Physics 123, Spring 2009
Homework #2
Due in P123 homework locker 4pm, Friday, February 6, 2009

Feel free to discuss the problems with me and/or each other. Each student must write up his/her own solutions separately.

Maxwell's equations in vacuum:

$\vec{\nabla} \cdot \vec{E} = 0$: Gauss's Law

$\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$: Faraday's Law

$\vec{\nabla} \times \vec{B} = +\frac{1}{c^2}\partial \vec{E}/\partial t$: Ampère's Law

Recall the vector derivative definitions:

$\vec{\nabla} \equiv \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y) + \hat{z}(\partial/\partial z)$

$\vec{\nabla} \cdot \vec{A} \equiv \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$

$\vec{\nabla} \times \vec{A} \equiv (\partial A_z/\partial y - \partial A_y/\partial z)\hat{x} + (\partial A_x/\partial z - \partial A_z/\partial x)\hat{y} + (\partial A_y/\partial x - \partial A_x/\partial y)\hat{z}$

1. A sinusoidal traveling wave (not necessarily a light wave) has frequency 880 Hz and velocity 440 m/sec. (a) At a given time, find the distance between any two locations that correspond to a difference in phase of $\pi/6$ radians. (b) At a fixed location, by how much does the phase (the argument of the sine function) change during a time interval of 1.0×10^{-4} sec?

2. We showed in class that a solution to Maxwell's equations in vacuum (i.e. with no charges or currents) looks like

$$\vec{E} = \vec{E}_0 \sin(\vec{k} \cdot \vec{x} - \omega t),$$

$$\vec{B} = \vec{B}_0 \sin(\vec{k} \cdot \vec{x} - \omega t)$$

where $\vec{E}_0$ and $\vec{B}_0$ are constant vectors. Use this solution to show that Gauss's Law implies that $\vec{k} \cdot \vec{E} = 0$, and that the second Maxwell's equation implies $\vec{k} \cdot \vec{B} = 0$, i.e. that the electric and magnetic fields are perpendicular to the direction of propagation of the light (transverse waves).

3. Similarly, use Faraday's Law to show $\vec{k}\times\vec{E} = \omega\vec{B}$ and that this implies $\vec{E}\cdot\vec{B} = 0$.

4. Use the result of problem 3 to show $|\vec{B_0}| = |\vec{E_0}|/c$. Show that this is consistent with Ampère's Law.

5. An electromagnetic wave has an electric field given by

$$\vec{E} = \hat{x}\ (225\ \mathrm{V/m})\ \sin\left[(0.077\ \mathrm{m}^{-1})\, z - (2.3\times 10^7 \mathrm{rad/s})\, t\right]$$

   (a) What are the wavelength and frequency of the wave?
   (b) Write down an expression for the magnetic field.

6. Suppose two waves of equal amplitude and frequency have a phase difference δ as they travel in the same medium. They can be represented as

$$D_1 = A\sin(kx - \omega t)$$

$$D_2 = A\sin(kx - \omega t + \delta).$$

   (a) Use the trigonometric identity $\sin\theta_1 + \sin\theta_2 = 2\sin[(\theta_1+\theta_2)/2]\cos[(\theta_1-\theta_2)/2]$ to show that the resultant wave is given by

$$D = 2A\cos(\delta/2)\ \ \sin(kx - \omega t + \delta/2).$$

   (b) What is the amplitude of this resultant wave? Is it sinusoidal, or not? (c) Show that constructive interference occurs if $\delta = 0, 2\pi, 4\pi, ...$ and destructive interference occurs if $\delta = \pi, 3\pi, 5\pi...$