Comments on the $e^+e^- \to \mu^+\mu^-$ cross sect.

) Comparison to what was advertised

a. Total cross section

$$\sigma = \frac{4}{3}\pi\frac{\alpha^2}{s}$$

is, except for the overall constant, just what we estimated in the "fun with orders of magnitude" section (see p. 1.38).

(well, 10 yrs ago)

Suppose $\sqrt{s} = 35$ GeV, as in an old expt. at DESY (see data below). Then

$35^2 \doteq 1225$

$$\sigma = \frac{4}{3}\pi\left(\frac{1}{137}\right)^2\frac{1}{(35\text{ GeV})^2}\underbrace{\left(\frac{389\,\mu b}{1\text{ GeV}^{-2}}\right)}_{=1}$$

$$\simeq 7\times10^{-5}\,\mu b \simeq 70\text{ pb}$$

(Actually, α is slighty bigger at this en.)

b. Differential cross section

Recall that our helicity argument predicted $\frac{d\sigma}{d\Omega} \sim 1+\cos^2\theta$, and that's what $|\overline{m}|^2$ looked like, and we claimed victory. As it turned out, $\frac{d\sigma}{d\Omega}$ had the same behavior, and we were vindicated. But this was because we have a 2-body final state. In general, there can be kinematic factors from phase space so that

$|\overline{\mathcal{M}}|^2$ and $\frac{d\sigma}{d(\text{whatever})}$ can have different behaviors.

(2) What about Z exchange?

In principle we should have included Z exchange:

So we would get a contribution from the square of this diagram and an interference term between this and the γ-exchange diagram. If the energy is small compared to m_Z, we can neglect it.

Why? First, note that the Z propagator looks like

$$\mu \overset{k}{\sim\sim} \nu \quad \frac{g^{\mu\nu} - k^\mu k^\nu / m_Z^2}{k^2 - m_Z^2} \qquad \text{neglecting } \Gamma_Z$$

The corresponding γ prop. has $\frac{g^{\mu\nu}}{k^2}$.

Considering just the denominators, we have

$$|\gamma|^2 \sim \frac{1}{(k^2)^2}$$

$$|Z|^2 \sim \left(\frac{1}{k^2 - m_Z^2}\right)^2$$

$$\gamma\text{-}Z \text{ interf} \sim \frac{1}{k^2(k^2 - m_Z^2)}$$

If $k^2 << M_Z^2$ ($\sqrt{s} \sim 1$ GeV, say), then the $|Z|^2$ and γ-Z int. terms are suppressed compared to the $|\gamma|^2$ contribution, because of the Z mass.

Another way of stating the same point is to note that in both cases, the exchanged γ or Z is virtual, or off mass shell. The denominator of the propagator simply measures how far off shell the particle is; that is, it measures the difference between the square of its momentum and its (mass)^2. [Aside: this is a general property of propagators, + is true for fermions + scalars too.] The closer the exchanged particle is to being on shell, the more the diagram contributes.

Hence, at <u>low</u> energies the Z doesn't contribute much because it is far off shell compared to the γ.
$\longleftrightarrow$ uncertainty principle

Q: What about <u>very</u> <u>high</u> energies, $\sqrt{s} >>$ all masses

<u>A:</u> The masses drop out, + it doesn't matter whether the exchanged particle is a Z or a γ; the mass becomes irrelevant.

One last comment about the Z. When you do include it, it gives a slight forward-backward asymmetry in $d\sigma/d\Omega$. The QED result doesn't care whether θ is near 0 or π, i.e.

it doesn't distinguish between the electron's direction + the positron's direction. The weak interactions do distinguish, + give an asymmetry about $\theta = \pi/2$;

σ : $\qquad\qquad$ $d\sigma/d\cos\theta$:

Perkins p. 198

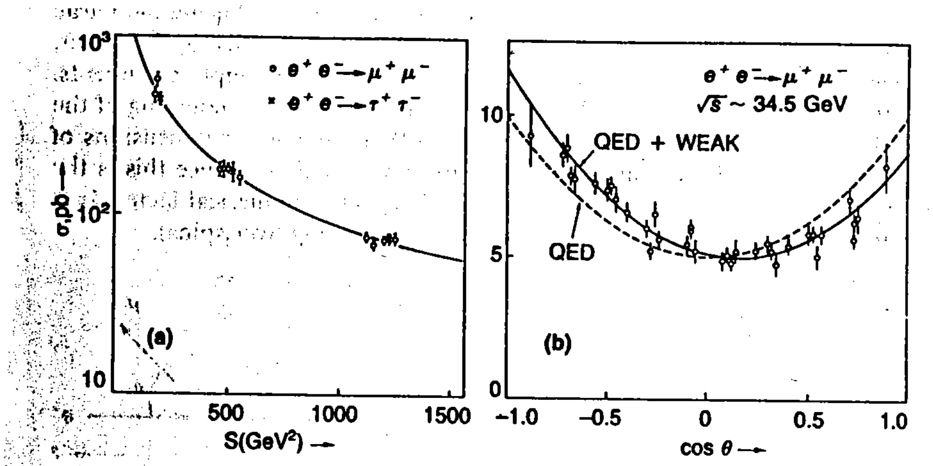


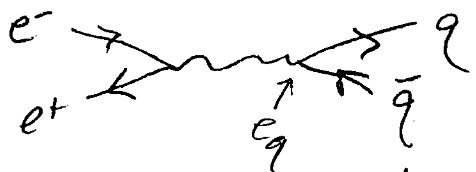
Figure 6.8 (a) Compilation of results (Wu, (1984)) on total cross-sections for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ from the e^+e^- collider PETRA at DESY. The QED prediction (6.31) is shown by the curve. For the total cross-section, neutral-current effects (Z^0-exchange) are small and unmeasurable. (b) The CMS angular distribution for the process $e^+e^- \to \mu^+\mu^-$ as measured at PETRA. The pure QED prediction (6.32) is shown by the dashed line. The full-line curve shows the small forward-backward asymmetry expected from combination of both γ- and Z^0-exchange. For more details, see Section 9.7.2.

We'll see why the asymmetry later.

③ $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

As discussed in PS81, measuring R gives evidence for color and counts the number of quarks below the collision energy.

Why? Because we can have



and the q and $\bar{q}$ turn into hadrons.

The right-hand vertex has a factor of e_q (quark chg.), so (neglecting masses) we have

$$R = N_c \sum e_q^2$$

where the sum is over all $q\bar{q}$ pairs whose production thresholds are below the collision energy.

If quarks have <u>color</u>, we must sum over the number of colors, giving an additional factor of N_c. Experimentally, we find $N_c = 3$.

[<u>Interesting aside</u>. This all sounds straightforward, but the world is more complicated than this, because the c quark and τ lepton have nearly the same mass ($m_c \sim 1.5$ GeV, $m_\tau \sim 1.8$ GeV). So when experiments produced $c\bar{c}$ pairs, they also produced $\tau^+\tau^-$ pairs at only a slightly higher energy. Not only that, but with all that mass, τ's can decay to hadrons (they can also decay purely leptonically). This led to a great deal of confusion until people realized there was a new quark <u>and</u> a new lepton. See Cahn & Goldhaber, ch. 9, for details.

<u>Q1</u>: Evidence for charm was found first, apart from any confusion with the τ, even though one of the expts could produce τ's. What was found? (This was 20 yrs ago, in Nov. '74.)

A1: What was found was not a jump in R but evidence for a $c\bar{c}$ bound state: the J/ψ resonance. In both e^+e^- collisions (at SLAC) and p-Be collisions (at Brookhaven) was observed resonance production of e^+e^- pairs w/ invariant mass $m_{ee} = 3.1$ GeV. They were seeing $J/\psi \to e^+e^-$

τ production wouldn't show up here.

Q2: τ's can decay to hadrons, and they can also decay leptonically to μ's or e's. Hence, $e^+e^- \to \tau^+\tau^-$ can give rise to hadrons, contributing to the numerator, or $\mu^+\mu^-$, contributing to the denominator. If you're doing the experiment, how can you get rid of τ contamination to measure R?

A2: τ decays always include a ν_τ (lepton number being conserved), which escapes undetected, carrying off some energy. So $e^+e^- \to \tau^+\tau^-$ will always have missing energy. By contrast, in

$$e^+e^- \to q\bar{q} \quad \& \quad e^+e^- \to \mu^+\mu^-$$

"everything" shows up in the detector (not counting events w/ final states close to the beam pipe). So eliminating events w/ missing energy can get rid of most τ events.

Comments on
Cross section calculations in real life

$e^+e^- \to \mu^+\mu^-$ in QED is just about the simplest cross section calculation you can do, which is why I chose to do it. There's only one diagram, it's a simple one, I neglected the masses, and there were only two particles in the final state. Hence it was not characteristic of real-life calculations of cross sections done for real-life experiments. Real-life complications include

- more, and more complicated, diagrams...
  ... and signs matter, because ~~for~~ interfering diagrams, the signs determine whether the interference is constructive or destructive

- More particles in the final state
  This is the biggee. In $e^+e^- \to \mu^+\mu^-$, our final state phase space had $d^3q_1\, d^3q_2\, \delta^4(\quad)$: a 6-dim. phase space reduced to 2 dimensions by the δ-fn, so we could do everything analytically. Each new particle in the final state gives an additional d^3p, but we get no new δ-fns.* 3-body final states can usually still be done analytically, but things get hairy fast.

*unless from decay of unstable part.

— Kinematic cuts

The actual final state phase space accessible in experiments is limited in practice. For example, particles close to the beam pipe aren't usually detected, and hadron jets with low transverse momentum (few GeV at Tevatron) aren't easily discernable. In addition, one usually makes cuts on measured quantities (e.g., omitting muons with $p_T < 20$ GeV, or outside a particular angular region) to reduce backgrounds. So not only do we have lots of integrals to deal with, we also have restrictions on limits of integration.

— stuff I'm sweeping under the rug for now, like radiative corrections.

Ignoring this, what's usually done in real life calc's? The squares of matrix elements are often calculated analytically, either by hand (as above) or with the help of symbolic manipulation programs like REDUCE, FORM, or SCHOONSHIP which can do Dirac algebra. The resulting $|\mathcal{M}|^2$, in invariant form (e.g. boxed eq'n on p. 2.55) is usually then put into a FORTRAN (yes, still FORTRAN mostly) program and the phase space integration is done numerically by Monte Carlo methods. This allows for cuts to be implemented in a fairly straightforward way.

Sometimes/often, instead of plugging $\sum_{\text{helicity}} |M|^2$ into the programs, one puts the individual M's for the individual helicities and lets the computer do the adding and squaring. There are (now standard) ways of setting up the helicity amplitudes that make this method more efficient. As we saw even with $e^+e^- \to \mu^+\mu^-$, it's a lot easier to write an amplitude than $|\overline{M}|^2$, especially with multiple, interfering diagrams.

Comments on QED in current HEP expts

QED has little mystery left. It's well understood, well tested, etc. In the spirit of "yesterday's discovery is today's background and tomorrow's callibration," it's pretty much tomorrow for QED. As discussed below, it's mostly used for callibration. An exception is done by Rochester people (led by Adrian Melissinos) at SLAC, using high intensity lasers to study nonlinear QED in various combinations of light/electron scattering.

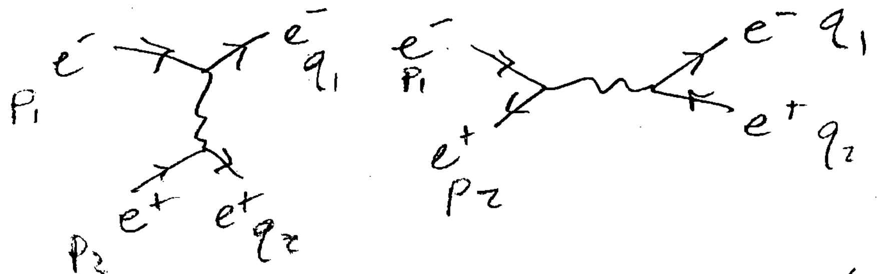
Otherwise, as far as I know,* the primary role of (primarily) QED processes in HEP expts involves taking advantage of their being well understood to use them to

measure luminosity in e^+e^- colliders by looking at low-angle Bhabha ( ) scattering ($e^+e^- \to e^+e^-$).

e.g. at CLEO, LEP, SLC...

Without going into details, we can make a few comments, and throw in some more kinematic nomenclature.

In QED, the diagrams that contribute to $e^+e^- \to e^+e^-$ in lowest order are

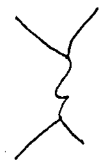


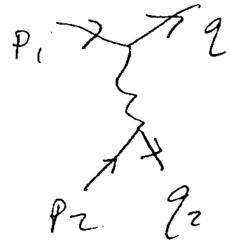
Where the initial momenta are $p_1 (e^-)$ + $p_2 (e^+)$, + final momenta are $q_1 + q_2$, in analogy w/ our $e^+e^- \to \mu^+\mu^-$ calculation.

In principle, Z exchange diagrams contribute; we'll mention them below.

---

* I'm not counting ongoing precision tests of QED such as measurements of magnetic moments, etc.

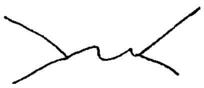
We can see that the  diagram dominates at low angles by considering the propogators. (neglect m_e)



$$\sim \frac{1}{(q_1-p_1)^2} \equiv (-)\frac{1}{2p_1\cdot q_1} = (-)\frac{1}{2p_1^0 q_1^0(1-\cos\theta)}$$

where θ is the scattering angle.
For θ near 0, $\cos\theta$ is near 1, and this diagram gives a large contribution.

But



$$\sim \frac{1}{(p_1+p_2)^2} = \frac{1}{E_{cm}^2} \quad ;\ \text{not so big.}$$

For Z exchange, we have

$$\frac{1}{(q_1-p_1)^2 - M_Z^2} \quad \text{and} \quad \frac{1}{(E_{cm}-M_Z)^2+\Gamma^2/4}$$

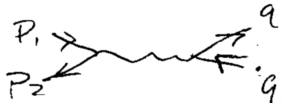
The 1st term is small compared to the corresponding one from QED, and the 2nd term gets suppressed by the width even near $E_{cm}=M_Z$. (We'll discuss this more later.)

This discussion is a good place to mention the Mandelstam variables s, t, u. These are relativistic invariants that are often used as kinematical variables. We've already seen s.

So consider a $2\to 2$ process (such as Bhabha scattering)

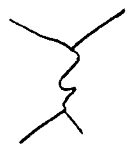
$$p_1, p_2 \to q_1, q_2$$

We can define

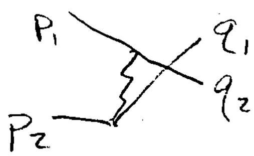
$s \equiv (p_1+p_2)^2$: This is just the cm energy of the collision, and is the square of the exchanged momentum in  This process is said to go through the s channel.

Note that $s = (p_1+p_2)^2 = (q_1+q_2)^2$ fr/ 4-mom. cons.

$t \equiv (p_1-q_1)^2$: This is (up to a sign) the square of the exchanged momentum in



; hence this is called a t-channel process. As we saw for Bhabha scattering, t is related to the cm scattering angle $(t = -2p_1^0 q_1^0 (1-\cos\theta))$. $dt \propto d\cos\theta$, + you often see cross sections expressed as $\frac{d\sigma}{dt}$, which is manifestly Lorentz invariant, instead of $d\sigma/d\cos\theta$, which is not.

$u \equiv (p_1-q_2)^2$: This corresponds to the momentum exchanged in  (not possible in Bhabha scatt.)

u is not independent of t and s.

To see this, note (keeping all masses):

$$
\begin{aligned}
s+t+u = \; & p_1^2 + p_2^2 \quad + \quad 2p_1 \cdot p_2 \\
+ \; & p_1^2 + q_1^2 \quad - 2p_1 \cdot q_1 \\
+ \; & p_1^2 + q_2^2 \quad \underbrace{- 2p_1 \cdot q_2}_{}
\end{aligned}
$$

$q_2 = p_1 + p_2 + q_2$

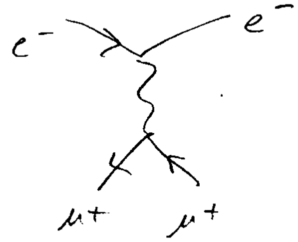
$$= -2p_1 \cdot (p_1 + p_2 - q_1)$$

$$= p_1^2 + p_2^2 + q_1^2 + q_2^2$$

$$\boxed{s+t+u = \sum_{\substack{\text{all} \\ \text{particles}}} m^2}$$

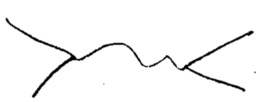
and if the particles are massless, $s+t+u=0$

So $e^-\mu^+ \to e^-\mu^+$:



$\sim \frac{1}{t}$

$e^+e^- \to \mu^+\mu^-$



$\sim \frac{1}{s}$