Physics 142, Fall 2013
Homework #4
Due in P142 homework locker 4pm, Friday, October 4, 2013

Feel free to discuss the problems with me and/or each other. Each student must write up his/her own solutions separately.

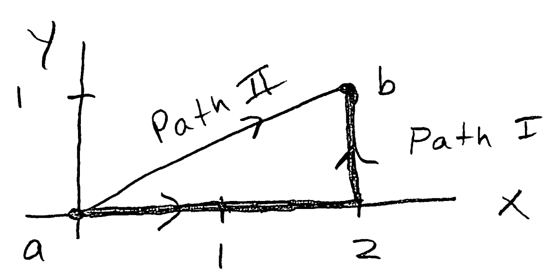
Unless otherwise indicated, problems are from Ohanian and Markert, Third Edition, Volume 2.

1. Calculate the curl of $\vec{v}$, $(\vec{\nabla} \times \vec{v})$ for the following vector functions:

   (a) $\vec{v} = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$
   (b) $\vec{v} = xy\hat{x} + 2yz^2\hat{y} + 3xz\hat{z}$
   (c) $\vec{v} = y^2\hat{x} + (2xy + z^2)\hat{y} - 2yz\hat{z}$

2. The electric field $\vec{E}$ has zero curl. We will see that $\vec{E}$ can be written as the gradient of a scalar function called the electrostatic potential; from this follows automatically that curl of $\vec{E}$ is zero. Show that the gradient of *any* scalar function has zero curl. That is, use the definitions of gradient and curl to show that
$$\vec{\nabla} \times (\vec{\nabla} f) = 0$$
for a scalar function $f(\vec{r})$.

3. (20 points) Demonstrate the path independence of the gradient integral for the function $f(x) = xy^2$ by computing
$$\int_a^b \vec{\nabla} f \cdot d\vec{l}$$
for paths I and II described below. Take $a = (0,0,0)$ and $b = (2,1,0)$.
Path I: $a \to (2,0,0)$ along the x axis, then $(2,0,0) \to b$ parallel to the y axis.
Path II: Straight line path from a to b. (Hint: There is a relation between dx and dy on this path.)



4. O&M chapter 23, problem 31.

5. O&M chapter 23, problem 32.