

Feel free to discuss the problems with me and/or each other. Each student must write up his/her own solutions separately.

Unless otherwise indicated, problems are from Giancoli, 4th edition.

1. Giancoli chapter 15, problem 4 (p. 418)
2. Giancoli chapter 15, problem 22 (p. 419)
3. Giancoli chapter 15, problem 31 (p. 419)
4. Giancoli chapter 15, problem 32 (p. 419)
5. Giancoli chapter 15, problem 34 (p. 419)

9. Two linear waves have the same amplitude and speed, and otherwise are identical, except one has half the wavelength of the other. Which transmits more energy? By what factor?
10. Will any function of  $(x - vt)$ —see Eq. 15-14—represent a wave motion? Why or why not? If not, give an example.
11. When a sinusoidal wave crosses the boundary between two sections of cord as in Fig. 15-19, the frequency does not change (although the wavelength and velocity do change). Explain why.
12. If a sinusoidal wave on a two-section cord (Fig. 15-19) is inverted upon reflection, does the transmitted wave have a longer or shorter wavelength?
13. Is energy always conserved when two waves interfere? Explain.
14. If a string is vibrating as a standing wave in three segments, are there any places you could touch it with a knife blade without disturbing the motion?

15. When a standing wave exists on a string, the vibrations of the incident and reflected waves cancel at the nodes. Does this mean that energy was destroyed? Explain.
16. Can the amplitude of the standing waves in Fig. 15-25 be greater than the amplitude of the vibrations that cause them (up and down motion of the hand)?
17. When a cord is vibrated as in Fig. 15-25 by hand or by a mechanical oscillator, the “nodes” are not quite true nodes (at rest). Explain. [Hint: Consider damping and energy loss from hand or oscillator.]
- \*18. AM radio signals can usually be heard behind a hill, although FM signals often cannot. That is, AM signals bend more than FM signals. Explain. (Radio signals, as we shall see, are carried by electromagnetic waves whose wavelength for AM is typically 200 to 600 m and for FM about 3 m.)
- \*19. If we knew that energy was being transmitted from one place to another, how might we determine whether the energy was being carried by particles (material objects) or by waves?

## Problems

### 15-1 and 15-2 Characteristics of Waves

1. (I) A fisherman notices that wave crests pass the bow of his anchored boat every 3.0 s. He measures the distance between two crests to be 8.0 m. How fast are the waves traveling?
2. (I) A sound wave in air has a frequency of 262 Hz and travels with a speed of 343 m/s. How far apart are the wave crests (compressions)?
3. (I) Calculate the speed of longitudinal waves in (a) water, (b) granite, and (c) steel.
4. (I) AM radio signals have frequencies between 550 kHz and 1600 kHz (kilohertz) and travel with a speed of  $3.0 \times 10^8$  m/s. What are the wavelengths of these signals? On FM the frequencies range from 88 MHz to 108 MHz (megahertz) and travel at the same speed. What are their wavelengths?
5. (I) Determine the wavelength of a 5800-Hz sound wave traveling along an iron rod.
6. (II) A cord of mass 0.65 kg is stretched between two supports 8.0 m apart. If the tension in the cord is 140 N, how long will it take a pulse to travel from one support to the other?
7. (II) A 0.40-kg cord is stretched between two supports, 7.8 m apart. When one support is struck by a hammer, a transverse wave travels down the cord and reaches the other support in 0.85 s. What is the tension in the cord?
8. (II) A sailor strikes the side of his ship just below the surface of the sea. He hears the echo of the wave reflected from the ocean floor directly below 2.8 s later. How deep is the ocean at this point?
9. (II) A ski gondola is connected to the top of a hill by a steel cable of length 660 m and diameter 1.5 cm. As the gondola comes to the end of its run, it bumps into the terminal and sends a wave pulse along the cable. It is observed that it took 17 s for the pulse to return. (a) What is the speed of the pulse? (b) What is the tension in the cable?
10. (II) P and S waves from an earthquake travel at different speeds, and this difference helps locate the earthquake “epicenter” (where the disturbance took place). (a) Assuming typical speeds of 8.5 km/s and 5.5 km/s for P and S waves, respectively, how far away did the earthquake occur if a particular seismic station detects the arrival of these two types of waves 1.7 min apart? (b) Is one seismic station sufficient to determine the position of the epicenter? Explain.

11. (II) The wave on a string shown in Fig. 15-33 is moving to the right with a speed of 1.10 m/s. (a) Draw the shape of the string 1.00 s later and indicate which parts of the string are moving up and which down at that instant. (b) Estimate the vertical speed of point A on the string at the instant shown in the figure.

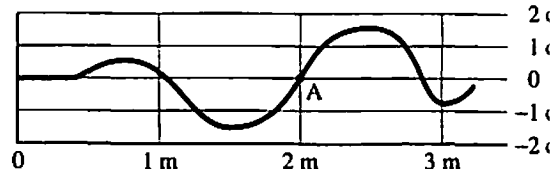


FIGURE 15-33 Problem 11.

12. (II) A 5.0 kg ball hangs from a steel wire 1.00 m in diameter and 5.00 m long. What would be the speed of a transverse wave in the steel wire?
13. (II) Two children are sending signals along a cord with mass 0.50 kg tied between tin cans with a tension of 180 N. It takes the vibrations in the string 0.50 s to go from one child to the other. How far apart are the children?
- \*14. (II) Dimensional analysis. Waves on the surface of the ocean do not depend significantly on the properties of the water such as density or surface tension. The “return force” for water piled up in the wave comes due to the gravitational attraction of the Earth. The speed  $v$  (m/s) of ocean waves depends on the acceleration due to gravity  $g$ . It is reasonable to expect that  $v$  might depend on water depth  $h$  and the wave’s wavelength  $\lambda$ . Assume the wave speed is given by the function  $v = Cg^\alpha h^\beta \lambda^\gamma$ , where  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $C$  are numbers with no dimension. (a) In deep water, the water depth below the surface does not affect the motion of waves at the surface. Use dimensional analysis (Section 1-7), determine the formula for the speed of surface waves in deep water. (b) In shallow water, the speed of surface waves is found experimentally to be independent of the wavelength (i.e.,  $\gamma = 0$ ). Use dimensional analysis, determine the formula for the speed of waves in shallow water.

### 15-3 Energy Transported by Waves

15. (I) Two earthquake waves of the same frequency travel through the same portion of the Earth, but one is carrying 3.0 times the energy. What is the ratio of the amplitudes of the two waves?
16. (II) What is the ratio of (a) the intensities, and (b) the amplitudes, of an earthquake P wave passing through the Earth and detected at two points 15 km and 45 km from the source.
17. (II) Show that if damping is ignored, the amplitude  $A$  of circular water waves decreases as the square root of the distance  $r$  from the source:  $A \propto 1/\sqrt{r}$ .
18. (II) The intensity of an earthquake wave passing through the Earth is measured to be  $3.0 \times 10^6 \text{ J/m}^2 \cdot \text{s}$  at a distance of 48 km from the source. (a) What was its intensity when it passed a point only 1.0 km from the source? (b) At what rate did energy pass through an area of  $2.0 \text{ m}^2$  at 1.0 km?
19. (II) A small steel wire of diameter 1.0 mm is connected to an oscillator and is under a tension of 7.5 N. The frequency of the oscillator is 60.0 Hz and it is observed that the amplitude of the wave on the steel wire is 0.50 cm. (a) What is the power output of the oscillator, assuming that the wave is not reflected back? (b) If the power output stays constant but the frequency is doubled, what is the amplitude of the wave?
20. (II) Show that the intensity of a wave is equal to the energy density (energy per unit volume) in the wave times the wave speed.
21. (II) (a) Show that the average rate with which energy is transported along a cord by a mechanical wave of frequency  $f$  and amplitude  $A$  is

$$\bar{P} = 2\pi^2 \mu v f^2 A^2,$$

where  $v$  is the speed of the wave and  $\mu$  is the mass per unit length of the cord. (b) If the cord is under a tension  $F_T = 135 \text{ N}$  and has mass per unit length  $0.10 \text{ kg/m}$ , what power is required to transmit 120-Hz transverse waves of amplitude 2.0 cm?

### 15-4 Mathematical Representation of Traveling Wave

22. (I) A transverse wave on a wire is given by  $D(x, t) = 0.015 \sin(25x - 1200t)$  where  $D$  and  $x$  are in meters and  $t$  is in seconds. (a) Write an expression for a wave with the same amplitude, wavelength, and frequency but traveling in the opposite direction. (b) What is the speed of either wave?
23. (I) Suppose at  $t = 0$ , a wave shape is represented by  $D = A \sin(2\pi x/\lambda + \phi)$ ; that is, it differs from Eq. 15-9 by a constant phase factor  $\phi$ . What then will be the equation for a wave traveling to the left along the  $x$  axis as a function of  $x$  and  $t$ ?
24. (II) A transverse traveling wave on a cord is represented by  $D = 0.22 \sin(5.6x + 34t)$  where  $D$  and  $x$  are in meters and  $t$  is in seconds. For this wave determine (a) the wavelength, (b) frequency, (c) velocity (magnitude and direction), (d) amplitude, and (e) maximum and minimum speeds of particles of the cord.
25. (II) Consider the point  $x = 1.00 \text{ m}$  on the cord of Example 15-5. Determine (a) the maximum velocity of this point, and (b) its maximum acceleration. (c) What is its velocity and acceleration at  $t = 2.50 \text{ s}$ ?
26. (II) A transverse wave on a cord is given by  $D(x, t) = 0.12 \sin(3.0x - 15.0t)$ , where  $D$  and  $x$  are in m and  $t$  is in s. At  $t = 0.20 \text{ s}$ , what are the displacement and velocity of the point on the cord where  $x = 0.60 \text{ m}$ ?

27. (II) A transverse wave pulse travels to the right along a string with a speed  $v = 2.0 \text{ m/s}$ . At  $t = 0$  the shape of pulse is given by the function

$$D = 0.45 \cos(2.6x + 1.2),$$

where  $D$  and  $x$  are in meters. (a) Plot  $D$  vs.  $x$  at  $t = 0$ . (b) Determine a formula for the wave pulse at any time assuming there are no frictional losses. (c) Plot  $D(x, t)$  vs.  $x$  at  $t = 1.0 \text{ s}$ . (d) Repeat parts (b) and (c) assuming the pulse is traveling to the left. Plot all 3 graphs on the same axes for easy comparison.

28. (II) A 524-Hz longitudinal wave in air has a speed 345 m/s. (a) What is the wavelength? (b) How much time required for the phase to change by  $90^\circ$  at a given point in space? (c) At a particular instant, what is the phase difference (in degrees) between two points 4.4 cm apart?
29. (II) Write the equation for the wave in Problem 28 traveling to the right, if its amplitude is 0.020 cm, and  $D = -0.020 \text{ cm}$  at  $t = 0$  and  $x = 0$ .
30. (II) A sinusoidal wave traveling on a string in the negative direction has amplitude 1.00 cm, wavelength 3.00 cm, a frequency 245 Hz. At  $t = 0$ , the particle of string at  $x = 0$  is displaced a distance  $D = 0.80 \text{ cm}$  above the origin and moving upward. (a) Sketch the shape of the wave at  $t = 0$ . (b) determine the function of  $x$  and  $t$  that describes the wave.

### 15-5 The Wave Equation

- \*31. (II) Determine if the function  $D = A \sin kx \cos \omega t$  is a solution of the wave equation.
- \*32. (II) Show by direct substitution that the following functions satisfy the wave equation: (a)  $D(x, t) = A \ln(x + vt)$  (b)  $D(x, t) = (x - vt)^4$ .
- \*33. (II) Show that the wave forms of Eqs. 15-13 and 15-15 satisfy the wave equation, Eq. 15-16.
- \*34. (II) Let two linear waves be represented by  $D_1 = f_1(x, t)$  and  $D_2 = f_2(x, t)$ . If both these waves satisfy the wave equation (Eq. 15-16), show that any combination  $D = C_1 D_1 + C_2 D_2$  does as well, where  $C_1$  and  $C_2$  are constants.
- \*35. (II) Does the function  $D(x, t) = e^{-(kx - \omega t)^2}$  satisfy the wave equation? Why or why not?
- \*36. (II) In deriving Eq. 15-2,  $v = \sqrt{F_T/\mu}$ , for the speed of a transverse wave on a string, it was assumed that the wave's amplitude  $A$  is much less than its wavelength. Assuming a sinusoidal wave shape  $D = A \sin(kx - \omega t)$ , show via the partial derivative  $v' = \partial D / \partial t$  that the assumption  $A \ll \lambda$  implies that the maximum transverse speed  $v'_{\text{max}}$  of the string itself is much less than the wave velocity. If  $A = \lambda/100$  determine the ratio  $v'_{\text{max}}/v$ .

### 15-7 Reflection and Transmission

37. (II) A cord has two sections with linear densities  $0.10 \text{ kg/m}$  and  $0.20 \text{ kg/m}$ , Fig. 15-34. An incident wave given by  $D = (0.050 \text{ m}) \sin(7.5x - 12.0t)$ , where  $x$  is in meters and  $t$  in seconds, travels along the lighter cord. (a) What is the wavelength on the lighter section of the cord? (b) What is the tension in the cord? (c) What is the wavelength when the wave travels on the heavier section?

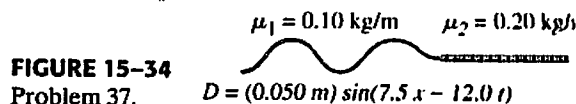


FIGURE 15-34  
Problem 37.