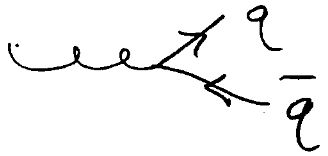
QCD and the Strong Interactions

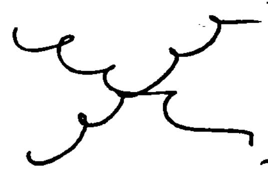
Now, finally, we come to the strong interactions as described by $SU(3)_c$. QCD arises from requiring invariance under local $SU(3)_{color}$, where, of the fermions, only the quarks transform. That is, only the quarks carry color; the leptons do not.

We know that requiring this invariance leads to the associated gauge bosons – the gluons. There are 8 generators for $SU(3)$ + hence 8 gluons. And because $SU(3)$ is nonabelian, we know that the gluons interact with each other, in addition to their interactions with the quarks.

So the QCD vertices are



$\sim g_s \gamma_\mu$ * color factor $\Big\}$ Works a lot like QED

and  and  $\Big\}$ we won't discuss these much.

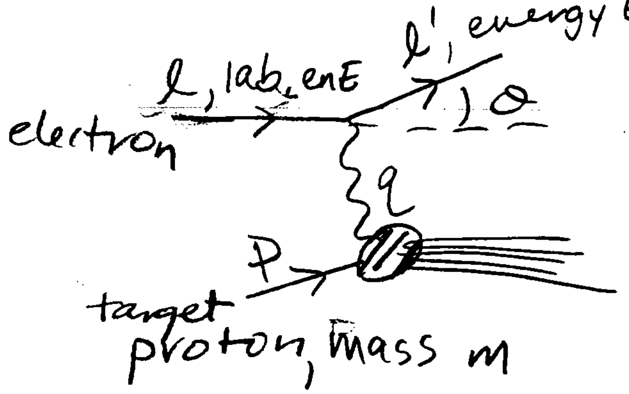
We could now discuss interactions of the quarks and gluons in pure QCD perturbation theory, like we did with QED. But we won't.

Instead, we recall that there are no free quarks and gluons — they're confined in hadrons. So we'll talk not only about QCD itself but the picture that's developed over the years (the parton model) of how to treat high energy interaction when quarks + gluons are involved. In other words, we'll talk about QCD in the context/framework relevant to high energy experiments. In fact, we'll talk more about the framework than about QCD itself. We start with

Deep Inelastic Scattering and the Parton Model

We'll ~~start~~ with deep inelastic electron-proton scattering. First some notation and kinematics.

Take p at rest in lab; e^- massless.



q = photon 4-mom $= \ell - \ell'$

$$Q^2 \equiv -q^2 = -(\ell-\ell')^2 = 4EE'\sin^2\theta/2$$

$$\nu \equiv \frac{q \cdot P}{m} = E - E'$$

For much of this I'm following Quigg, Chap 7.

* For a thorough, definitive discussion of all this see the soon-to be published *QCD + Collider Phys* by Ellis, Stirling, + Webber

We can define dimensionless variables x + y which we'll use below.

$$x = \frac{Q^2}{2q \cdot P} = \frac{Q^2}{2m\nu}$$

$$y = \frac{\nu}{E} = 1 - \frac{E'}{E}$$

We can write the $\overline{|M|^2}$ for this process as

$$\frac{1}{2} \sum_{spins} |M|^2 = \frac{e^4}{(q^2)^2} L^{\mu\nu} W_{\mu\nu}$$

where

- $L^{\mu\nu}$ comes from the electron part of the diagram

It's just like the $A_{\mu\nu}$ in $e^+e^- \to \mu^+\mu^-$; from that calc, we know (with the $\frac{1}{2}$ fr/ spin-avg. included)

$$L^{\mu\nu} = 2\left[\ell^\mu \ell'^\nu + \ell^\nu \ell'^\mu - g^{\mu\nu} \ell \cdot \ell'\right]$$

- $W^{\mu\nu}$ comes from the other vertex and contains all the hadronic stuff we don't understand

We can't calculate $W^{\mu\nu}$ from 1st principles (or 2nd or 3rd...), but we can use symmetries and Lorentz covariance to write it in the most general form we can. It must be a tensor because it has to contract w/ $L^{\mu\nu}$ to give a scalar. We can have only combinations of $g_{\mu\nu}$, p_μ, q_μ, $\epsilon_{\mu\nu\rho\sigma}$.
Current conservation and symmetry of $L^{\mu\nu}$ constrain the form $W_{\mu\nu}$ can take (see e.g. Quigg) and when all is said and done, we can write

$$W_{\mu\nu} = -W_1\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right] + \frac{W_2}{m^2}\left[p_\mu - \frac{p\cdot q}{q^2}q_\mu\right]\left[p_\nu - \frac{p\cdot q}{q^2}q_\nu\right]$$

W_1 and W_2 are form factors to be determined
- They're Lorentz invariant
- They depend, in principle, on Q^2 + ν.
  Since they have to be Lorentz inv. it's easiest to express them in terms of invariants. Q^2 is okay, but ν is not.
  We'll use $x \equiv \frac{Q^2}{2m\nu}$ instead $\Rightarrow W_1(x, Q^2)$
  $W_2(x, Q^2)$

Oops, ν is OK. x is just standard.

We can take the $\overline{|M|}^2$ and put it into the differential cross section for scattering the electron into energy E' and solid angle Ω';

$$\frac{d\sigma}{dE'd\Omega'} = \frac{\alpha^2}{4E^2\sin^2\frac{\theta}{2}}\left[2W_1(x,Q^2)\sin^2\frac{\theta}{2} + W_2(x,Q^2)\cos^2\frac{\theta}{2}\right]$$

NB. All hadronic phase space int. in the W's

Now, going back to our original expressions (p.5.2–5.3) we note that we can exchange the variables E' and $\cos\theta'$ for Q^2 and ν or, equivalently, the dimensionless var's x and y.

Also, let $F_1 \equiv M W_1$
$F_2 \equiv \nu W_2$

In principle, functions of x, Q^2

Then we have

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2 x y}\left[\frac{[1+(1-y)^2]}{2}\,2xF_1 + (1-y)\left[F_2 - 2xF_1\right] - \frac{M^2}{Q^2}x^2y^2F_2\right]$$

F_1 + F_2 are called the (electromagnetic) <u>structure functions</u> of the nucleon.

[Note: For neutrino scattering, we get a similar expression, with different structure fns]

Although in principle F_1 + F_2 are functions of x + Q^2, i.e., they depend on the details of the kinematics, an amazing thing happens in what's called the <u>Bjorken limit</u>; For $Q^2, q\cdot p \to \infty$ with x fixed, $\boxed{F_1 + F_2 \text{ depend only on } x\,!}$

This is called <u>Bjorken scaling</u>, and was seen in DIS experiments in the late 60's.

This means that if you measure, e.g. F_1 by fitting the cross section on the previous page, F_1's with the same x are all the same, even if Q^2 (which is the square of the momentum transfer from the electron to the proton) is different in each case. F_1 only depends on x + not the momentum transfer!

This is very important, because it implies that the photon is scattering off something pointlike in the proton. Why? Recall that the F's are dimensionless, and can only depend on dimensionless things. So if F were a function of Q^2 (or Q), it would have to be via Q/Q_0, and Q_0 would set the scale for whatever's going on inside the proton. But if what's going on has no scale – i.e., the proton has pointlike constituents – then there can be no Q^2 dependence.

[Foreshadowing: QCD predicts that scaling is not exact – that there is weak Q^2 dependence in the structure functions. This scaling violation is seen in current experiments but the uncertainties in the original SLAC experiments were too large for it to show up there. Just as well!]

Bjorken + Feynman immediately figured out the significance of this observation of scaling – that there were pointlike constituents of the proton. These are, of course, the quarks. Generically, the proton's constituents are called partons. Gluons can be partons too.

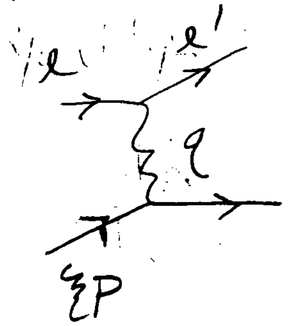
So let's see what the structure functions are in the context of the parton model. [At this level, it's called the "naive parton model". Below, when we introduce complications from QCD interactions, it becomes the "QCD-improved parton model"]

So we work in the "infinite momentum frame", where we can neglect the proton mass. We have

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left[1+(1-y)^2\right]F_1 + \frac{1-y}{x}\left[F_2 - 2xF_1\right]\right]$$

note change of var, fr/above

Now consider the photon scattering off a quark with charge e_q and fraction ξ of the proton's momentum. Let's call the cross section for this single process $\hat{\sigma}$. You can calculate this in QED just as we did $e^+e^- \to \mu^+\mu^-$:



$$\Rightarrow \frac{d^2\hat{\sigma}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left[1+(1-y)^2\right]\frac{e_q^2}{2}\,\delta(x-\xi)\right]$$

If we identify this with the expression on page 5.7, we see that this process gives the contributions to F_1 & F_2

$$\hat{F}_1 = x e_q^2\,\delta(x-\xi) \quad \& \quad \hat{F}_2 = 2x\hat{F}_1$$

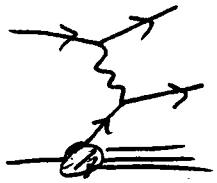
$\Rightarrow$ x is identified with the quark's momentum fraction $\Rightarrow$ F_2 probes a quark constituent with momentum fraction x.

Now the reason we put the "^" on σ and the F's is that this is just for one quark and one momentum fraction. To get the whole thing, we have to sum over all of them.

In our picture (the naive parton model), the virtual photon scatters incoherently off the quarks. (Works and is consistent w/ asymptotic freedom)

So let

$q(\xi)\,d\xi$ = probability that a quark q carries momentum fraction between ξ + $\xi + d\xi$



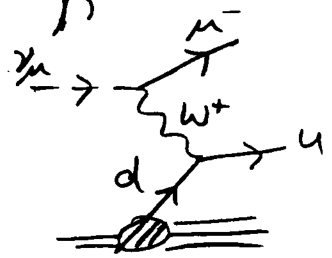
Then

$$F_2(x) = \sum_q \int_0^1 d\xi\; q(\xi) \times e_q^2\,\delta(x-\xi)$$

$$= \sum_q e_q^2\, x\, q(x)$$

So $$F_2(x) = x\left[\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{1}{9}s(x) + \frac{4}{9}\bar{u}(x) + \ldots\right]$$

[Similarly, ν scattering gives, e.g. from



$$F_2^\nu(x) = 2x\left[d(x) + s(x) + \bar{u}(x) + \bar{c}(x)\right]$$

etc.]

Note that there's no way of knowing a priori what the quark constituents are and what their momentum distributions $q(x)$ ("parton distributions") are. We have to figure all that out from the results of experiments.

[The story is told well in <u>The Hunting of the Quark</u> by Michael Riordan]

So the picture we have now, based on what we've seen in experiments, is

$$\text{proton} = \underset{\text{Valence quarks}}{u\,u\,d} + \underset{\text{"sea"}}{\overset{\text{infinite}}{q\bar{q}}} + \text{gluons}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$u_v(x),\ d_v(x) \qquad S(x) \qquad g(x)$$

With

$$u = u_v + S$$

$$d = d_v + S$$

$$\bar{u} = \bar{d} \simeq s = \bar{s} = S$$

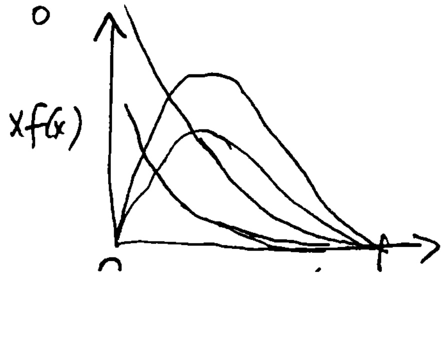
and the sum rules

$$\int_0^1 dx\, u_v(x) = 2\int_0^1 dx\, d_v(x) = 2$$

$$\sum_{q,\bar{q}} \int_0^1 dx\, x\, q(x) \simeq 0.5 = \text{fraction of mom. carried by quarks}$$

meas.

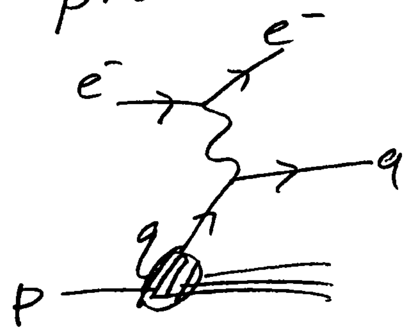
$$\Rightarrow \int_0^x dx\, x\, G(x) \simeq 0.5$$ Half of the momentum is carried by the gluons!



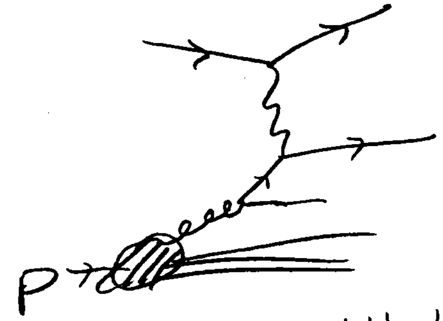
Scaling violation: QCD-improved parton model

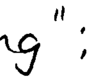
We can't calculate the structure functions or parton distributions from first principles in QCD, but QCD does tell us that there are small violations of scaling — that the parton distributions depend logarithmically on Q^2. In fact QCD does predict this Q^2 evolution quantitatively.

To see how the Q^2 dependence comes about, imagine deep inelastic scattering from a quark in the proton:



Now suppose the quark really came from



This is a small but measurable QCD correction. You can think of similar things that could happen. The bottom line is that how much you probe this structure depends on how much energy you put in, i.e. on Q^2. It results in Q^2-dependence of the parton distributions due to "parton splitting":  and . This is described by the DGLAP (Dokshitzer Gribov Lipatov Altarelli Parisi) eq'ns:

quark density

$$\frac{d}{d\ln Q^2} q_i(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left\{P^{qq}\left(\frac{x}{y}\right)q_i(y,t)+P^{qg}\left(\frac{x}{y}\right)g(y,t)\right\}$$

"splitting function"

+ similarly

$$\frac{d}{d\ln Q^2} g(x,t) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left\{P^{gq}\left(\frac{x}{y}\right)\sum_i q_i(y,t)+P^{gg}\left(\frac{x}{y}\right)g(y,t)\right\}$$

where $P^{ab}(z)$ = probability of finding parton a in parton b with momentum fraction z

= parton splitting function

Time doesn't permit deriving this here, but a beautiful, physically-motivated derivation can be found in one of my all-time favorite physics papers, Altarelli + Parisi, Nucl. Phys. B 126 (1977) 298.

Comments:

- This Q^2-evolution (= scaling violation) is observed in experiments
- The same evolution eq'ns govern the fragmentation functions that describe how quarks turn into hadrons
- splitting fns govern parton showers

Hadron-hadron interactions

If we're interested in hadron collisions — as many of us are — then we have to compute the cross section involving the quarks and gluons, and fold them in with the parton distributions. For definiteness, we'll consider $p\bar{p}$:

$$\sigma(p\bar{p} \to c + \underbrace{X}_{= \text{anything: the remnants of the proton...}}) = \int_0^1 f_a^p(x_a) f_b^{\bar{p}}(x_b)\, \hat{\sigma}(\overbrace{ab \to cX}^{\text{"hard subprocess"}})\, dx_a\, dx_b$$

So, for example, in $t\bar{t}$ production we need $\hat{\sigma}(q\bar{q} \to t\bar{t})$ and $\hat{\sigma}(gg \to t\bar{t})$

One thing we have to keep in mind in hadron collisions is that the cm frame for the hard subprocess is not, in general, the lab frame, because the colliding partons don't usually have equal values of x. And because the remnants of the p and $\bar{p}$ go down the beampipe, we don't detect them and so we don't even know what the hard subprocess cm frame was; we just know that its moving along the beam axis.

Not only that, but the events from different collisions are all from different cm frames! That means that when we combine them to measure cross sections, <u>the cross sections had better be invariant under longitudinal boosts</u> (= Lorentz transformations along the beam direction).

We recall from before that $d\sigma$ is Lorentz invariant, so we just have to make sure that the variables we use for differential cross sections are longitudinal-boost-invariant.

What is typically done, and what does the job, is to use, instead of p_x, p_y and p_z,

p_T = transverse momentum

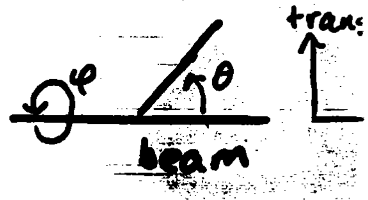
y = rapidity

φ = azimuthal angle

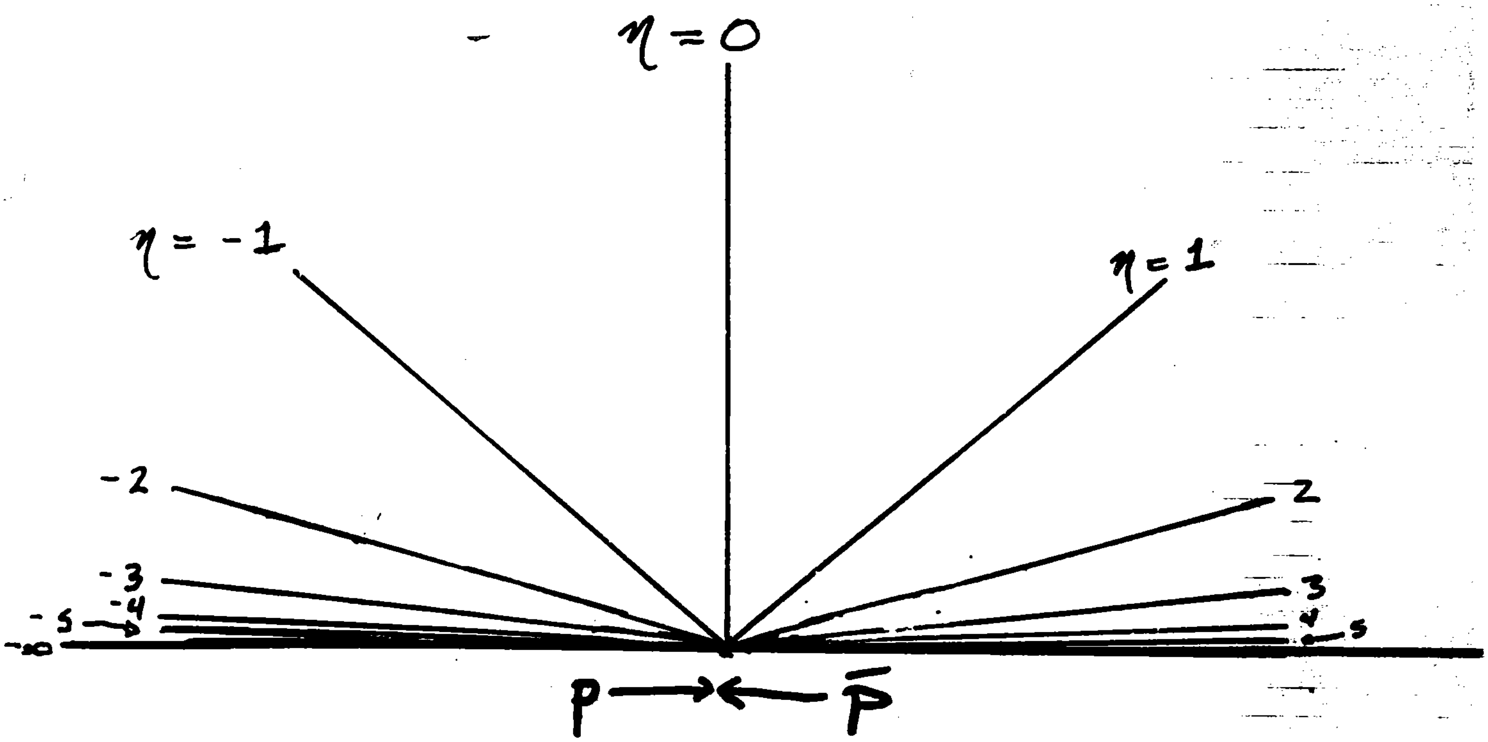
See next page for definitions.

Kinematical Variables for use in hadron collisions

$$\boxed{\begin{array}{l} \text{rapidity} = y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \\ \text{pseudo-rapidity} = \eta = -\ln\left(\tan\frac{\theta}{2}\right) \end{array}}$$



N.B.: $y = \eta$ for massless particles



$$\boxed{\begin{array}{l} \Delta R = \text{angular distance} = \sqrt{(\Delta\eta)^2 + (\Delta\varphi)^2} \\ p_T = \text{mom. transverse to beam} = |\vec{p}|\sin\theta \end{array}}$$

Running Coupling Const. + Asymptotic Freedom

Once again, time doesn't permit a proper and detailed treatment of renormalization and running of coupling constants, but it can be found in any field theory book, and Quigg has a very nice discussion.

Suffice it to say that so far we've mostly concerned ourselves with processes at tree level, but loop diagrams also contribute, and they give rise to "radiative corrections" to processes. These corrections give Q^2 dependence.

To compute loop diagrams we need one more Feynman rule. (We're not actually going to compute anything; I'm just including it for completeness.) The rule is

For 4-momentum q in a loop, integrate over $\frac{d^4q}{(2\pi)^4}$. If the particle in the loop is a fermion, take the trace and multiply by -1.

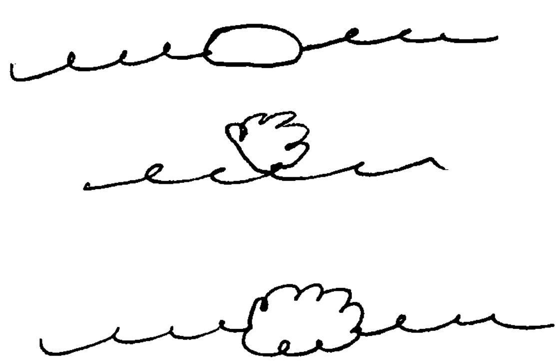
In general, the resulting integral is divergent and we must introduce a regularization scheme to remove the infinity and leave the physical, finite part.

The most common such scheme is <u>dimensional regularization</u>, in which we let the dimensionality of spacetime deviate slightly from 4. The divergences then show up as poles in the deviation(s) ϵ from 4 dimensions.

$\overline{MS}$ is a dimensional regularization scheme.

Now I'll just state the result for the calculation of the 1-loop corrections to the strong coupling constant α_s.

The diagrams that contribute are



quark loop

gluon loops.

and the quark loop and gluon loop pieces contribute with opposite signs. In QCD in the standard Model, the gluon contribution is bigger.

We find, that for a given value of $\alpha_s(\mu^2)$ at some scale μ^2, the value of α_s at another scale q^2 is given by

$$\frac{1}{\alpha_s(q^2)} = \frac{1}{\alpha_s(\mu^2)} + \left(\frac{\overset{\text{gluon contrib.}}{11N} - \overset{\text{quark contrib.}}{2n_f}}{12\pi}\right)\ln\left(\frac{-q^2}{\mu^2}\right)$$

where N is for $SU(N)$ and n_f is the number of quark flavors. $N=3$ and $n_f=6$ in the SM,

So for large q^2, α_s becomes small and we have asymptotic freedom. We discussed some of the consequences in the intro. section.

N.B. Running of α_s observed!

This is just the tip of the iceberg. For more good stuff see <u>QCD and Collider Physics</u> by Ellis, Stirling and Webber, to be published in the next few months.