are called photons. This explains the three anomalies:

1. Higher intensity just means more photons, not more energy/photon, if the frequency is fixed. More photons to absorb means more ejected electrons.

2. Cutoff frequency ν_0. Because the electrons are bound in the metal, there is a minimum amount of energy required to eject the electrons from the metal. It's called the work function W_0. Photons with frequencies such that $h\nu < W_0$ will not eject electrons, so there's no effect for $\nu < W_0/h$

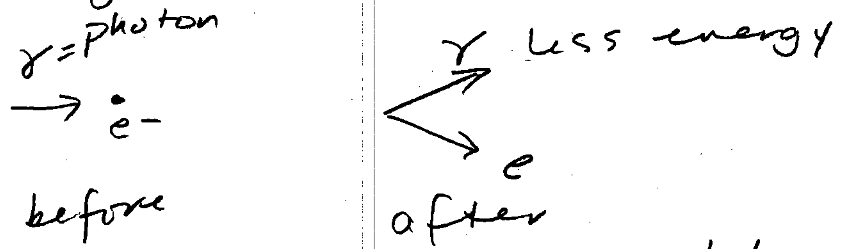
3. KE dependence on frequency. The energy available to the electron after absorbing a photon is

$$KE_{max} = h\nu - W_0$$

which increases with frequency. It's KE_{max} because some energy may be lost to collisions on the way out.

Compton effect (1923): More evidence for light as particle

In the Compton effect, light scatters off electrons just as if it were a particle: the conservation of energy and momentum works just like with billiard balls:



For the photon just take $p = h/\lambda$ and $E = h\nu$ and $\lambda\nu = c$, and the rest of the calc. goes as for particles.

So the upshot of all this is that under some circumstances the description of light as a particle is unavoidable. We can't drop the wave picture, though, because we still need waves to describe things like interference and diffraction.

Atomic spectra + stability ; Bohr atom (1913)

By 1911, Rutherford determined existence of nucleus (α part. scattered off gold foil).

$\Rightarrow$ electrons outside central nucleus.

stability ~~They~~ can't be at rest, else they would fall in fr/ electric attraction to pos. nucleus.

$\Rightarrow$ atom would collapse.

So let them orbit abt nucleus, like planets.

Oops, accelerating charged particles radiate:

Fr/ Physics 516, Electric field of moving charge is

$$\vec{E}(\vec{x},t) = \frac{e}{4\pi\epsilon_0}\left[\frac{\hat{n}-\vec{v}/c}{\gamma^2(1-\frac{\vec{v}}{c}\cdot\hat{n})^3 R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{n}\times\{(\hat{n}-\vec{v}/c)\times\dot{\vec{v}}/c}{1-\vec{v}/c\cdot\hat{n}}\right]_{ret}$$

where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ $R = |\vec{x}-\vec{r}(\tau_0)|$ + $\hat{n} = \frac{\vec{x}-\vec{r}(\tau_0)}{R}$

$\vec{r}(\tau_0)$ = position of particle at time τ_0 ; τ_0 defined by $\vec{x}-\vec{r}(\tau_0) = c(t-\tau_0)$

Upshot: accel. particle produces $\vec{E}$ field.

$\vec{E}$ field has energy $\Rightarrow$ particle loses energy

$\Rightarrow$ should spiral in to nucleus

* Why is the atom stable?

Spectrum

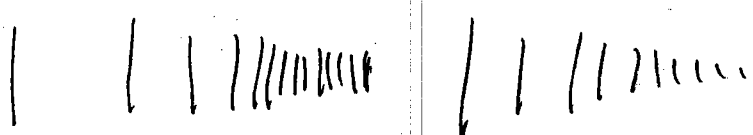
Atoms get <u>excited</u> (e.g. through collisions) + go to higher energy. Excess energy given up as <u>radiation</u>, i.e., light.

Now, if <u>all energies</u> are possible, get <u>all wavelengths</u> ⇒ if we spread out accoding to wavelength (e.g. w/ prism) the result (= "wavelength spectrum") is <u>continuous</u> → a big smear of light.

<u>Wrong!</u> You get only <u>discrete</u> wavelengths (= lines on image of spectrum)

And they have <u>funny spacing</u> ⇒ they get closer + closer together as wavelength decreases.

They look like


see book for better version.

etc

+ You can find a formula for the wavelengths for each series. The first one, in the visible + near UV was found by <u>Balmer</u> for <u>Hydrogen</u>:

$$\frac{1}{\lambda} = \underset{\substack{\uparrow \\ \text{Rydberg const}}}{R_H} \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \qquad n = 3, 4, \ldots$$

One n for each line in the spectrum.

Some other guys (Lyman, Paschen, Brackett, etc) found some other series in the UV + IR,

$$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad n = 2, 3 \ldots$$

$$\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad 4, 5,$$

$$\vdots$$

Bohr came up w/a model to explain this. Part was right, part was wrong, but it was an important step in the right direction.

- What was right was that he said the differences In these expressions come from energy differences— the radiations f/ changes in energy levels,

$$\Delta E = h\nu = hc\left(\frac{1}{\lambda}\right)$$

The integers come in because the energy levels are quantized, not arbitrary. Only certain discrete energy levels, ("stationary states") are possible. That means only discrete transitions are possible, hence the discrete spectral lines.

The energy levels are

$$E = -\left(\frac{m_e Z^2 e^4}{(4\pi\epsilon_0) 2\hbar^2}\right)\frac{1}{n^2} \quad n = 1, 2, 3 \ldots$$

Hence transitions give

$$\Delta E = \frac{hc}{\lambda} = (\quad)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right):$$

What Bohr got wrong was where this quantization came from. He said that electrons move in circular orbits about the nucleus (classical orbits!) and that only orbits are possible in which angular momentum is quantized,

viz., $$L = \frac{nh}{2\pi}$$

[$L = mvr$ and $\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r}$ give r and v. Then $E = K + V$.

The minus sign comes fr/ the attractive force.]

We'll see when we solve the Schrödinger eq'n where the quantum theory says the quantization comes from.

De Broglie + Particles as Waves   1923

If radiation can act like particles, part's should
act like waves

$\boxed{\lambda = h/p}$ de Broglie wavelength

[Small: for 1kg baseball, v = 10 m/sec
$\Rightarrow 6.6 \times 10^{-25}$ Å

For 100 eV electron, 1.2 Å]

How to reveal? Diffraction? Need $\lambda \sim a$
$\Rightarrow$ beam of electrons "shine" on Xtal

conf. Davisson-Germer (1927)

results show interference + correct λ

JJ Thompson disc. e^- in 1897

|| [G.P. Thompson son in 1927 discovered e^- diffraction

Prettier: Double slit expt

All matter does this, but for most, λ so small
can't tell

Wave-part. duality — Bohr: principle of complementarity
exptally, can see one or other,
not both.

Matter: wave is prob. amplitude fn

Heisenberg uncertainty principle

$$\boxed{\Delta p_x \Delta x \geq \frac{\hbar}{2}} \qquad \hbar = h/2\pi$$

Uncertainty in simultaneous meas. of p_x + x

⇒ *inherent* limitation

⇒ Not meaningful to say definite position + mom.

Similarly, $\Delta t \, \Delta E \geq \hbar/2$

⇒ No more exact determinism.

Now *probabilistic* *interpretations* take over.