

Feel free to discuss the problems with me and/or each other.

1. We found that the coupling of a fermion f to the Z was given by

$$-i \frac{g}{\cos \theta_W} \bar{\psi}_f \gamma^\mu \left[\frac{1}{2} (1 - \gamma^5) T^3 - \sin^2 \theta_W Q \right] \psi_f Z_\mu.$$

The corresponding vertex factor (obtained here by stripping off the wave functions $\bar{\psi}_f$, ψ_f , and Z_μ) is often reexpressed in the general form

$$-i \frac{g}{\cos \theta_W} \gamma^\mu \frac{1}{2} (c_V^f - c_A^f \gamma^5).$$

- (a) What are the general expressions for c_V^f and c_A^f ?
- (b) What are c_V^f and c_A^f for $f =$ (i) neutrino, (ii) charged lepton (iii) up-type quark, and (iv) down-type quark? Assume $\sin^2 \theta_W = 0.23$.
- (c) The partial width for the decay of the Z into a fermion-antifermion pair is proportional to $c_V^2 + c_A^2$. Does this account for the difference between the branching ratios obtained by simple counting of states and those listed in the Particle Data Booklet?

2. Verify the form of the Lagrangian given in class for spontaneous breaking of a local U(1). Start with the gauge invariant Lagrangian

$$\mathcal{L} = (\partial^\mu + ieA^\mu)\Phi^*(\partial_\mu - ieA_\mu)\Phi - \mu^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Show that the substitution $\Phi(x) = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$ gives the Lagrangian on page 4.9 of the lecture notes. What are the interaction terms?

3. Because of radiative corrections, the value of coupling constants depends on the energy at which they are probed; they are said to “run” with energy. The strong coupling constant $\alpha_s(Q^2)$ as measured at the energy scale Q is given in terms of its value at another scale μ by (to lowest order)

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f) \log(Q^2/\mu^2)},$$

where n_f is the number of quark flavors with masses below the scale Q .

- (a) We see that α_s decreases as Q^2 gets large (“asymptotic freedom”). The flip side is that as Q^2 decreases, α_s increases without bound. We define the Q^2 scale at which $\alpha_s(Q^2)$ gets arbitrarily large (the denominator goes to zero) as Λ^2 . Find Λ^2 in terms of μ^2 and use it to show that we can write

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \log(Q^2/\Lambda^2)}.$$

- (b) Assuming $\Lambda = 200$ MeV, find $\alpha_s(m_Z^2)$, where $M_Z = 90$ GeV and $n_f = 5$. What is $\alpha_s((1 \text{ TeV})^2)$ ($n_f = 6$)?