

Weak Interactions + Electroweak Theory

3.1

we come to the $SU(2)$ part of $SU(3) \times SU(2) \times U(1)$, also known as the weak interactions.

(As you'll see below, this identification isn't exact, but we'll come back to that...)

Plan: We're going to discuss this in two parts:

I. Weak Interactions

This will be a sort of historical discussion of the development of our understanding of the weak interactions. It won't be complete, or even particularly systematic, but I'll at least try to make it organized. What I'm trying to do is

1. Emphasize the interplay between experiment and development of the theory, and
2. Introduce some of the important/interesting/standard phenomena + characteristics of the weak interactions along the way

Ref's: For more info, see Perkins Chap 7 + Cahn Chap. 6 (also 3). I'm sort of following Cahn, w/ stuff thrown in fr/ other places, including Halzen + Martin + an excellent set of lecture notes by Mel Shachet of Univ. of Chicago.

II. Electroweak Theory → p. 3.31

This will be the more systematic + theoretical

part, where we write down the $SU(2) \times U(1)$ gauge theory, + show how the EM and weak bosons + interactions we know + love come out. (We'll give electroweak symmetry breaking + Higgs physics its own section.)

Ref's: For more info, see Cahn Chap. 12; Perkins Chap 9 (although it might not help much); Quigg Chap 6 (esp. sec 6.3); Halzen + Martin Chap 13; Aitchison + Hey Chap. 14. I'll be drawing from Quigg, A+H, + H+M.

end of plan

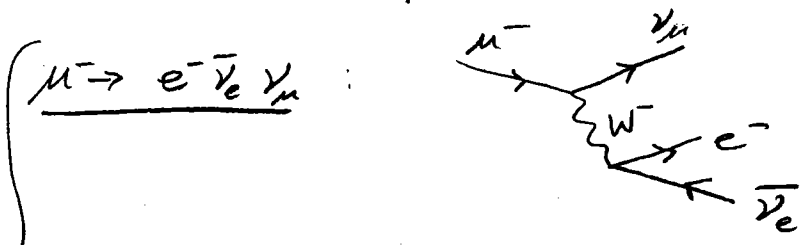
↳ Weak Interactions

We understand the weak int's now in terms of W + Z bosons interacting w/ fermions (+ each other + photons), as mentioned above already. The W 's + Z 's, of course, weren't actually seen until the 80's, and before that it took some clever work to get the $V-A$ structure of the theory to come out. a.k.a. left-handed a.k.a. $1-\gamma_5$

Notice how, in the beginning, experimental info. came from particle decays (mostly nuclear β decay at first) then π , μ , + K decays - π 's + K 's being among the lightest + hadrons) and eventually more from scattering experiments as accelerators became more important. Also note that we start w/ charge-changing interactions; neutral currents come later.

Keep this **example** (μ decay) in mind through the following. The structure of processes like μ decay is what we're working our way up to. Note that β decay is very similar.

for comparison of structure only.



$$M \sim \left[\frac{g}{\sqrt{2}} \bar{u}_\mu \gamma^\alpha \frac{1}{2} (1 - \gamma^5) u_\mu \right] \frac{1}{M_W^2 - q^2} \left[\frac{g}{\sqrt{2}} \bar{u}_e \gamma_\alpha (1 - \gamma^5) u_{\nu_e} \right]$$

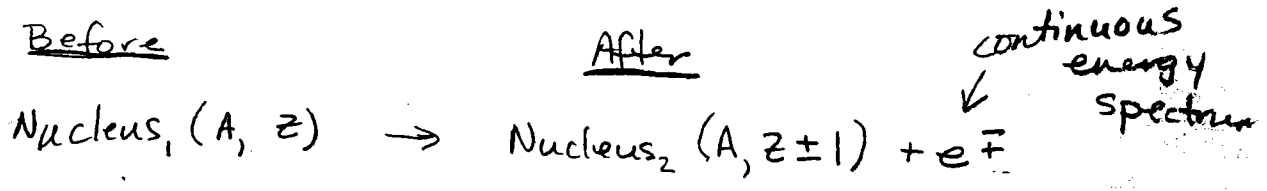
Comments

- This is for comparison of structure, so don't get hung up on details too much.
- u 's & \bar{u} 's are spinors; subscripts refer to the particle. u_μ is initial muon spinor, not 4-vector. Don't worry about using u instead of v for antiparticles here.
- The propagator term has a piece missing that's not important for μ decay but matters at higher energy. Don't worry about it now.
- We won't reproduce the whole thing in this sec; but we're especially interested in the $\gamma^\alpha (1 - \gamma^5)$ which I'll usually write as $\gamma^\mu (1 - \gamma^5)$.

N.B. I'm following Shocket's notes fairly closely below.

β-decay + Fermi Theory

Back up to the early 30's when all that was known about weak interactions was that some nuclei decayed by emitting β-radiation, i.e., electrons + positrons. Here's what was observed



Without even worrying about the dynamics, there are two **problems** here:

1. There's one more fermion on the R.H.S.
 ⇒ ang. momentum not conserved
2. A 2-body decay implies fixed energy of the final state particles, e.g. the e⁻, but a continuous spectrum was observed. Only at the endpoint (max. energy) did the electron have the expected energy; otherwise it had too little.
 ⇒ energy being lost

(Bohr, being open minded, was willing to give up energy conservation except on average.)

1931
↓

The **solution** came from Pauli, who invented the neutrino;

a massless (or very light) fermion that participates in weak decays but otherwise doesn't do much. ...

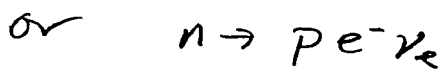
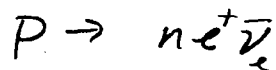
... so it carried away energy without being detected. Sounds farfetched, but it worked!

[Obvious parallels w/ dark matter candidates, esp. WIMPs, or Weakly Interacting Massive Particles, whose grav. int. effects we can see, but otherwise don't do much...]

Just after this, Chadwick discovered the neutron (1932), so Fermi had everything he needed to construct a theory of β -decay, which he took to be a contact interaction (i.e., it all happened at a point:) (4-Fermi)



The decay inside a nucleus could be



Even though $m_n > m_p$, both kinds of β decay can happen once you account for nuclear binding energy in the various states.

Success #1: We've got the four fermions represented by the four spinors (no, not a band fr/ the 60's) in \mathcal{M} on p. 3.3. Sort of. I'm assuming

1. The $n + \bar{\nu}_n$ can be replaced by quarks. okay
2. The quarks can be represented by the $p + n$ (or vice versa) not really okay, but good enough for now.

Fermi modeled his weak interaction theory after 3.4
 EM, i.e., he made it a current-current
 interaction. Recall that the interaction \mathcal{L} in
 EM looks like

$$A^\mu J_\mu^{EM} \quad \text{where } J_\mu^{EM} = e \bar{\psi} \gamma_\mu \psi$$

so that in something like $e^+e^- \rightarrow \mu^+\mu^-$ we end
 up with a M.E. that looks like

$$M^{EM} \sim \frac{1}{q^2} (e \bar{u}_e \gamma_\mu u_e) (e \bar{u}_\mu \gamma^\mu u_\mu)$$

\swarrow Lorentz index
 \uparrow particles

which we can write as

$$M^{EM} \sim \frac{e^2}{q^2} j_\mu^{(e)} j_\mu^{(\mu)}, \quad j^\mu = \bar{u} \gamma^\mu u$$

Fermi tried

$$M^{\beta \text{ decay}} = G_F (\bar{u}_\mu \gamma^\mu u_p) (\bar{u}_e \gamma_\mu u_n)$$

\uparrow
 Fermi
 const

which also looks like a current-current interaction.
 Comparing to M_1^{EM} we note that G_F has dimensions
 of (Energy)⁻².

Success #2: Fermi came close, actually. He ^{almost} got the
 overall structure correct, at least as far as the γ^μ 's.
 And G_F is correct for the low energies of the time.

Interlude: β spectrum, Kurie plots, + neutrino mass

(3.7)

The neutrino mass can be investigated by looking at the energy spectrum of the electron in β -decay.

Just from kinematics, it is obvious that the heavier the neutrino is, the lower ^{is} the maximum energy for the electron. (A massive ν has minimum energy m_ν . A massless ν has no minimum energy, making more available to the electron.)

So the **shape** of the energy spectrum near the maximum electron energy is sensitive to neutrino mass; measuring this spectrum is one of the ways of looking for ν mass.

The usual thing to look at is a Kurie plot, in which the electron spectrum is plotted in a funny but useful form. It's based on the fact that, if the matrix element is constant, i.e. independent of the momentum, then after integrating over everything except the electron momentum p_e we have

$$d\Gamma = N(p_e) dp_e \propto p_e^2 (Q - E_e)^2 dp_e$$

where $N(p_e)$ is the number of electrons w/ momentum between $p_e + p_e + dp_e$, + Q is the energy lost by the decaying nucleus.

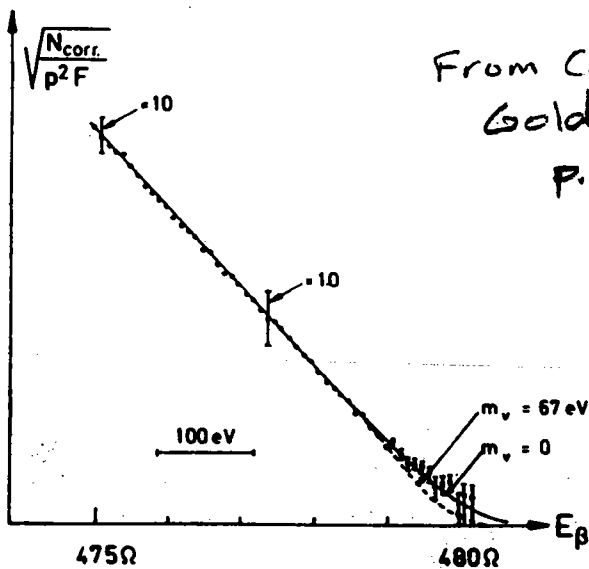
So then,

$$\frac{1}{p_e} \left(\frac{dN}{dp_e} \right)^{1/2} = \sqrt{\frac{N(p_e)}{p_e^2}} \propto (Q - E_e)$$

In other words, $\sqrt{\frac{N}{p_e^2}}$ vs. E_e is a straight line.
Kurie plot

In real life $N(p_e) dp_e = p_e^2 (Q - E_e)^2 F(E_e)$,

where $F(E_e)$ is the "Fermi function" — it contains the energy dependence of the matrix element, & depends on the particular nucleus.



Usually $\sqrt{\frac{N}{p_e^2 F}}$ vs E_e is plotted.

Deviations from the expected endpoint behavior would show up if $m_\nu \neq 0$.

Figure 6.1: The Kurie plot for the beta decay of tritium showing the portion of the electron spectrum near the end point at 18.6 keV. As pointed out by Fermi in his 1934 paper setting out the principles of beta decay, if the neutrino mass is nonzero there will be a deviation of the plot from linearity near the end point. By studying this region with extreme care, Bergkvist was able to set an upper limit of 60 eV on the mass of the neutrino (more precisely, the electron-antineutrino) [K. E. Bergkvist, *Nucl. Phys.* B39, 317 (1972)]. The x-axis of the Figure shows the magnet setting of the spectrometer. The interval corresponding to 100 eV is indicated, as well as two sample error bars with a magnification of 10. The curves expected, including the effects of the apparatus resolution, for neutrino masses of 67 eV and 0 eV are shown. Without the resolution effects, the curve for 0 eV would be a straight line, while the 67 eV curve would fall more abruptly to zero.

Recent historical note: This isn't just ancient history. The β spectrum of tritium is still being examined for evidence of ν masses. John Simpson created a stir about 10 years ago when he reported evidence for a 17 KeV neutrino. In the late 80's the excitement increased when Hime + Jelley, in a separate experiment, saw a similar effect. Many experimentalists joined in. Some saw the effect, but most did not. An interesting fact was that the experiments fell into two classes according to the type of detector used (they were measuring electron energies; remember): solid state or magnetic spectrometer. All of the positive results showed up in expts with solid state detectors. Meanwhile negative results accumulated in other, very rigorous, expts. Eventually Jelley himself (or maybe Hime? or both?) was able to show that properties of the detector itself could give rise to effects that mimic a 17 KeV neutrino.

Note that this is very different from the recent Los Alamos expt, which looks for neutrino oscillations, i.e. $\nu_\mu \rightarrow \nu_e$. This can happen only if the ν 's are massive, and the rate depends on the mass + mixing angle. In that expt a ν_μ beam (from $\pi \rightarrow \mu \nu$) was incident on a target (water I think). They looked for $\nu_\mu \rightarrow \nu_e$ then $\nu_e + p \rightarrow e + n$; Terentov light from the e 's, is observed. There's no preprint yet as far as I know.

I might be
remembering this wrong.
end of interlude

So, where were we? ^{P. 3.4} Fermi has his theory, which 3.10 has some but not all of the features we need.

Actually, it was obvious pretty quickly that Fermi's proposal couldn't account for everything, because it only allowed for $\Delta J = 0$ transitions, in which angular momentum of the nucleus is conserved.

But $\Delta J = 1$ transitions (known as Gamow-Teller transitions) also occurred:

<u>Fermi</u>	<u>Gamow-Teller</u>
$N_1 \rightarrow N_2 \begin{matrix} e\nu \\ \underbrace{\quad} \\ S=0 \end{matrix}$	$N_1 \rightarrow N_2 \begin{matrix} e\nu \\ \underbrace{\quad} \\ S=1 \end{matrix}$
$\Delta \vec{J} = 0$	$\Delta \vec{J} = 1$
$J=0 \rightarrow J=0$ allowed	$J=0 \rightarrow J=0$ forbidden
<u>Ex:</u> $O^{14} \rightarrow N^{14} + e^- + \nu$	$He^4 \rightarrow Li^6 + e^- + \bar{\nu}$
$J^P \quad 0^+ \quad 0^+$	$0^+ \quad 1^+$

So we have to back up + look more systematically at the possibilities. Now, the ME for the interaction has to be a Lorentz invariant, so we'll examine what the possibilities are for each "half" of it. For which we need...

Bilinear covariants, which is a fancy way of saying Dirac matrices sandwiched in between Dirac wave functions in such a way that they have definite transformation properties under Lorentz boosts, i.e., they form scalars, vectors, etc.

The bilinear covariants are

Scalar S	$\bar{\Psi}\Psi$	
Pseudoscalar P	$\bar{\Psi}\gamma^5\Psi$	odd under space inversion
Vector V	$\bar{\Psi}\gamma^\mu\Psi$	space components odd under space inv.
Axial vector A	$\bar{\Psi}\gamma^\mu\gamma^5\Psi$	space comp. even under space inv.
Tensor T	$\bar{\Psi}\sigma^{\mu\nu}\Psi$	

$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$

It's left to the reader to show (or look up in Bjorken + Drell or favorite field theory book) that these perform as advertised under Lorentz transformations.

So our friend the EM current $j_\mu^{EM} = \bar{\Psi}\gamma_\mu\Psi$ is a vector, and Fermi's theory is a pure vector interaction. As foreshadowing, note that γ^5 switches a thing's behavior under space inversion so if you have both (thing) and (γ^5 thing), you have parity nonconservation.

So, it's clear fr/ Gamow-Teller int's that we have to sample from more than the V's to get a theory of the weak interactions; then parity violation is revealed, making it even more clear.

Parity Violation + V-A

(3.12)

Life becomes exciting again in the 50's (well, ^{particle} physics becomes exciting) with the θ - τ puzzle. It had been taken as a given that parity was conserved in the weak interactions; there was no reason not to, and there's nothing a physicist loves like a good symmetry. And parity, or space inversion, is one of the best. Or was thought to be.

Then came the τ - θ puzzle. τ + θ were particles w/ the following decay modes (Note: this τ is not the τ lepton):

$$\begin{aligned} \tau^+ &\rightarrow \pi^+ \pi^- \pi^+ & P = - \\ \theta^+ &\rightarrow \pi^+ \pi^0 & P = + \end{aligned}$$

The puzzle was that they had the same mass + lifetime — they looked like different decay modes of the same particle. But that couldn't be — they had different parity. Lee + Yang showed that, after all, it could be. Turned out there was ample evidence for P conservation in EM + strong interactions, but it hadn't been tested at all in weak interactions.

So we know what happened: C.S. Wu checked it out, + she + collaborators demonstrated P violation in β -decay. θ was also confirmed in $\pi^+ \rightarrow \mu^+ \nu \rightarrow e \nu \nu$ decays.

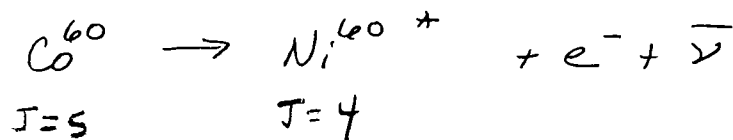
1720

(3.13)

Wu's parity violation experiment is worth mentioning further because the approach is typical of what's done when looking for violations of symmetries. The point is to find an observable that changes (e.g., ^{changes} sign) under the symmetry transformation. If the interaction obeys the symmetry, its rate or cross section should not depend on the observable. If it does, then the rate changes under the transformation and the interaction is not invariant.

For \mathcal{P} , we need a variable that's odd under space inversions.

The decay was



This may be a lie. (Note: This is a Gamow-Teller transition. You don't want a Fermi transition, because they're known to conserve \mathcal{P} .)

(3) Momenta change signs under \mathcal{P} , & we have 3 to work with in the final state. Also, we want a scalar quantity that's \mathcal{P} -odd. Try

$$\vec{p}_e \cdot \vec{p}_\nu \times \vec{p}_N$$

This is standard. Only problem is that these momenta are coplanar, so the whole thing is zero. Since $0 = -0$, it doesn't help.

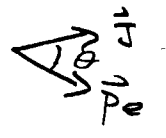
But \vec{J} is even under \mathcal{P} ,* so if we combine it with momentum, we have an odd variable.

$$* \vec{J} = \vec{x} \times \vec{p}$$

Specifically, they looked at

$$\langle \vec{J} \rangle \cdot \vec{p}_e$$

where $\langle \vec{J} \rangle$ is the average value of the angular momentum of the ^{60}Co nucleus. If the rate depends on $\langle \vec{J} \rangle \cdot \vec{p}_e$, then P is violated. Note that $\langle \vec{J} \rangle$ is in the direction of the applied magnetic field, so changing \vec{B} changes $\langle \vec{J} \rangle$. More specifically, they looked for dependence on θ :



$$I(\theta) = 1 + \alpha \frac{\langle \vec{J} \rangle \cdot \vec{p}_e}{E_e}$$

↑
to be measured

$$= 1 + \alpha \frac{v}{c} \cos \theta$$

For P conservation, $\alpha = 0$. Any nonzero value of α
 $\Rightarrow \not{P}$; maximal $\not{P} \Leftrightarrow \alpha = -1$. We found $\alpha = -1$
 \Rightarrow maximal parity violation. (indep. of v)

Note also that ang. mom. cons \Rightarrow electron + antineutrino have spins along \vec{J} .
 $^{60}\text{Co} \xrightarrow{J=5} \text{Ni}^{60*} \xrightarrow{J=4} e^- + \bar{\nu}$
 $s = 1/2$ for both

For $v/c \rightarrow 1$, electron spin must be antiparallel to \vec{p}_e (else $1 - \cos \theta = 0$ and rate vanishes).

true in β decay

\Rightarrow electrons in β decay have negative helicity
 Furthermore, positrons have positive helicity in β -decay

Not only that, but Goldhaber + friends \rightarrow

showed that ν 's in β decay also have negative helicity, + $\bar{\nu}$'s have positive helicity.

That means that the weak interactions involve, for the leptons at least, left-handed particles and right-handed antiparticles. To project out these states, we throw in the negative helicity projection operator $\frac{(1-\gamma_5)}{2}$

Success #3: $(1-\gamma_5)$ in lepton part of \mathcal{M} .

Of course, that didn't mean automatically that we had $\gamma^\mu(1-\gamma_5)$; we could have the other types of operator as well.

Further investigations involving comparing consequences of the various types of decays for the different types of couplings finally narrowed things down. For example, in $\pi \rightarrow \mu \nu$ or $\pi \rightarrow e \nu$, only axial (A) + pseudoscalar (P) couplings can contribute.

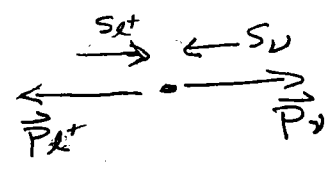
To choose between them, note that

$$\begin{array}{lll} A & \text{has } \ell^+ \text{ helicity } & +\frac{v}{c} \\ P & \text{" " " " } & -\frac{v}{c} \end{array}$$

That has implications for the ratio of

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu)}$$

Consider cons. of ang. mom. π has spin 0, so spins of $l + \nu$ must be equal + opposite. But ν is massless, + must have spin pointing opposite to momentum:



That means the positive lepton has to have its spin opposite to its momentum. No problem for P_l . But for Axial int., as $\frac{v}{c} \rightarrow 1$, that doesn't happen. The extent to which it can happen is the extent to which $\frac{v}{c}$ can deviate from 1, i.e., it depends on how massive the particle is. (This is another way of saying that only massless particles are helicity eigenstates - see your favorite discussion of the Dirac eq'n.)

The upshot is that for Axial interactions, $\pi \rightarrow e\nu$ should be suppressed compared to $\pi \rightarrow \mu\nu$ because $m_\mu \gg m_e$.

That's exactly what happens. As you can see in the particle data book, $\pi^+ \rightarrow \mu^+ \nu$ is by far the dominant decay.

From this and other, similar considerations, the form of the weak interactions* as we know them emerged, including

* not least for charged cc mes. + interactions

Successes #4 + 5: The weak interactions

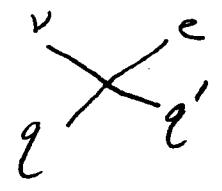
are of the form V-A, i.e., $\gamma^\mu(1-\gamma_5)$, for all fermions, and the coupling constant is the same for all of them. (Note this last point had to wait until the quark model to come through.)

Eventually, the idea of W exchange came in. It was more or less inevitable, because the Fermi theory is neither renormalizable nor unitary. Renormalizability may seem obscure but unitary isn't - we certainly believe in conservation of probability.

As an example, let's consider ν scattering, e.g.

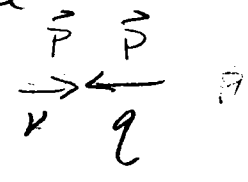
$$\nu q \rightarrow \bar{\nu} q'$$

Recall that our order of magnitude estimate showed for this cross sect $\sigma \sim G^2 s$



where s is the cm energy. This works fine for low energy, and something like this comes out of the Fermi theory. But as s increases, σ increases without bound! This can't happen: it violates unitarity.

To be more specific, in the cm frame

$$\sigma_{tot} = \frac{G^2 s}{\pi} = \frac{4 G^2 |\vec{p}|^2}{\pi}$$


Now, since this is a point interaction, there's no orbital angular momentum. Remember partial wave analysis fr/ PS81? The inelastic cross sect. can be written

$$\sigma = \frac{\pi \lambda^2}{2} \sum_l (2l+1) (1 - \chi_l^2)$$

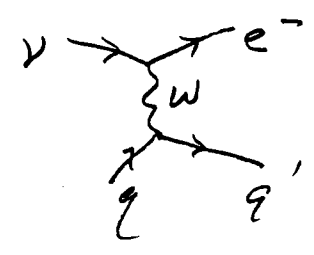
initial deg. of fr. \nearrow $l=0$ only for point interaction $\chi = 1/p$ $\underbrace{\hspace{2cm}}_{\text{max}=1}$

so $\sigma < \frac{\pi \lambda^2}{2} = \pi / 2 p^2$

$\Rightarrow p < \sqrt{\frac{\pi}{252} G} = 309 \text{ GeV}$ from unitarity.

Now there's nothing to stop us from using higher energies than that, so there must be something in the theory that stops the c.s. from blowing up.

As it turns out it's not a contact interaction, but is understood in terms of W exchange



and in place of G_F we have the W prop:

$$\frac{-q^2 + q^2 q^{22}/M_W^2}{q^2 - M_W^2} \quad W \text{ propagator}$$

the $q^2 q^2$ part gives a log divergence we don't care too much about

So $G_F^2 s$ becomes $\left(\frac{q^2}{q^2 - M_W^2} \right)^2 q^2 \rightarrow \frac{q^2}{q^2}$ for $q^2 \gg M_W^2$

↑ coupling

which behaves fine at high energies.

What should we see in ν scattering? A σ that increases with energy, but eventually slows down at higher energy. Until recently, expts could only show the $\sigma \sim s$ behavior.

Now, at HERA, evidence for the W propagator (or, equivalently, finite M_W) shows up;

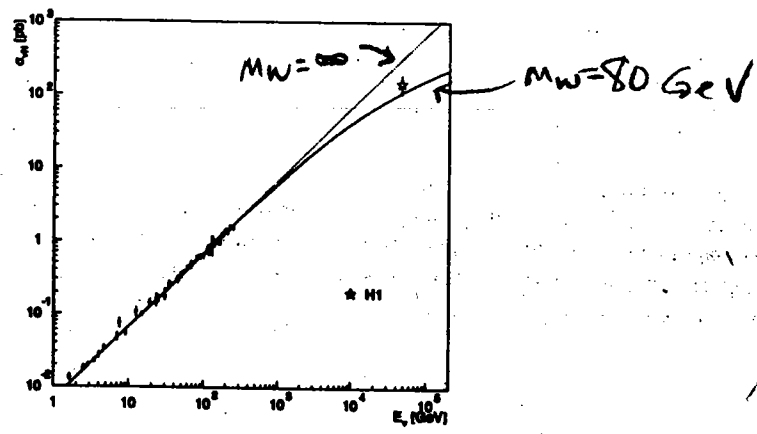


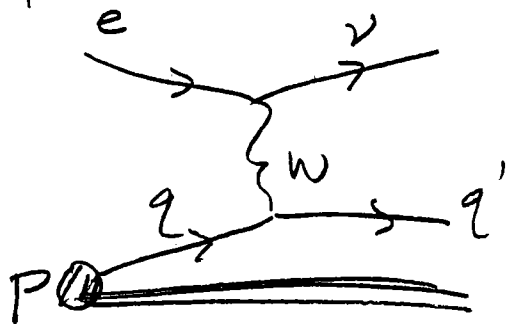
Figure 8: The energy dependence of the νN cross section. The crosses represent the low energy neutrino data while the full and the open stars refer to the HERA measurements, which for the purpose of this comparison, have been converted to a νN cross section. The H1 point correspond to an equivalent fixed target energy of 50 TeV. The straight line is the extrapolation from low energies assuming $m_W = \infty$ while the curve represents the predicted cross section including the W propagator.

(Ref: H1 collab, T. Ahmed et al, Phys. Lett. B324 (1994) 241) (3.20)

What's shown in the plot is the cs for ν -nucleon scattering vs E_ν . The hard process in νN scatt. is actually νq scattering, so what we said above applies.

The low energy points are from νN scattering in fixed target expts. (there are also some from H1 at ~~HERA~~ down there).

The high energy point is from H1 at HERA. Recall that HERA is an ep collider (27 GeV e's, 820 GeV p's, what they looked at was



and they had to translate back to the equivalent fixed target energy to make the comparison. Note that the high energy point corresponds to a fixed target E_ν of 50 TeV!

Comments

- This works. The H1 data agree w/ the fixed target data in the region of overlap.
- The highest energy point shows the cross section turning over, and is consistent with finite m_W but not $m_W \rightarrow \infty$

Quarks, GIM mechanism, + CKM matrix

3.21

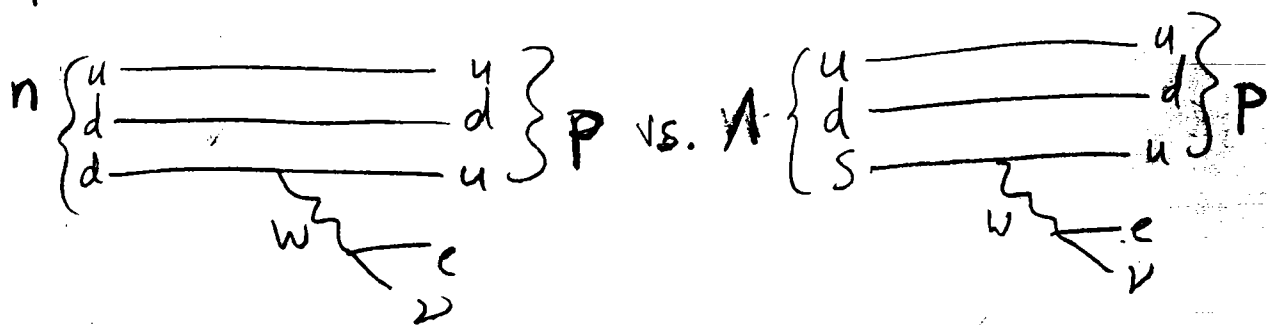
We've got the structure of the charge-changing interactions (those w/ W^\pm exchange) down, but there are two more ^{related} things we need to address before going on: quarks + neutral currents.

Cabibbo angle as a youth: we've mentioned β -decay involving nucleons $n + p$.^{*} But there were also known to be strange particles, of which the Λ was the lightest baryon. The Λ should also exhibit β -decay, e.g.

$$\Lambda \rightarrow p e^- \bar{\nu}$$

If we assume that $\Lambda \rightarrow p e^- \bar{\nu}$ has the same ME as for $n \rightarrow p e^- \bar{\nu}$, + calculate the branching ratio, it comes out an order of magnitude bigger than the measured value. Cabibbo came to the rescue.

Compare $n + \Lambda$ decay:



Only difference is whether s or d turns into u quark. Now, under strong + EM interactions, s + d are completely distinct, but under

* But no charm yet; it doesn't show up until later.

the weak int's they have the same quantum no's, so maybe they mix (like $K^0 + \bar{K}^0$ in $K_S + K_L$). Cabibbo proposed that some linear combination of d + s is a weak eigenstate + is connected to the u quark by the weak interactions:

$$d' = \cos \theta_c |d\rangle + \sin \theta_c |s\rangle$$

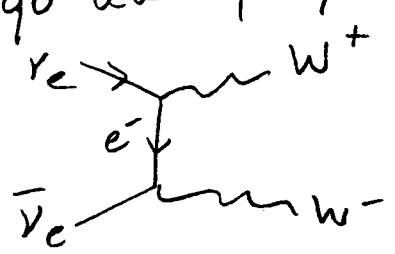
Where the mixing angle θ_c is called the Cabibbo angle and $|d\rangle + |s\rangle$ are the strong + EM quark eigenstates.

From the $\Lambda \rightarrow p e \bar{\nu}$ BR measurement we find

$$\sin \theta_c \approx 0.22$$

Not only that but $\cos \theta_c = 0.98$ accounts for a slight but measured suppression in $n \rightarrow p e \bar{\nu}$ compared to $n \rightarrow p e \bar{\nu}$ which I failed to mention before.

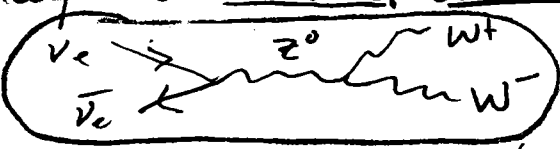
Cabibbo angle achieves its potential: Now, you may ask, what about the other, orthogonal linear combination of $|d\rangle + |s\rangle$? Well, it's the early 60's and as yet there's nothing for it to couple to. ^{*} But it'll find something. First we have to consider ν scattering again. All the problems do not go away w/ W exchange. Consider



The contribution from longitudinally polarized W's

* ν is analogous to right-handed electrons

(yes, they can be long, polarized, unlike the photon, because they have a mass) diverges with increasing energy. This can only be fixed if there's another, interfering diagram to provide cancellation. Exchange of a heavy fermion that couples to ν_e would do it, but nobody's ever seen one. What saves the day is neutral currents, i.e., Z^0 exchange.

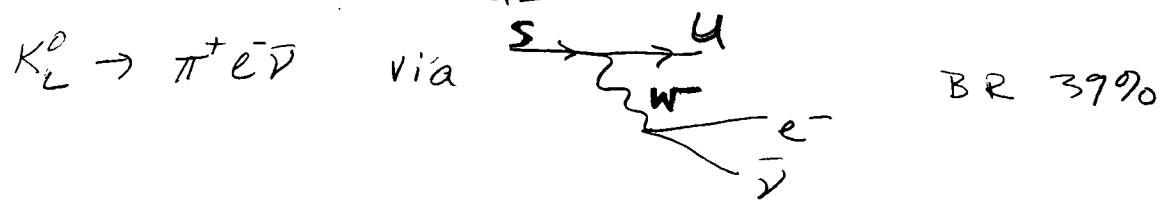


Aside: There's a pattern here: physical cross sections that look divergent + have to be saved by contributions from exchange of a new particle. It happens yet again, and longitudinal W 's are again a culprit. $W_L W_L \rightarrow W_L W_L$ violates unitarity at high energies unless you can throw in Higgs exchange. (That's not all the Higgs is good for, of course). Requiring partial wave unitarity (cf. p.3.18) places an upper limit on the Higgs mass of order 1 TeV.

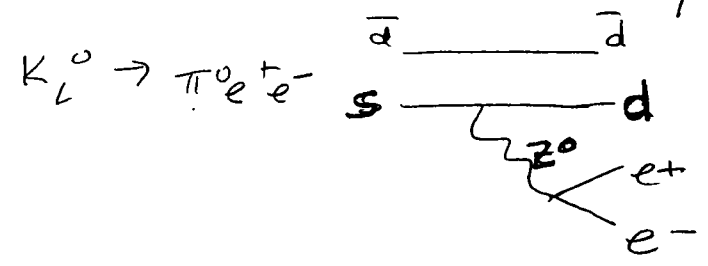
So, neutral currents save the day, + evidence for them was found at CERN in 1973 in νN scattering; see Cahn Chap. 12 (ref 12.1). (N.B. Prof Perkins himself was on this expt...)

But neutral currents also muck up the works, because they would couple to other fermions than ν 's, giving all kinds of non-charge-changing reactions.

So, for example, if we have (and we do) the charge-changing decay

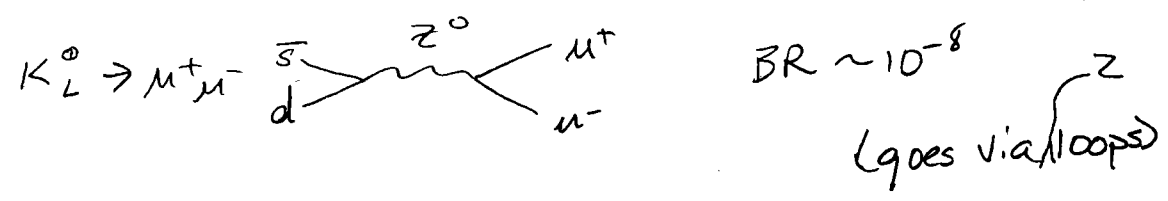
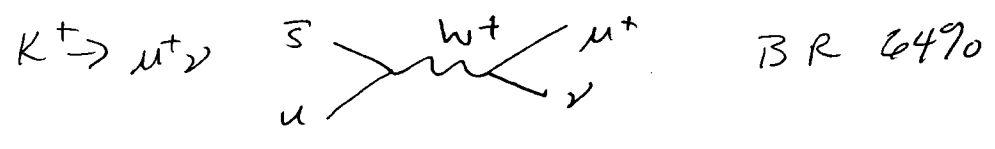


We should also have the charge-preserving reaction



with comparable strength. But it's never been seen, and the experimental upper limit on its BR is ~10^-6!

Then there's a similar situation with, e.g., charged K's in annihilation diagrams:



So strangeness-changing neutral currents don't seem to exist. How do we get rid of them? Glashow, Iliopoulos, + Maiani followed a time-honored tradition and invented a new particle - the charm quark - and the first of the FCNC's (flavor-changing neutral currents) went away via the GIM mechanism.

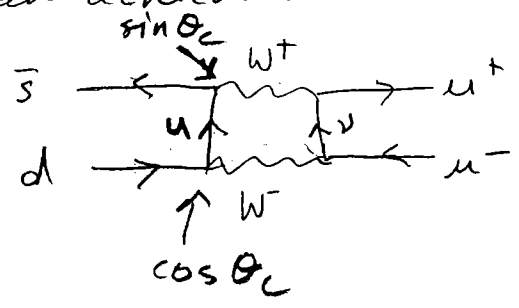
How GIM works: Remember $d' = \cos\theta_c |d\rangle + \sin\theta_c |s\rangle$ 3.25

is the thing that couples to the u via the weak interactions. What about the orthogonal state

$$s' = -\sin\theta_c |d\rangle + \cos\theta_c |s\rangle ?$$

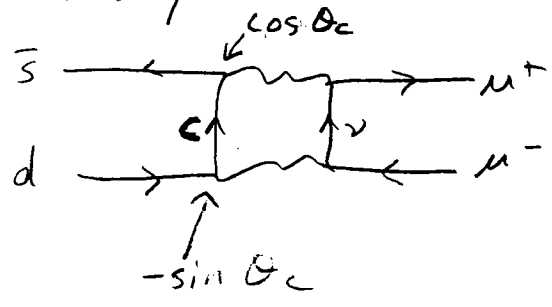
So far it's useless, but GIM suggested that if we give it a new quark c to interact with, we can get cancellations and eliminate the problem.

And in fact, the problem is more subtle than I've implied. The Z turns out not to mix flavors, which eliminates the problems on the previous page. The real problem is that multiple W exchange (via a loop) can achieve the same thing, as in $K_L \rightarrow \mu^+ \mu^-$:



$$-\sin\theta_c \cos\theta_c * (\text{ampl})$$

This should give a BR much bigger than the measured one of $\sim 10^{-8}$ so we still need a cancellation. This comes from the diagram where a c quark instead of a u quark is exchanged:

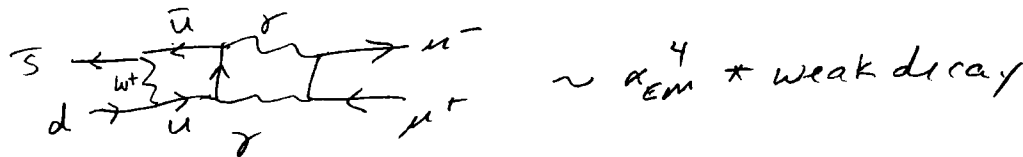


$$-\cos\theta_c \sin\theta_c * (\text{ampl})$$

\Rightarrow cancellation

Note: Since $\cos\theta_c$ is near 1 and $\sin\theta_c$ is small, $s+d$ couple most strongly to members of their own generation.

- The measured BR is consistent with



(Note that the cancellation on the previous page isn't exact because of the mass difference between the $u+c$. The difference in amplitudes is pretty much negligible, though.)

- A few years after this was all proposed, charm was discovered (via the J/psi) in 1974.

* - The convention in writing diagrams is that the quark lines refer to the strong + EM eigenstates, and the appropriate factors relating them to the weak eigenstates get thrown in explicitly as part of the vertex factors. Hence d' + s' are the weak eigenstates, but it's always d + s in the diagrams.

- This can be expressed in matrix form. Being sloppy with overall factors, we can write down the charged currents for leptons as

$$J_{leptons}^\mu \sim \bar{\mu} \gamma^\mu (1 - \gamma_5) \nu_\mu + \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e + h.c.$$

Let's be sloppy, drop the h.c. + just refer to $\gamma^\mu (1 - \gamma_5)$ as O ; we don't care about the Lorentz structure at the moment.

Now let's combine the leptons into column + row vectors, where the row vectors imply hermitian conjugation.

(Bear with me. We'll get to the quarks.)

So we have

$$J_{\text{leptons}} \sim (\bar{e} \ \bar{\mu}) \theta \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$= (\bar{e} \ \bar{\mu}) \theta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

(Each of e, μ, ν is a spinor; we've buried the Lorentz + spinor structure)

The matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ tells how the charged lepton EM eigenstates couple weakly to the neutrinos. The matrix happens to be diagonal in this case. Note that it has to be unitary because it's just a rotation (though a trivial one here).

For the quarks, things get more interesting because of the difference between the strong + weak eigenstates. But still, we just have a rotation described by a single parameter θ_c :

$$J_{\text{quarks}} \sim (\bar{u} \ \bar{c}) \theta \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Cabibbo joined by Kobayashi + Maskawa: 3 generations + CKM

So things are looking good, and they start to look even better when Kobayashi + Maskawa point out that if there were six quarks instead of four, the matrix would be 3x3. It would still be unitary of course, but when all is said and done (and some phase freedom is absorbed into quark wave fns) would have . . .

... four degrees of freedom:

(3, 28)

3 euler angles + one complex phase

The beauty part is that the phase can account for CP violation! (It can't actually explain it, but at least it can parametrize it.). The rest, as they say, is history. We have 6 quarks, with

$$J_{\text{quarks}} \sim (\bar{u} \ \bar{c} \ \bar{t}) \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{=V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Tim's talk

V is

the CKM matrix. It's unitary ($V^T V = I$) and complex. The diagonal elements are very close to 1, and the farther away you get from the diagonal, the smaller they get in magnitude, so that cross-generational transitions are suppressed. Unitarity gives various constraints, among which is that the absolute squares of any row or column must sum to one; think of the previous sentence in light of that. The off-diagonal elements of $V^T V$ must be 0 from unitarity; that condition is exactly the 3-generational extension of the cancellation provided by the GIM mechanism.

In fact, the upper left part of V , $\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$

is, to an excellent approximation, $\begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$; that's

why the GIM mechanism worked in the first place.

This ^{+ unitarity,} simplifies that $V_{tb} \approx 1$ and the other V_{ti} 's and V_{ib} 's are very small. (N.B. The values of these parameters are determined experimentally; nobody knows why they are what they are.)

Aside: That $V_{tb} \approx 1$ is why top can be considered to decay exclusively to Wb .

A standard parameterization is

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij}, \quad s_{ij} \equiv \sin\theta_{ij}$$

Three angles $\theta_{12}, \theta_{13}, \theta_{23}$ + 1 phase δ .

θ_{12} is Cabibbo angle, + $\theta_{23} + \theta_{13} \ll 1$. So

approximately, keeping only 1st order

$$V \approx \begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} & c_{12} & s_{23} \\ s_{12}s_{23} - c_{12}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}e^{i\delta} & 1 \end{pmatrix}$$

3.30

The most common approx^{parameterization} used these days for the CKM matrix V is not the one at the bottom of the previous page. Instead, it's one due to Wolfenstein (fr/ 1983):

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

The param's λ, A, ρ, η are real. The $\rho - i\eta$ in the corners is what allows CP violation to be incorporated.

$$\lambda \approx \sin \theta_c \approx 0.22 \quad \text{fr/ strange part. decays}$$

$$A = 0.79 \pm 0.06 \quad \text{fr/ } b \rightarrow c \text{ decays } (V_{cb} = A\lambda^2)$$

$$\sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09 \quad \text{fr/ charmless } b \text{ decays}$$

phase of V_{ub} , i.e., relative sizes of $\rho + \eta$, not known well + only obtainable indirectly.

For more info, see e.g. Rosner's excellent plenary talk fr/ DPF94 (EFI 94-38; hep-ph/9408349).

Precision meas. of CKM matrix elements give info about θ and possible new physics. (E.g., deviations fr/ unitarity may indicate existence of additional generations).