- Feel free to discuss the problems with me and/or each other.

1. Show that an electron and positron (each with mass m_e) cannot annihilate into a (massless) photon and still conserve energy. Similarly, show that an electron cannot radiate a photon and conserve energy while remaining on shell. Are these results still true when $m_e \to 0$? What about an electron and positron annihilating to create a Z^0 with mass m_Z? If it is possible, what condition must the initial momenta satisfy? (Hint: Assume all particles are on shell and then consider conservation of energy and momentum.)

2. The W and Z^0 bosons have the following major decay modes:

$$W^+ \to e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau, u\bar{d}, c\bar{s};$$

$$Z^0 \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \nu_e\bar{\nu_e}, \nu_\mu\bar{\nu_\mu}, \nu_\tau\bar{\nu_\tau}, u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}.$$

   We can estimate the branching ratios by counting states in a way similar to how we estimate the ratio of quark to muon creation rates in e^+e^- collisions.
   (a) Estimate the W and Z branching ratios for the decays listed above. Compare to the Particle Data Booklet listing to see how reasonable the anwers are; you may have to combine some modes.
   (b) The top quark almost always decays to a b quark and a W^+, which means that the W branching ratios determine those of the top. $t\bar{t}$ events are labeled, according to the decays of the two W's, as dilepton events, lepton+jets, or all jet events. What percentage of each type of event do you expect? What are the dilepton and lepton+jet event percentages if you don't include the τ's? Why do you think you would leave out the τ's?

3. Determine how far the following particles travel in the lab before decaying, and comment on the implications for seeing these particles in detectors. Consult the particle data group (in the booklet or online at pdg.lbl.gov) for the relevant masses and lifetimes (or decay widths as the case may be).
   (a) A top quark with (3-)momentum equal to its mass.
   (b) A π^0 meson with energy 15 GeV.
   (c) A muon with energy 50 GeV.
   (d) A 30 GeV K_S^0 meson.
   (e) A 30 GeV K_L^0 meson. (If you didn't know already, this problem should tell you what the subscripts L and S stand for.)
   (f) A B^+ meson with total energy 5.290 GeV. (This is the energy of a B from decay of an $\Upsilon(4S)$ at rest, as in the CLEO experiment.)
   (g) An electron with energy 100 GeV.

4. Contravariant four-vectors transform under Lorentz transformations according to
$$a'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} a^{\nu}.$$
For transformation to a frame moving with velocity v along the $+z$ direction, what are the values of $\frac{\partial x'^{\mu}}{\partial x^{\nu}}$ for each μ, ν? We can consider $\frac{\partial x'^{\mu}}{\partial x^{\nu}}$ to be a 4×4 matrix A, with rows and columns labeled by μ, ν. Show that, for this example, $\det A = 1$.