Physics 123, Spring 2009
Homework #10
Due in P123 homework locker 4pm, Friday, April 24, 2009

Feel free to discuss the problems with me and/or each other. Each student must write up his/her own solutions separately.
Unless otherwise indicated, problems are from Giancoli, 4th edition.

Note that the book's presentation differs slightly from that in lecture, so you should be sure to read the relevant sections in Giancoli before doing the problems.

1. Giancoli, chapter **37**, **question** 21, page 1011 (vol. III).
2. Giancoli, chapter 37, **question** 25.
3. Giancoli, chapter 37, problem 54.
4. Giancoli, chapter 37, problem 65.
5. Giancoli, chapter 37, problem 70.
6. Giancoli, chapter 37, problem 97.

He proposed the photoelectric effect as a test for the photon theory of light. In the **photoelectric effect**, the photon theory says that each incident photon can strike an electron in a material and eject it if the photon has sufficient energy. The maximum energy of ejected electrons is then linearly related to the frequency of the incident light.

The photon theory is also supported by the **Compton effect** and the observation of electron–positron **pair production**.

The **wave–particle duality** refers to the idea that light and matter (such as electrons) have both wave and particle properties. The wavelength of an object is given by

$$\lambda = \frac{h}{p}, \qquad (37\text{–}7)$$

where p is the momentum of the object ($p = mv$ for a particle of mass m and speed v).

The **principle of complementarity** states that we must be aware of both the particle and wave properties of light and of matter for a complete understanding of them.

Early models of the atom include the plum-pudding model, and Rutherford's planetary (or nuclear) model of an atom which consists of a tiny but massive positively charged nucleus surrounded (at a relatively great distance) by electrons.

To explain the **line spectra** emitted by atoms, as well as the stability of atoms, **Bohr's theory** postulated that: (1) electrons bound in an atom can only occupy orbits for which the angular momentum is quantized, which results in discrete values for the radius and energy; (2) an electron in such a **stationary state** emits no radiation; (3) if an electron jumps to a lower state, it emits a photon whose energy equals the difference in energy between the two states; (4) the angular momentum L of atomic electrons is quantized by the rule

$$L = \frac{nh}{2\pi}, \qquad (37\text{–}10)$$

where n is an integer called the **quantum number**. The $n = 1$ state is the **ground state**, which in hydrogen has an energy $E_1 = -13.6\,\text{eV}$. Higher values of n correspond to **excited states**, and their energies are

$$E_n = -(13.6\,\text{eV})\frac{Z^2}{n^2}. \qquad (37\text{–}14b)$$

Atoms are excited to these higher states by collisions with other atoms or electrons, or by absorption of a photon of just the right frequency.

De Broglie's hypothesis that electrons (and other matter) have a wavelength $\lambda = h/mv$ gave an explanation for Bohr's quantized orbits by bringing in the wave–particle duality: the orbits correspond to circular standing waves in which the circumference of the orbit equals a whole number of wavelengths.

Questions

1. What can be said about the relative temperatures of whitish-yellow, reddish, and bluish stars? Explain.
2. If energy is radiated by all objects, why can we not see most of them in the dark?
3. Does a lightbulb at a temperature of 2500 K produce as white a light as the Sun at 6000 K? Explain.
4. Darkrooms for developing black-and-white film were sometimes lit by a red bulb. Why red? Would such a bulb work in a darkroom for developing color photographs?
5. If the threshold wavelength in the photoelectric effect increases when the emitting metal is changed to a different metal, what can you say about the work functions of the two metals?
6. Explain why the existence of a cutoff frequency in the photoelectric effect more strongly favors a particle theory rather than a wave theory of light.
7. UV light causes sunburn, whereas visible light does not. Suggest a reason.
8. The work functions for sodium and cesium are 2.28 eV and 2.14 eV, respectively. For incident photons of a given frequency, which metal will give a higher maximum kinetic energy for the electrons?
9. (*a*) Does a beam of infrared photons always have less energy than a beam of ultraviolet photons? Explain. (*b*) Does a single photon of infrared light always have less energy than a single photon of ultraviolet light?
10. Light of 450-nm wavelength strikes a metal surface, and a stream of electrons emerges from the metal. If light of the same intensity but of wavelength 400 nm strikes the surface, are more electrons emitted? Does the energy of the emitted electrons change? Explain.
11. Explain how the photoelectric circuit of Fig. 37–4 could be used in (*a*) a burglar alarm, (*b*) a smoke detector, (*c*) a photographic light meter.
12. If an X-ray photon is scattered by an electron, does the photon's wavelength change? If so, does it increase or decrease?
13. In both the photoelectric effect and in the Compton effect, a photon collides with an electron causing the electron to fly off. What then, is the difference between the two processes?
14. Consider a point source of light. How would the intensity of light vary with distance from the source according to (*a*) wave theory, (*b*) particle (photon) theory? Would this help to distinguish the two theories?
15. If an electron and a proton travel at the same speed, which has the shorter de Broglie wavelength? Explain.
16. Why do we say that light has wave properties? Why do we say that light has particle properties?
17. Why do we say that electrons have wave properties? Why do we say that electrons have particle properties?
18. What are the differences between a photon and an electron? Be specific: make a list.
19. In Rutherford's planetary model of the atom, what keeps the electrons from flying off into space?
20. How can you tell if there is oxygen near the surface of the Sun?
21. When a wide spectrum of light passes through hydrogen gas at room temperature, absorption lines are observed that correspond only to the Lyman series. Why don't we observe the other series?
22. Explain how the closely spaced energy levels for hydrogen near the top of Fig. 37–26 correspond to the closely spaced spectral lines at the top of Fig. 37–21.

23. Is it possible for the de Broglie wavelength of a "particle" to be greater than the dimensions of the particle? To be smaller? Is there any direct connection?

24. In a helium atom, which contains two electrons, do you think that on average the electrons are closer to the nucleus or farther away than in a hydrogen atom? Why?

25. How can the spectrum of hydrogen contain so many lines when hydrogen contains only one electron?

26. The Lyman series is brighter than the Balmer series because this series of transitions ends up in the most common state for hydrogen, the ground state. Why then was the Balmer series discovered first?

27. Use conservation of momentum to explain why photons emitted by hydrogen atoms have slightly less energy than that predicted by Eq. 37–9.

28. Suppose we obtain an emission spectrum for hydrogen at very high temperature (when some of the atoms are in excited states), and an absorption spectrum at room temperature, when all atoms are in the ground state. Will the two spectra contain identical lines?

Problems

37–1 Planck's Quantum Hypothesis

1. (I) Estimate the peak wavelength for radiation from (*a*) ice at 273K, (*b*) a floodlamp at 3500 K, (*c*) helium at 4.2 K, (*d*) for the universe at $T = 2.725$ K, assuming blackbody emission. In what region of the EM spectrum is each?

2. (I) How hot is metal being welded if it radiates most strongly at 460 nm?

3. (I) An HCl molecule vibrates with a natural frequency of 8.1×10^{13} Hz. What is the difference in energy (in joules and electron volts) between successive values of the oscillation energy?

4. (II) Estimate the peak wavelength of light issuing from the pupil of the human eye (which approximates a blackbody) assuming normal body temperature.

5. (III) Planck's radiation law is given by:

$$I(\lambda, T) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

where $I(\lambda, T)$ is the rate energy is radiated per unit surface area per unit wavelength interval at wavelength λ and Kelvin temperature T. (*a*) Show that Wien's displacement law follows from this relationship. (*b*) Determine the value of h from the experimental value of $\lambda_P T$ given in the text. [You may want to use graphing techniques.] (*c*) Derive the T^4 dependence of the rate at which energy is radiated (as in the Stefan-Boltzmann law, Eq. 19–17), by integrating Planck's formula over all wavelengths; that is, show that

$$\int I(\lambda, T)\, d\lambda \propto T^4.$$

37–2 and 37–3 Photons and the Photoelectric Effect

6. (I) What is the energy of photons (in joules) emitted by a 104.1-MHz FM radio station?

7. (I) What is the energy range (in joules and eV) of photons in the visible spectrum, of wavelength 410 nm to 750 nm?

8. (I) A typical gamma ray emitted from a nucleus during radioactive decay may have an energy of 380 keV. What is its wavelength? Would we expect significant diffraction of this type of light when it passes through an everyday opening, such as a door?

9. (I) About 0.1 eV is required to break a "hydrogen bond" in a protein molecule. Calculate the minimum frequency and maximum wavelength of a photon that can accomplish this.

10. (I) Calculate the momentum of a photon of yellow light of wavelength 6.20×10^{-7} m.

11. (I) What minimum frequency of light is needed to eject electrons from a metal whose work function is 4.8×10^{-19} J?

12. (I) What is the longest wavelength of light that will emit electrons from a metal whose work function is 3.70 eV?

13. (II) What wavelength photon would have the same energy as a 145-gram baseball moving 30.0 m/s?

14. (II) The human eye can respond to as little as 10^{-18} J of light energy. For a wavelength at the peak of visual sensitivity, 550 nm, how many photons lead to an observable flash?

15. (II) The work functions for sodium, cesium, copper, and iron are 2.3, 2.1, 4.7, and 4.5 eV, respectively. Which of these metals will not emit electrons when visible light shines on it?

16. (II) In a photoelectric-effect experiment it is observed that no current flows unless the wavelength is less than 520 nm. (*a*) What is the work function of this material? (*b*) What is the stopping voltage required if light of wavelength 470 nm is used?

17. (II) What is the maximum kinetic energy of electrons ejected from barium $(W_0 = 2.48\text{ eV})$ when illuminated by white light, $\lambda = 410$ to 750 nm?

18. (II) Barium has a work function of 2.48 eV. What is the maximum kinetic energy of electrons if the metal is illuminated by UV light of wavelength 365 nm? What is their speed?

19. (II) When UV light of wavelength 285 nm falls on a metal surface, the maximum kinetic energy of emitted electrons is 1.70 eV. What is the work function of the metal?

20. (II) The threshold wavelength for emission of electrons from a given surface is 320 nm. What will be the maximum kinetic energy of ejected electrons when the wavelength is changed to (*a*) 280 nm, (*b*) 360 nm?

21. (II) When 230-nm light falls on a metal, the current through a photoelectric circuit (Fig. 37–4) is brought to zero at a stopping voltage of 1.84 V. What is the work function of the metal?

22. (II) A certain type of film is sensitive only to light whose wavelength is less than 630 nm. What is the energy (eV and kcal/mol) needed for the chemical reaction to occur which causes the film to change?

23. (II) The range of visible light wavelengths extends from about 410 nm to 750 nm. (*a*) Estimate the minimum energy (eV) necessary to initiate the chemical process on the retina that is responsible for vision. (*b*) Speculate as to why, at the other end of the visible range, there is a threshold photon energy beyond which the eye registers no sensation of sight. Determine this threshold photon energy (eV).

46. (II) Neutrons can be used in diffraction experiments to probe the lattice structure of crystalline solids. Since the neutron's wavelength needs to be on the order of the spacing between atoms in the lattice, about 0.3 nm, what should the speed of the neutrons be?

47. (II) An electron has a de Broglie wavelength $\lambda = 6.0 \times 10^{-10}$ m. (*a*) What is its momentum? (*b*) What is its speed? (*c*) What voltage was needed to accelerate it to this speed?

48. (II) What is the wavelength of an electron of energy (*a*) 20 eV, (*b*) 200 eV, (*c*) 2.0 keV?

49. (II) Show that if an electron and a proton have the same nonrelativistic kinetic energy, the proton has the shorter wavelength.

50. (II) Calculate the de Broglie wavelength of an electron in a TV picture tube if it is accelerated by 33,000 V. Is it relativistic? How does its wavelength compare to the size of the "neck" of the tube, typically 5 cm? Do we have to worry about diffraction problems blurring our picture on the screen?

51. (II) After passing through two slits separated by a distance of 3.0 μm, a beam of electrons creates an interference pattern with its second-order maximum at an angle of 55°. Find the speed of the electrons in this beam.

* 37–8 Electron Microscope

*52. (II) What voltage is needed to produce electron wavelengths of 0.28 nm? (Assume that the electrons are nonrelativistic.)

*53. (II) Electrons are accelerated by 3450 V in an electron microscope. Estimate the maximum possible resolution of the microscope.

37–10 and 37–11 Bohr Model

54. (I) For the three hydrogen transitions indicated below, with n being the initial state and n' being the final state, is the transition an absorption or an emission? Which is higher, the initial state energy or the final state energy of the atom? Finally, which of these transitions involves the largest energy photon? (*a*) $n = 1$, $n' = 3$; (*b*) $n = 6$, $n' = 2$; (*c*) $n = 4$, $n' = 5$.

55. (I) How much energy is needed to ionize a hydrogen atom in the $n = 3$ state?

56. (I) (*a*) Determine the wavelength of the second Balmer line ($n = 4$ to $n = 2$ transition) using Fig. 37–26. Determine likewise (*b*) the wavelength of the third Lyman line and (*c*) the wavelength of the first Balmer line.

57. (I) Calculate the ionization energy of doubly ionized lithium, Li^{2+}, which has $Z = 3$.

58. (I) Evaluate the Rydberg constant R using the Bohr model (compare Eqs. 37–8 and 37–15) and show that its value is $R = 1.0974 \times 10^7\ \text{m}^{-1}$.

59. (II) What is the longest wavelength light capable of ionizing a hydrogen atom in the ground state?

60. (II) In the Sun, an ionized helium (He^+) atom makes a transition from the $n = 5$ state to the $n = 2$ state, emitting a photon. Can that photon be absorbed by hydrogen atoms present in the Sun? If so, between what energy states will the hydrogen atom transition occur?

61. (II) What wavelength photon would be required to ionize a hydrogen atom in the ground state and give the ejected electron a kinetic energy of 20.0 eV?

62. (II) For what maximum kinetic energy is a collision between an electron and a hydrogen atom in its ground state definitely elastic?

63. (II) Construct the energy-level diagram for the He^+ ion (like Fig. 37–26).

64. (II) Construct the energy-level diagram (like Fig. 37–26) for doubly ionized lithium, Li^{2+}.

65. (II) Determine the electrostatic potential energy and the kinetic energy of an electron in the ground state of the hydrogen atom.

66. (II) An excited hydrogen atom could, in principle, have a diameter of 0.10 mm. What would be the value of n for a Bohr orbit of this size? What would its energy be?

67. (II) Is the use of nonrelativistic formulas justified in the Bohr atom? To check, calculate the electron's velocity, v, in terms of c, for the ground state of hydrogen, and then calculate $\sqrt{1 - v^2/c^2}$.

68. (II) A hydrogen atom has an angular momentum of $5.273 \times 10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$. According to the Bohr model, what is the energy (eV) associated with this state?

69. (II) Assume hydrogen atoms in a gas are initially in their ground state. If free electrons with kinetic energy 12.75 eV collide with these atoms, what photon wavelengths will be emitted by the gas?

70. (II) Suppose an electron was bound to a proton, as in the hydrogen atom, but by the gravitational force rather than by the electric force. What would be the radius, and energy, of the first Bohr orbit?

71. (II) *Correspondence principle*: Show that for large values of n, the difference in radius Δr between two adjacent orbits (with quantum numbers n and $n - 1$) is given by

$$\Delta r = r_n - r_{n-1} \approx \frac{2r_n}{n},$$

so $\Delta r/r_n \rightarrow 0$ as $n \rightarrow \infty$ in accordance with the correspondence principle. [Note that we can check the correspondence principle by either considering large values of n ($n \rightarrow \infty$) or by letting $h \rightarrow 0$. Are these equivalent?]

General Problems

72. If a 75-W lightbulb emits 3.0% of the input energy as visible light (average wavelength 550 nm) uniformly in all directions, estimate how many photons per second of visible light will strike the pupil (4.0 mm diameter) of the eye of an observer 250 m away.

73. At low temperatures, nearly all the atoms in hydrogen gas will be in the ground state. What minimum frequency photon is needed if the photoelectric effect is to be observed?

74. A beam of 125-eV electrons is scattered from a crystal, as in X-ray diffraction, and a first-order peak is observed at $\theta = 38°$. What is the spacing between planes in the diffracting crystal? (See Section 35–10.)

75. A microwave oven produces electromagnetic radiation at $\lambda = 12.2$ cm and produces a power of 860 W. Calculate the number of microwave photons produced by the microwave oven each second.

76. Sunlight reaching the Earth has an intensity of about $1350\,\mathrm{W/m^2}$. Estimate how many photons per square meter per second this represents. Take the average wavelength to be 550 nm.

77. A beam of red laser light ($\lambda = 633\,\mathrm{nm}$) hits a black wall and is fully absorbed. If this light exerts a total force $F = 6.5\,\mathrm{nN}$ on the wall, how many photons per second are hitting the wall?

78. The Big Bang theory states that the beginning of the universe was accompanied by a huge burst of photons. Those photons are still present today and make up the so-called cosmic microwave background radiation. The universe radiates like a blackbody with a temperature of about 2.7 K. Calculate the peak wavelength of this radiation.

79. An electron and a positron collide head on, annihilate, and create two 0.755-MeV photons traveling in opposite directions. What were the initial kinetic energies of electron and positron?

80. By what potential difference must (*a*) a proton ($m = 1.67 \times 10^{-27}\,\mathrm{kg}$), and (*b*) an electron ($m = 9.11 \times 10^{-31}\,\mathrm{kg}$), be accelerated to have a wavelength $\lambda = 6.0 \times 10^{-12}\,\mathrm{m}$?

81. In some of Rutherford's experiments (Fig. 37–17) the α particles ($\mathrm{mass} = 6.64 \times 10^{-27}\,\mathrm{kg}$) had a kinetic energy of 4.8 MeV. How close could they get to the center of a silver nucleus ($\mathrm{charge} = +47e$)? Ignore the recoil motion of the nucleus.

82. Show that the magnitude of the electrostatic potential energy of an electron in any Bohr orbit of a hydrogen atom is twice the magnitude of its kinetic energy in that orbit.

83. Calculate the ratio of the gravitational force to the electric force for the electron in a hydrogen atom. Can the gravitational force be safely ignored?

84. Electrons accelerated by a potential difference of 12.3 V pass through a gas of hydrogen atoms at room temperature. What wavelengths of light will be emitted?

85. In a particular photoelectric experiment, a stopping potential of 2.70 V is measured when ultraviolet light of wavelength 380 nm is incident on the metal. If blue light of wavelength 440 nm is used, what is the new stopping potential?

86. In an X-ray tube (see Fig. 35–26 and discussion in Section 35–10), the high voltage between filament and target is V. After being accelerated through this voltage, an electron strikes the target where it is decelerated (by positively charged nuclei) and in the process one or more X-ray photons are emitted. (*a*) Show that the photon of shortest wavelength will have

$$\lambda_0 = \frac{hc}{eV}.$$

(*b*) What is the shortest wavelength of X-ray emitted when accelerated electrons strike the face of a 33-kV television picture tube?

87. The intensity of the Sun's light in the vicinity of Earth is about $1350\,\mathrm{W/m^2}$. Imagine a spacecraft with a mirrored square sail of dimension 1.0 km. Estimate how much thrust (in newtons) this craft will experience due to collisions with the Sun's photons. [*Hint:* Assume the photons bounce perpendicularly off the sail with no change in the magnitude of their momentum.]

88. Photons of energy 9.0 eV are incident on a metal. It is found that current flows from the metal until a stopping potential of 4.0 V is applied. If the wavelength of the incident photons is doubled, what is the maximum kinetic energy of the ejected electrons? What would happen if the wavelength of the incident photons was tripled?

89. Light of wavelength 360 nm strikes a metal whose work function is 2.4 eV. What is the shortest de Broglie wavelength for the electrons that are produced as photoelectrons?

90. Visible light incident on a diffraction grating with slit spacing of 0.012 mm has the first maximum at an angle of 3.5° from the central peak. If electrons could be diffracted by the same grating, what electron velocity would produce the same diffraction pattern as the visible light?

91. (*a*) Suppose an unknown element has an absorption spectrum with lines corresponding to 2.5, 4.7, and 5.1 eV above its ground state, and an ionization energy of 11.5 eV. Draw an energy level diagram for this element. (*b*) If a 5.1-eV photon is absorbed by an atom of this substance, in which state was the atom before absorbing the photon? What will be the energies of the photons that can subsequently be emitted by this atom?

92. Light of wavelength 424 nm falls on a metal which has a work function of 2.28 eV. (*a*) How much voltage should be applied to bring the current to zero? (*b*) What is the maximum speed of the emitted electrons? (*c*) What is the de Broglie wavelength of these electrons?

93. (*a*) Apply Bohr's assumptions to the Earth–Moon system to calculate the allowed energies and radii of motion. (*b*) Given the known distance between Earth and the Moon, is the quantization of the energy and radius apparent?

94. Show that the wavelength of a particle of mass m with kinetic energy K is given by the relativistic formula $\lambda = hc/\sqrt{K^2 + 2mc^2K}$.

95. A small flashlight is rated at 3.0 W. As the light leaves the flashlight in one direction, a reaction force is exerted on the flashlight in the opposite direction. Estimate the size of this reaction force.

96. At the atomic-scale, the electron volt and nanometer are well-suited units for energy and distance, respectively. (*a*) Show that the energy E in eV of a photon, whose wavelength λ is in nm, is given by

$$E = \frac{1240\,\mathrm{eV\cdot nm}}{\lambda\,(\mathrm{nm})}.$$

(*b*) How much energy (eV) does a 650-nm photon have?

***97.** Three fundamental constants of nature—the gravitational constant G, Planck's constant h, and the speed of light c—have the dimensions of $[L^3/MT^2]$, $[ML^2/T]$, and $[L/T]$, respectively. (*a*) Find the mathematical combination of these fundamental constants that has the dimension of time. This combination is called the "Planck time" t_P and is thought to be the earliest time, after the creation of the universe, at which the currently known laws of physics can be applied. (*b*) Determine the numerical value of t_P. (*c*) Find the mathematical combination of these fundamental constants that has the dimension of length. This combination is called the "Planck length" λ_P and is thought to be the smallest length over which the currently known laws of physics can be applied. (*d*) Determine the numerical value of λ_P.

98. Imagine a free particle of mass m bouncing back and forth between two perfectly reflecting walls, separated by distance ℓ. Imagine that the two oppositely directed matter waves associated with this particle interfere to create a standing wave with a node at each of the walls. Show that the ground state (first harmonic) and first excited state (second harmonic) have (non-relativistic) kinetic energies $h^2/8m\ell^2$ and $h^2/2m\ell^2$, respectively.