Physics 582, Fall 2006
Homework # 2
Due in class, Tuesday, November 14, 2006

- Feel free to discuss the problems with me and/or each other.

1. These questions refer to the Dirac Lagrangian and QED as discussed in class.

   (a) Show that, for a free Dirac field, the probability current $j^\mu = \bar{\psi}\gamma^\mu\psi$ is conserved, *i.e.*, show that $\partial_\mu j^\mu = 0$. (Hint: You will need the equation of motion for $\bar{\psi}$. You can get it from the Euler-Lagrange equations or from taking the Hermitian conjugate of the equation for ψ. Be careful with signs!)

   (b) Now suppose that the fermion has charge e, and we have a QED current $j^\mu_{QED} = e\bar{\psi}\gamma^\mu\psi$. Show that this current is conserved in QED ($\partial_\mu j^\mu_{QED} = 0$).

   (c) Show that the QED Lagrangian is invariant under local gauge transformations.

   (d) Show that a mass term for the photon, which would be of the form $m^2 A^\mu A_\mu$ is not gauge invariant.

2. Consider the cross section calculation for $e^+e^- \to \mu^+\mu^-$; when we did the calculation in class we neglected all masses. Show that, when you include the muon mass (but still neglecting the electron mass), the differential cross section in the center of mass is (with $E = E_{cm}/2 = \sqrt{s}/2$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s^2}\left(\frac{|\vec{p}\prime|}{E}\right)(E^2 + m^2 + |\vec{p}\prime|^2 cos^2\theta)$$

   and the total cross section is

$$\sigma = \frac{4\pi\alpha^2}{s^2}\left(\frac{|\vec{p}\prime|}{E}\right)(E^2 + m^2 + \frac{1}{3}|\vec{p}\prime|^2).$$

   Check that these reduce to the correct cross sections in the massless limit.