Physics 142, Fall 2013
Homework #3
Due in P142 homework locker 4pm, Friday, September 27, 2013

Feel free to discuss the problems with me and/or each other. Each student must write up his/her own solutions separately.

Unless otherwise indicated, problems are from Ohanian and Markert, Third Edition, Volume 2.

1. Rochester is at about 43 degrees north latitude. Find the surface area of the earth which lies north of Rochester (approximately). To do this, use spherical coordinates to calculate the outside surface area of a spherical "cap" whose edge reaches Rochester. (Don't forget that latitude is measured from the equator; θ is measured from the vertical, in this case the north pole.) Assume the sphere has radius R. Take $R = 6.4 \times 10^6$ m and give a numerical answer. What fraction of the earth's surface does this represent?

2. Find the gradients ($\vec{\nabla} f$) of the following functions:

   (a) $f(x, y, z) = x^2 + y^3 + z^4$
   (b) $f(x, y, z) = x^2 + y^3 x^3$
   (c) $f(x, y, z) = e^x \sin(y) \ln(z)$

3. What are the dimensions of $\vec{\nabla} r$ where $r = \sqrt{x^2 + y^2 + z^2}$? Find $\vec{\nabla} r$ and express it in terms of the unit vector $\hat{\mathbf{r}}$.

4. The height of a certain hill (in meters) is given by

   $$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12),$$

   where y is the distance in kilometers north, and x is the distance east, from some reference point.

   (a) What is $\vec{\nabla} h(x, y)$?
   (b) Where is the top of the hill located?
   (c) How high is the hill?
   (d) How steep is the slope (in meters of height per kilometer of distance) at a point 1 km north and 1 km east of the reference? In what direction is the slope steepest at that point?

5. Calculate the divergence of $\vec{v}$, $(\vec{\nabla} \cdot \vec{v})$ for the following vector functions:

   (a) $\vec{v} = 2x^2\hat{x} + xz^2\hat{y} - xyz\hat{z}$
   (b) $\vec{v} = xy\hat{x} + 2yz^2\hat{y} + 3xz^2\hat{z}$
   (c) $\vec{v} = y^2\hat{x} + (2x + z^2)\hat{y} - 2xy\hat{z}$