In fact we can express the exclusion principle in a stronger way as follows:

> A system with multiple electrons is described by a total eigenfunction that is completely antisymmetric.

What does this ("completely antisymmetric") mean for more than two electrons? It means the wave fn must change sign when we exchange <u>any</u> pair of particle labels. That means we get more than two terms when we have more than two particles.

How do we find such wave functions? We don't have to use brute force fortunately; we can use "Slater determinants."

Notice that for two particles, ψ_A can be written

$$\psi_A = \frac{1}{\sqrt{2}}\left[\psi_\alpha(1)\psi_\beta(2) - \psi_\beta(1)\psi_\alpha(2)\right]$$

$$= \frac{1}{\sqrt{2!}}\begin{vmatrix} \psi_\alpha(1) & \psi_\alpha(2) \\ \psi_\beta(1) & \psi_\beta(2) \end{vmatrix}$$

This generalizes, for example for 3 particles we have

$$\psi_A = \frac{1}{\sqrt{3!}}\begin{vmatrix} \psi_\alpha(1) & \psi_\alpha(2) & \psi_\alpha(3) \\ \psi_\beta(1) & \psi_\beta(2) & \psi_\beta(3) \\ \psi_\gamma(1) & \psi_\gamma(2) & \psi_\gamma(3) \end{vmatrix}$$

which when we expand it out is equal to

$$\psi_A = \frac{1}{\sqrt{3!}}\Big\{\psi_\alpha(1)\big[\psi_\beta(2)\psi_\gamma(3) - \psi_\gamma(2)\psi_\beta(3)\big]$$
$$- \psi_\beta(1)\big[\psi_\alpha(2)\psi_\gamma(3) - \psi_\gamma(2)\psi_\alpha(3)\big]$$
$$+ \psi_\gamma(1)\big[\psi_\alpha(2)\psi_\beta(3) - \psi_\beta(2)\psi_\alpha(3)\big]\Big\}$$

$$= \frac{1}{\sqrt{3!}}\Big\{\psi_\alpha(1)\psi_\beta(2)\psi_\gamma(3) - \psi_\alpha(1)\psi_\gamma(2)\psi_\beta(3)$$
$$- \psi_\beta(1)\psi_\alpha(2)\psi_\gamma(3) + \psi_\beta(1)\psi_\gamma(2)\psi_\alpha(3)$$
$$+ \psi_\gamma(1)\psi_\alpha(2)\psi_\beta(3) - \psi_\gamma(1)\psi_\beta(2)\psi_\alpha(3)\Big\}$$

and the $\frac{1}{\sqrt{3!}}$ guarantees that the normalization comes out right. In the homework you'll verify that this is really antisymmetric under exchange of particle labels.

Are there any other particles besides electrons that require antisymmetric total wave functions? And are there any that require symmetric w.f.'s? We know the answer to at least one of these questions has to be yes, because indistinguishability requires wave functions that have definite symmetry properties. In fact the answer to <u>both</u> questions is yes. Pauli showed using very general considerations in relativistic quantum field theory that a particle's

<u>spin</u> determines whether its multiparticle systems have to have symmetric or antisymmetric total wave functions. Recall that the electron has spin ½ (in units of $\hbar$). All spin ½ particles — in fact all particles with half-odd-integer spin — must have antisymmetric total wave functions. These particles are called "Fermions." Integer-spin particles must have symmetric total wave functions, and they're called "Bosons"

Some examples

	particle	spin	
Fermions:	electron	½	antisymm total w.f.
	quark	½	
	neutrino	½	
	proton	½	
	neutron	½	
Bosons:	photon	1	symmetric total w.f.
	$W^{\pm}$, Z	1	
	gluon	1	
	pion	0	
	Higgs boson	0	

We can get symmetric total wave functions like we got the antisymmetric ones, only with + signs instead of − signs everywhere.

The Periodic Table

Now we're going to skip ahead to sec. 9.7. The exclusion principle and what we've learned about hydrogen and multielectron atoms can help us explain the periodic table, developed by Mendeleev in 1869.

There are 100 or so chemical elements, and in the mid-to-late 1800's it was thought that the corresponding atoms were all distinct elementary particles. But even before the electron + nucleus were discovered, there were some hints that there may be a common structure underneath. There were regularities in properties of the atoms, e.g. which other atoms they'd bond with or whether they'd bond at all. One modern illustration is the pattern of ionization energies of the elements:

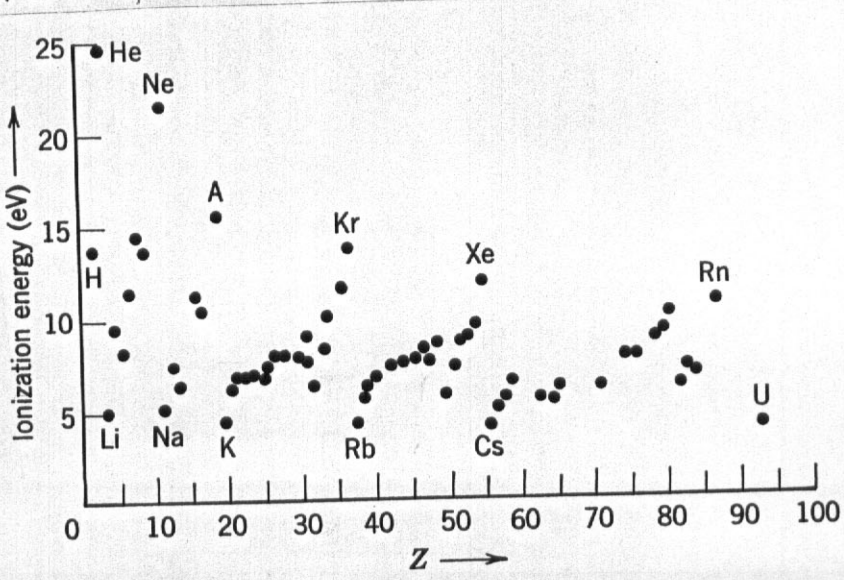

Figure 9-15 The measured ionization energies of the elements.

The repetition of the pattern is striking.

In 1869 Mendeleev figured out a way to organize the elements according to their common properties, which resulted in the Periodic table, as shown on p.6.14.1 (fig 9-13, p.330 in book). The elements with similar properties appear in columns in the periodic table. For example

- 1st column = alkalis, valence +1; they bind w/ the halogens in the next-to-last column
- last column = noble gases; they don't do much of anything* & are sometimes called inert
- etc. See inorganic chemistry for details.

We now understand the regularities in terms of the energy levels of the electrons in the atoms, and which states are filled in the ground state.

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* but check out superfluid Helium-3

6.14.1

	s^1	s^2
1s	1 H	
2s	3 Li	4 Be
3s	11 Na	12 Mg
4s	19 K	20 Ca
5s	37 Rb	38 Sr
6s	55 Cs	56 Ba
7s	87 Fr	88 Ra

	d^1	d^2	d^3	d^4	d^5	d^6	d^7	d^8	d^9	d^{10}
3d	21 Sc	22 Ti	23 V	24 Cr $4s^1 3d^5$	25 Mn	26 Fe	27 Co	28 Ni	29 Cu $4s^1 3d^{10}$	30 Zn
4d	39 Y	40 Zr	41 Nb $5s^1 4d^4$	42 Mo	43 Tc	44 Ru $5s^1 4d^7$	45 Rh $5s^1 4d^8$	46 Pd $5s^0 4d^{10}$	47 Ag $5s^1 4d^{10}$	48 Cd
5d	57 La Lantha-nides	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt $6s^1 5d^9$	79 Au $6s^1 5d^{10}$	80 Hg
6d	89 Ac Actinides									

	p^1	p^2	p^3	p^4	p^5	p^6
1s						2 He
2p	5 B	6 C	7 N	8 O	9 F	10 Ne
3p	13 Al	14 Si	15 P	16 S	17 Cl	18 A
4p	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5p	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6p	81 Ti	82 Pb	83 Bi	84 Po	85 At	86 Rn
7p						

	f^1	f^2	f^3	f^4	f^5	f^6	f^7
4f Lanthanides	58 Ce $5d^0 4f^2$	59 Pr $5d^0 4f^3$	60 Nd $5d^0 4f^4$	61 Pm $5d^0 4f^5$	62 Sm $5d^0 4f^6$	63 Eu $5d^0 4f^7$	64 Gd $5d^1 4f^7$
5f Actinides	90 Th $6d^2 5f^0$	91 Pa $6d^1 5f^2$	92 U $6d^1 5f^3$	93 Np $6d^1 5f^4$	94 Pu $6d^1 5f^5$	95 Am $6d^1 5f^6$	96 Cm $6d^1 5f^7$

	f^8	f^9	f^{10}	f^{11}	f^{12}	f^{13}	f^{14}
4f Lanthanides	65 Tb $5d^0 4f^9$	66 Dy $5d^0 4f^{10}$	67 Ho $5d^0 4f^{11}$	68 Er $5d^0 4f^{12}$	69 Tm $5d^0 4f^{13}$	70 Yb $5d^0 4f^{14}$	71 Lu $5d^1 4f^{14}$
5f Actinides	97 Bk $6d^1 5f^8$	98 Cf $6d^0 5f^{10}$	99 Es $6d^0 5f^{11}$	100 Fm $6d^0 5f^{12}$	101 Md $6d^0 5f^{13}$	102 No $6d^0 5f^{14}$	103 Lw $6d^1 5f^{14}$

Figure 9-13 The periodic table of the elements, showing the electron configuration for each element.

To see how this works we need some nomenclature. We'll describe the states according to their n & l quantum numbers (for our purposes all m_l & m_s for a given n&l can be considered degenerate in energy). We just give the number for n, but l is (for historical reasons) identified by a letter as follows:

$l =$	0	s
	1	p
	2	d
	3	f
	4	g
	5	h
	$\vdots$	

So the following states are:

$$1s \Leftrightarrow n=1,\ l=0$$

$$3d \Leftrightarrow n=3,\ l=2$$

$$4p \Leftrightarrow n=4,\ l=1$$

etc.

Now we can think about atoms with multiple electrons. We want to minimize the energy, but the exclusion principle tells us the electrons can't all pile up in the $n=1$ state. In fact, the most we can have are 2 electrons (with quantum numbers $n=1, \ell=0, m_\ell=0, m_s=\pm 1/2$), then we have to move on to $n=2$, etc. So here's how it works as we start through the periodic table

$Z=1$, hydrogen $1s^1$ ← 1 e^- in the 1s state

$Z=2$ helium $1s^2$ ← 2 electrons in the 1s state

Now the $n=1$ "shell" is full, so we move on to $n=2$

$Z=3$ lithium ${}^3Li$ $1s^2 2s^1$ outer e^- $n=2, \ell=0$

4 berillium ${}^4Be$ $1s^2 2s^2$ $n=2, \ell=0$

Now we move on to $n=2, \ell=1$: the 2p subshell

${}^5B$ $1s^2 2s^2 2p^1$

⋮

${}^{10}Ne$ $1s^2 2s^2 2p^6$

+ now $n=2$ is all full up.

Compare to the structure of the periodic table. . . (6.17)

Now we fill the $n=3$ shells. So the $3s$ + $3p$ get their 2 + 6 elements respectively. Next we'd expect to find $n=3$, $l=2$, i.e. $3d$. But it turns out (because of the interactions among the electrons themselves) that the $4s$ state is next, <u>then</u> $3d$

So after

$$Z=18 \quad {}^{18}A \quad 1s^2 2s^2 2p^6 3s^2 3p^6$$

We get

$$ {}^{19}K \quad \text{———— "— } 3p^6 1s^1 $$

$$ {}^{20}Ca \quad \ldots\ldots \quad 3p^6 1s^2 $$

$$ {}^{21}Sc \quad \ldots \quad 3p^6 1s^2 3d^1 $$

etc. We now understand the structure of the periodic table, and that the regularities were telling us something about the structure of the atom. There are lots of interesting implications of all this (e.g. noble gases are relatively inert because they have filled shells), some of which are discussed in the text, + which are worth pursuing.

I'll leave you with the year 2000 version of the periodic table, + the question of What structure might be hiding there? 6.18

quarks:

	$(\text{isospin})_z$	electric charge
$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{1}{2}$ $-\frac{1}{2}$	$+2/3$ $-1/3$
leptons $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1