Limitations of the Standard Model

(1.27)

The SM works beautifully, + is consistent with all experiments performed to date. It has limitations, though, and we don't believe it's the final theory of everything. In no particular order, here is a partial list of open questions to keep theorists (+ exptalists) busy for some time:

- $\cancel{CP}$

  CP violation isn't explained in the SM; it's simply put by hand into the CKM matrix

- Fermion masses

  related?

  Why do the fermions have the masses they do?
  Why is top so heavy? Are ν's massless?

- Fermion generations

  Why 3 generations of fermions?

- Fine tuning / hierarchy / naturalness

  Higgs —○— — $\Rightarrow$ Quadratic divergences in Higgs mass radiative corrections require delicate cancellations of huge numbers: this seemingly arbitrary fine-tuning is "unnatural."

  Related: If the SM is all there is, why the huge gap btwn the electroweak scale (10^2 GeV) and the Planck scale (10^{19} GeV), where gravity presumably comes in.

- EWSB + triviality

  Single scalar field theory (i.e. the theory that describes the Higgs) doesn't exist! Renormalization group calculations show that the Higgs self coupling ultimately $\to 0$. That means the Higgs has no self-interactions, which are needed to break the EW symmetry.

- Dark matter / dark energy

  90% of the mass of the universe is non-luminous, + we don't know the 1st thing about it. (Well, the first thing is that it's there but nonluminous + not usual SM stuff. We don't know the next thing about it.)

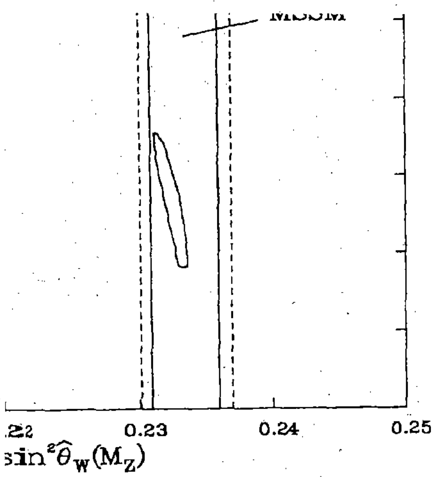
- Gravity

  How does a quantum th. of gravity fit in? What is a quantum theory of gravity?

- Unification

  Are the strong, weak, + EM forces aspects of a single unified force? Not in the SM, if we mean unification of coupling constants; see fig., p. 1.29

Ideally, we'd like a theory that explains everything, w/ no free parameters. String theory is a candidate, but making testable predictions is very difficult. Some



$s^2(M_Z)$ from $\alpha(M_Z)$ and $\alpha_s(M_Z)$ in MSSM) GUT's, compared with the at 90% C.L. The smaller ranges of (M_Z) and $\alpha(M_Z)$ only, while the larger v and high scale uncertainties added ions for $M_{SUSY}=M_Z$ and 1 TeV (see own for comparison.

the introduction of additional at the high scale, would allow cuss the high-scale thresholds in Sec. V, where we introduce ilar to those introduced for the hroughout this paper we display in a transparent form, which en- our discussion and to use the re- mmarize our conclusions in Sec.

O-LOOP PREDICTIONS

running of the couplings in any ntermediate scale, we can reduce grand desert and account for all desert boundaries by properly s. If one uses a two-loop β func- en one-loop threshold corrections The normalized couplings are

$$t+\theta_i-\Delta_i \quad \text{for } i=1,2,3\ , \qquad (9)$$

$_G/M_Z)$, M_G is the grand serves as the high-scale boundary the coupling at that point.

$$\left.\frac{\alpha_j(M_G)}{\alpha_j(M_Z)}\right|$$

be calculated to a precision consistent with the θ_i. Our ignorance of their exact values suggests that they should be reasonably parametrized and estimated within a given model, and then translated into theoretical uncertainties on any predictions. This will be carried out in the following sections. We will also show that for reasonable masses for the sparticles, the MSSM can be treated as a two-scale model with all mass effects included in the threshold corrections.

At the Z threshold (which serves as the low-scale boundary of the desert), we have

$$\frac{1}{\alpha_i(M_Z)}=\frac{3}{5}\frac{1-s^2(M_Z)}{\alpha(M_Z)},\frac{s^2(M_Z)}{\alpha(M_Z)},\frac{1}{\alpha_s(M_Z)}$$

$$\text{for } i=1,2,3\ ,$$

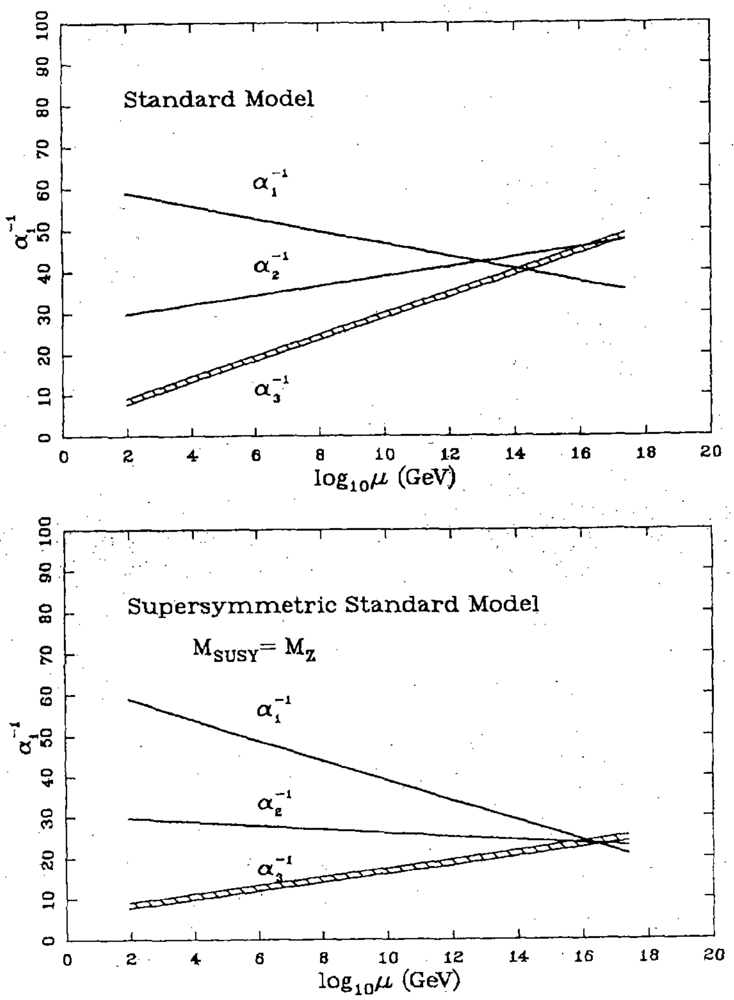


FIG. 3. The running coupling in ordinary (SM) and SUSY (MSSM) GUT's, assuming $s^2(M_Z)=0.2324\pm0.0006$, $1/\alpha(M_Z)=127.9\pm0.2$ (the larger uncertainty compared to (5) is due to m_t), and $\alpha_s(M_Z)=0.120\pm0.010$, for $M_{SUSY}=M_Z$. The uncertainties from threshold effects are best seen in Figs. 1 and 2.

Anatomy of a Cross Section + Fun with Orders of Magnitude

Before getting into detailed calculations, let's review what goes into them, then play with numbers.

In the beginning is the Lagrangian (or lagrangian density, where $L = \int d^3x \mathcal{L}$). The whole theory is contained in the Lagrangian. Let's assume f_i, f_j, f_k are some generic fields described by $\mathcal{L}$. $\boxed{\text{Schematically}}$* we have 3 types of terms in $\mathcal{L}$. They look like:

1. mass terms: $m f_i^2$ or $m^2 f_i^2$

   (m vs. m^2 depends on fermion vs boson)

2. Kinetic energy terms: $\partial f_i \cdot \partial f_i$ or $f_i \partial f_i$

3. interaction terms $g f_i f_j f_k$

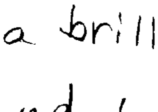
   (May contain derivatives, or >3 fields, or both.)

*I'm playing fast + loose w/ absolute values, antiparticles, overall factors, matrix structures, ...

All the physics is there. To calculate a cross section or decay rate, we use $\mathcal{L}$ to obtain transition probabilities from transition amplitudes (= transition matrix elements)

Basically, we have $\mathcal{M} \sim \langle a | e^{i \int d^4x \, \mathcal{L}} | b \rangle$, and a & b pick out the relevant terms in $\mathcal{L}$. Expanding the exponential gives a power series in the coupling constant, and Feynman diagrams are a brilliant way of skipping the gory algebra and just using the rules to write down $\mathcal{M}$ directly to whatever order you like. Again, schematically, the terms in $\mathcal{L}$ translate into Feynman rules as follows (the rules are in momentum space so derivatives become powers of momenta):

mass + kinetic $\Longrightarrow$ propagator ——— $\dfrac{\sim i}{p^2 - m^2}$ (e.g.)

Interaction $\Longrightarrow$ vertex  g

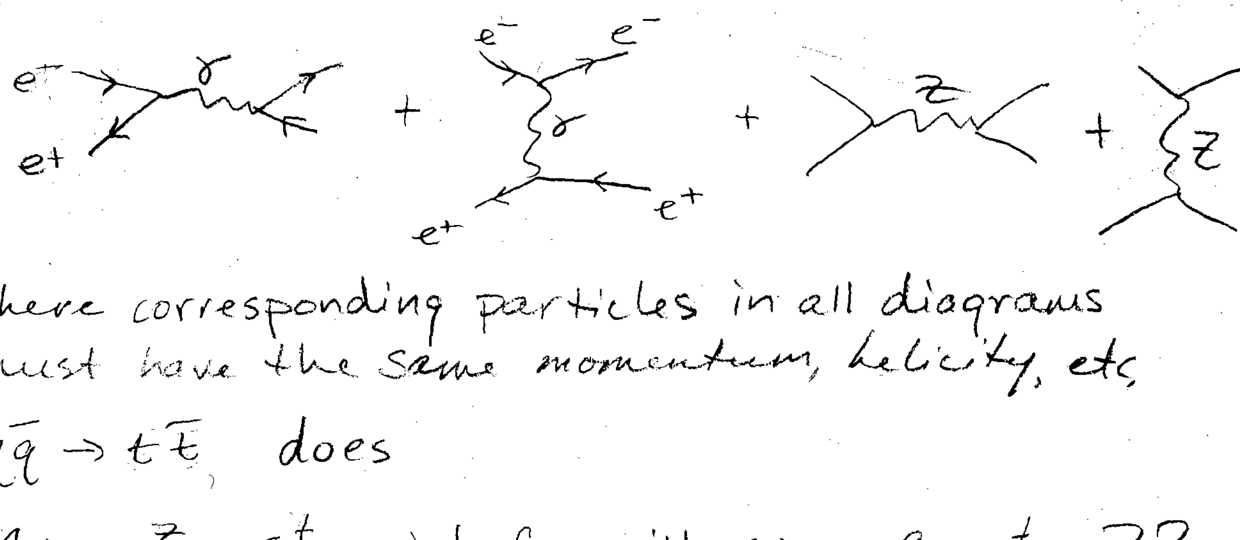
The coupling constant is g; vertices get hooked together with propagators, & each vertex gives one power of the coupling constant. The Feynman rules give the $\mathcal{M}$ that corresponds to the diagram.

[<u>Historical aside</u>: This was all worked out first in QED. Schwinger was King of the gory algebra, and Feynman came up w/ the diagram method which, mysteriously, gave the same answers. Dyson was the only guy who understood both + he showed that the two approaches were completely equivalent.]

Now, having been taught quantum mechanics from birth, we know that these amplitudes are complex numbers and that amplitudes for

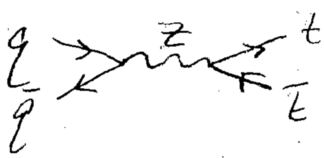
all diagrams with the <u>same</u> initial and final state must be added and then squared. The cross terms give interference between diagrams.

<u>Ex</u>: In $e^+e^- \to e^+e^-$ to lowest order, we must have

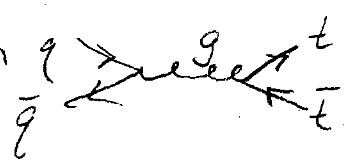


Where corresponding particles in all diagrams must have the same momentum, helicity, etc.

In $q\bar{q} \to t\bar{t}$ does



interfere with



??

All that remains is to combine the square of the amplitude with the initial flux, + sum over final states (i.e., integrate over final state phase space as necessary.) Schematically,

$$d\sigma = (\text{flux})\, |\mathcal{M}|^2 \, d\underbrace{\text{lips}}_{\substack{\text{Lorentz inv phase space} \\ \frac{d^3p}{2E}}} \, \delta^4(\;)$$

4-mom cons.

$$\boxed{d\sigma = \text{flux} * |\mathcal{M}|^2 * \text{phase space} * \delta\text{-fn}}$$

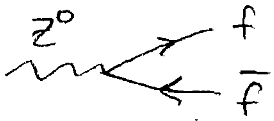
Then integrate over everything except the variables you're interested in, e.g. for $\frac{d\sigma}{dp_T}$, integrate over everything except p_T.

Decay widths: That was for cross sections. For decay widths you do exactly the same thing, except there's a different flux factor ($\frac{1}{m}$ in rest frame of decaying particle of mass m):

$$d\Gamma = \text{flux} * |\mathcal{M}|^2 * \text{phase space} * \delta\text{-fn}$$

How does this relate to a particle's lifetime? We have to distinguish between partial + total widths, because the same particle can have different decay modes.

For concreteness, let's consider the Z^0. Almost all of its decays are to fermion-antifermion pairs, so we'll ignore everything else. So we can have



where $f\bar{f} = \nu_e\bar{\nu}_e$, $\nu_\mu\bar{\nu}_\mu$, $\nu_\tau\bar{\nu}_\tau$, e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$ (top not allowed kinematically)

For each decay mode i we calculate Γ_i:

$$\Gamma_i = \int d\Gamma = \int (\ldots)\ |\mathcal{M}|^2\ (\ldots)$$

Each Γ_i is called a "**partial width**," because it accounts for only part of the decays. The "**total width**" is the sum of the partial widths:

$$\Gamma_{tot} = \sum \Gamma_i$$

and the <u>total</u> width gives the lifetime:

$$\tau = \frac{1}{\Gamma_{tot}}$$

Note that <u>there is no such thing as a partial lifetime</u>!!! The quantity $1/\Gamma_i$ is totally meaningless. There is one and only one lifetime, and

et ox [illegible] 1

if you measure the lifetime in two different modes, you get the same thing.

But the partial width does give some information: the "branching ratio" or branching fraction

$$BR_i = \frac{\Gamma_i}{\Gamma_{tot}}$$

is the fraction of times the decaying particle goes to mode i. Makes sense, given that, when you calculate the width, you're calculating a transition probability and then summing over final states by integrating over phase space.

Comments

- This is the same width that appears in the Breit-Wigner resonance formula. More on this when we discuss physics at the Z^0.
- You can guess the partial widths/branching ratios by simple counting of states for heavy particles decaying to light ones, similar to what you saw with isospin. You'll do this for W's & Z's in the homework.

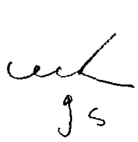
Fun w/ Orders of Magnitude

Here are a few back-of-the-envelope examples to give us a feel for orders of magnitude associated with the various interactions,

Note before we start: the basic vertex for each interaction has a coupling associated with it. Since matrix elements get squared, the square of the coupling is the "minimum" factor associated with a process, and it's the square of the coupling (modulo extra factors) that is the usual expansion parameter. So:

EM  $e^2 \to \alpha \simeq \frac{1}{137}$

weak  $g^2 \to G_F \simeq 10^{-5}\,\text{GeV}^{-2}$

strong  $g_s^2 \to \alpha_s \simeq 0.1$

Because these couplings α, G_F, α_s go into the square of the matrix elements, they (or their squares) determine relative sizes of cross sections + lifetimes, all other things being equal.

We'll do a few examples, the first few of which are taken from T.D. Lee, Part. Phys. + Intro to Field Theory.
These are basically dimensional analysis.

Ex. 1 <u>Strong interaction cross section: pp at high en.</u>

At high energy, for strong interactions, we can use the "black disk" model and take the geometrical cross section

$$\sigma_{pp} \sim \pi r_p^2$$

$r_p \sim 1\,fm = 10^{-13}\,cm$, the typical size for a hadron.

$$\rightarrow \quad \sigma_{pp} \sim 3 \times 10^{-26}\,cm^2 \simeq 30\,mb \quad (1\,barn = 10^{-24}\,cm^2)$$

Comparing to Table 1.3 in Perkins, we see that this is indeed a typical strong interaction c.s.

Note that σ is constant, indep. of energy. The total pp cs shown in Perkins Fig. 4.15 bears this out.

We can go even further, noting that the proton is made up of three quarks, which at high energy are approximately free. Assuming that the cross section is proportional to no. of incident quarks, we predict

$$\sigma_{np} \simeq \sigma_{pp} \simeq \sigma_{\bar{p}p} \simeq \sigma_{\bar{n}p}$$

and $\dfrac{\sigma_{\pi p}}{\sigma_{pp}} \simeq \dfrac{\sigma_{Kp}}{\sigma_{pp}} \simeq \dfrac{2}{3}$

It works; $\sigma_{NN} \simeq 45\,mb$ and $\sigma_{\pi^\pm p} \simeq 25\,mb$ and $\sigma_{K^\pm p} \simeq 20\,mb$

So, the bigger the coupling, the bigger the cross section. Because lifetimes ~ 1/width, lifetimes are shorter for stronger interactions and longer for weaker interactions. From table 1.3 in Perkins, we have

interaction	typical lifetime
EM	10^{-20} sec
Weak	10^{-8} sec ($\rightarrow \sim 10^{-12}$ sec for D, B mesons)
strong	10^{-23} sec

This has interesting consequences. Suppose we create an unstable particle. How far does it go before it decays? Recall that τ is the lifetime in the rest frame of the particle. In the lab frame, in which the particle is moving, the lifetime is longer by a factor of $\gamma = E/m$:

$$T_{lab} = \gamma \tau$$

So the distance the particle travels before decaying, is

$$d = \gamma \beta \tau$$

where β = velocity of the particle. We'll assume energetic particles + take $\beta = 1$, so

$$d \simeq \gamma \tau$$

Now go back to the lifetime table, + translate into distances, assuming $1 \lesssim \gamma \lesssim 100$:

interaction	distance
EM	$\sim 10^{-10} - 10^{-8}$ cm
Weak	$\sim 100 - 10^{4}$ cm ($\rightarrow \sim 10^{-2} \rightarrow 1$ cm for B's, D's)
Strong	$\sim 10^{-13} - 10^{-11}$ cm

Strong + EM decays happen, for all practical purposes, at the point of production*, but <u>weakly decaying particles can travel macroscopic distances before decaying</u>. This is the idea behind silicon vertex detectors, which allow us to detect a separation, if any, between the production and decay points of decaying particles, especially B mesons. Very useful for identifying B's in studies of B mixing, CP, B decays, + top studies.

*That's why π^0's look like electrons or photons.

Q: what about muons? or even K's?

Ex. 4 Assoc. production + subsequent decay of strange part's

The facts mentioned above:

- diff. in lifetime ranges for diff. interactions
- ability of weakly decaying particles to travel macroscopic distances

Led to the discovery of strangeness in the 50's.

Observations of cosmic ray interactions in cloud chambers showed (for what we call strange part's)

- associated production via strong int.
  Some particles were produced only in association w/ other particles, e.g.

  $$\pi^- p \to K^0 \Lambda^0$$

  but never e.g. $\pi^- p \to K^0 n$. The production mechanism was determined to be strong int. from large cross section.

- subsequent weak decay
  These part's traveled finite distance before decaying (see illustr., next page)

  $\Rightarrow$ long lifetime

  $\Rightarrow$ weak decay

  N.B. Only charged particles leave tracks. K + Λ neutral, but can be reconstructed fr/ their decay products.

[See Cahn + Goldhaber, Chap. 3, for detailed discussion.]

From Aitchison + Hey:

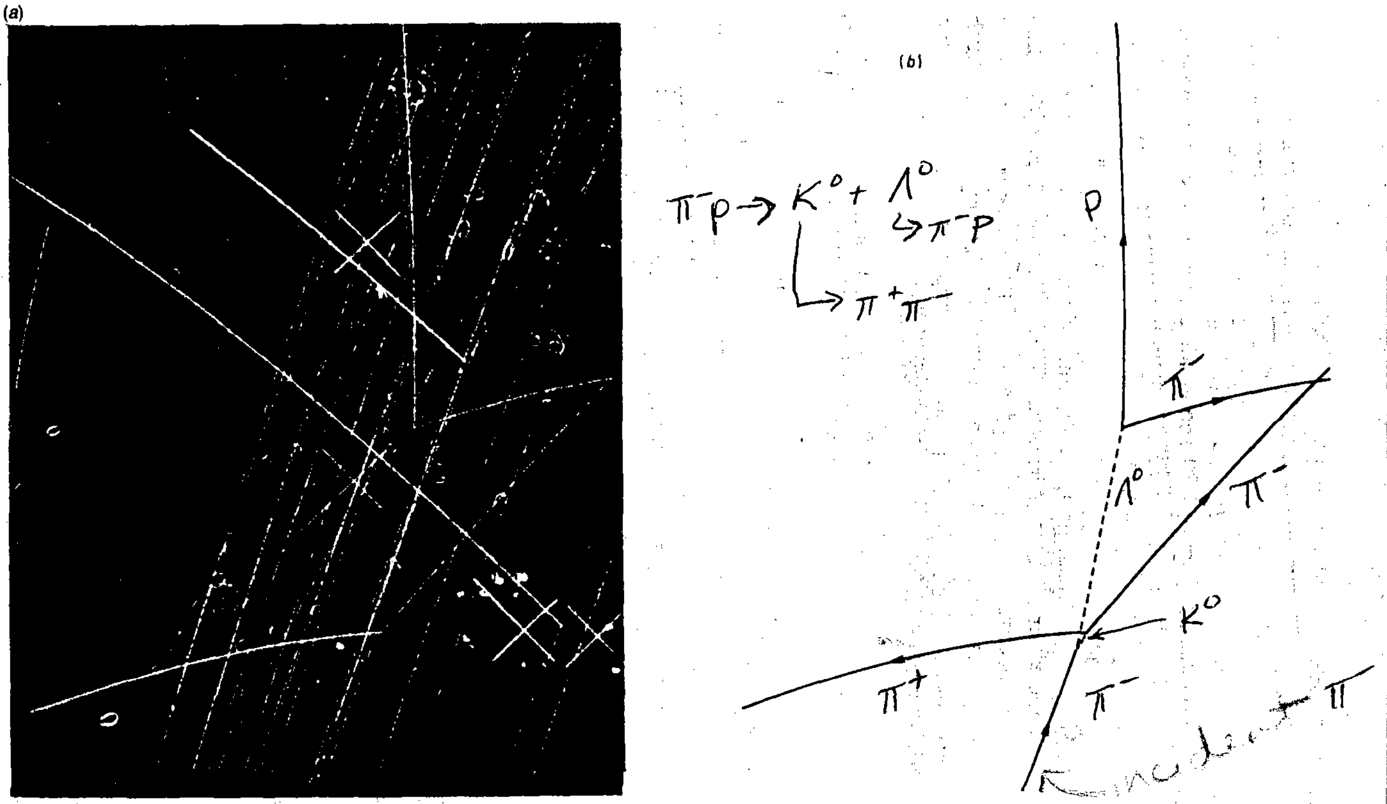
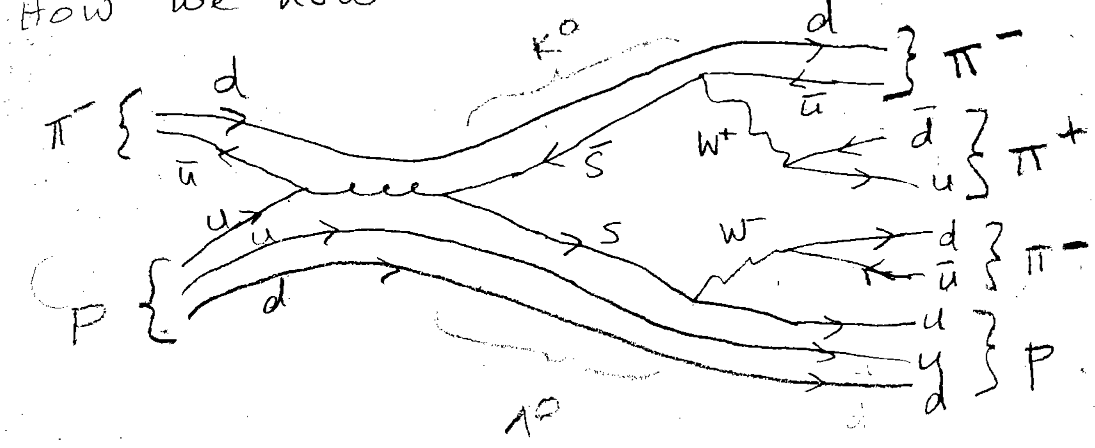


Figure 1.30 Associated production of strange particles.

Why these were not produced singly, + why they didn't decay strongly, was solved by inventing <u>strangeness</u>, which is conserved in strong int's but not in weak.

How we now view it:



Ex. 5 Top decay

These orders of magnitude provide pretty good rules of thumb, but there are exceptions. E.g. t decay:

$$t \to W^+ b^*$$

is weak but has a very short lifetime:

$$\Gamma(t \to Wb) \simeq 175\ \text{MeV} \left(\frac{m_t}{m_W}\right)^3$$

For $m_t = 174$ GeV, the full tree-level Γ is 1.5 GeV. That's a lifetime

$$\tau \sim 10^{-24}\ \text{sec},$$

very small compared to usual weak lifetimes. (Why? Because top is so heavy, + decays to a real W.)

Significance: Compare lifetime to hadronization time ($\sim O(10^{-23})$ sec). Top doesn't have time to hadronize before decaying! So there are no top mesons, baryons, or toponium ($t\bar{t}$ bound states). For once it's good to use parton level!

* almost 100% of the time.