# II. Electroweak Theory

Having taken a historical tour of some highlights of weak interaction physics, we're now going back to the group theory: the SU(2) x U(1) part. We'll need some (but not a lot) of the Lie algebra machinery from last semester. I refer you to Tipton's notes or Georgi's *Lie Algebras in Particle Physics* or chapter 2 of Halzen + Martin for a review.

First a preview. Remember what we did for the case of EM, that is, what we did to get the full Lagrangian density with interactions? If, for example, we started out with a free fermion, we had

$$\mathcal{L}_{free} = \bar{\psi}(i\not{\partial} - m)\psi$$

cf pp. ~2.17, 2.39 etc

To add EM interactions, we took

$$\left.\begin{array}{l} ① \ \partial^\mu \rightarrow D^\mu = \partial^\mu + iqA^\mu \\ ② \ \mathcal{L} \rightarrow \mathcal{L} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \end{array}\right\} \text{For E+M}$$

where $A^\mu$ is the photon field and $F^{\mu\nu} = (\partial^\mu A^\nu - \partial^\nu A^\mu)$

That's all we do for any gauge symmetry, except that we'll have to generalize a little to allow for non-abelian groups.

(Hint: The self-interactions of the gauge bosons will come from the generalization of $F^{\mu\nu}$!)

<u>One more comment</u>: Since the EM + weak interactions are unified, what we're about to do now <u>includes</u> what we already did for EM. We'll see the EM interaction fall out of this. (Also, the U(1) we start with is not the U(1) we end up with. You'll see what I mean below.)

<u>Yet another comment</u>: We said that the charged-current weak interaction (i.e., the one involving W's, not Z's) was V-A, which means it couples only to lefthanded particles. Only the interactions with W's have this pure V-A/left-handed character. In particular

> Z's couple to right-handed fermions
> (with <u>small</u> strength compared to left-handed)
>
> $\gamma$'s don't care about handedness at all
> (they couple equally to right- + left-handed fermions)
>
> (ditto for gluons)

These are exptal facts that have to be accounted for in electroweak unification.

Enough preliminaries. We're going to do two things:

1. Show how to write down a gauge theory of interactions for general nonabelian symmetries
2. Write down the $SU(2)_L \times U(1)_Y$ theory and show how the EM + weak interactions come out.

## 1. Non-abelian gauge theory

As I said, this is analogous to what we did for $U(1)$ + E&M, only now the symmetry group can be non-abelian. We'll make this general and not specify a group yet, but you can keep in mind as examples $SU(2)$ for the weak interaction and $SU(3)$ for QCD.

So consider a group $G$ associated with a symmetry transf. The transf. is given by $U$ (an element of the group.)

Schematically, $\psi \to U\psi$

(~~Trivial~~ ex: $U = e^{i\alpha}$ for $U(1)$; $\alpha$ is param of. transf.)

In general we write

$$U = e^{i\alpha_a T_a} \qquad \text{(sum over } a\text{)}$$

Where
- $T_a$ are the **generators** of the transf.
  (e.g. $J_1, J_2, J_3$ for spin $SU(2)$)
- How many there are depends on the group. There are $N^2-1$ for $SU(N)$
  i.e., 3 for $SU(2) \leftrightarrow W^\pm, W^0$ ← see below
  8 for $SU(3) \leftrightarrow$ gluons
- $\alpha_a$ are param's of the transf. which in general can depend on spacetime

position x. If they do, it's a local transf. If they're constant, it's a global transf. We won't actually care about particular values of the $\alpha$'s; we just care that $\mathcal{L}$ be invariant under arbitrary $\alpha_a(x)$.

More about generators etc

- In $SU(N)$, $S \equiv$ special (det $U = 1$) $\Rightarrow \mathrm{tr}(T_a) = 0$ all a

  $U \equiv$ unitary $U^\dagger U = 1$

  n.b. $U^\dagger U = 1 \Rightarrow \alpha_a T_a = \alpha_a^* T_a^\dagger$ so take $T = T^\dagger$ so $\alpha$'s real

- Non-abelian $\Leftrightarrow$ T's don't commute

$$[T_a, T_b] = i f_{abc} T_c \qquad \text{(sum over } c\text{)}$$

In words, the commutator of any two generators is a linear combination of all the T's. The $f_{abc}$, which tell you what the combination is, are called the structure constants, and depend on the group. $f_{abc}$ are antisymmetric under exchange of any pair of indices.

Ex: $f_{abc} = \epsilon_{ijk}$ for $SU(2)$ $i,j,k = 1,2,3 = N^2-1$

- The T's are linearly indep. traceless hermitian matrices. There's no fixed set of them for any given group (they just have to satisfy the commutation relns) but standard choices are

(2 spin states) SU(2): Pauli spin matrices $\vec{\sigma}$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(3 colors) SU(3): Gell-Mann matrices $T_a = \lambda_a/2$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

These matrices act on wave functions whose component correspond to the associated symmetry. E.g., for $SU(2)_L$, it's weak isospin $\pm 1/2$ e.g. $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ or $\begin{pmatrix} u \\ d \end{pmatrix}$

For $SU(3)_c$, it's color (R, G, B) e.g. $\begin{pmatrix} u_R \\ u_G \\ u_B \end{pmatrix}$.

You often see $\begin{pmatrix} \nu \\ e \end{pmatrix}$ but we don't usually write; usually we just throw in a color index because the 3 color states are otherwise indistinguishable.

[Aside: You may notice that I've already assumed that the matter particles fit into the fundamental representation of the groups: N dimensional for SU(N). If you've forgotten what that means, don't worry about it; I just wanted to point out that I'm making an assumption.]

Back to the Lagrangian. Suppose we have a Dirac particle with

$$\mathcal{L}_{free} = \bar{\psi}(i\not{\partial} - m)\psi$$

and we want invariance under a local symmetry transf. For simplicity, let's take it to be infinitessimal ($\alpha$'s small)

$$U = e^{i\alpha_a T_a} \simeq 1 + i\alpha_a T_a$$

So $\psi \to (1 + i\alpha_a(x) T_a)\psi$ and $\partial_\mu\psi \to (1 + i\alpha_a T_a)\partial_\mu\psi + iT_a\psi\partial$

If we do as in QED, we get gauge fields (one corresponding to each generator; there was just one in QED but there are $N^2-1$ in $SU(N)$)

$$G_\mu^a \to G_\mu^a - \frac{1}{g}\partial_\mu\alpha_a$$

← This will be changed

and covariant derivative

$$\boxed{D_\mu = \partial_\mu + igT_a G_\mu^a}$$

what we'd do in analogy w/ QED

where $g$ is the coupling strength; the group stuff is buried in the T's. So plugging $D_\mu$ into $\mathcal{L}$ we get

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - g(\bar{\psi}\gamma^\mu T_a\psi)G_\mu^a$$

If we apply a gauge transf, the $\bar{\psi}\not{\partial}\psi$ piece works fine, and we get cancellation from the $G_\mu^a$ piece as in QED.

But there's a new problem due to the fact that the $T_a$'s don't commute. Recall $\psi \to (1 + i\alpha_b T_b)\psi$ + $\bar\psi \to \bar\psi(1 - i\alpha_b T_b)$ ($T$ hermitian). Then

$$\bar\psi \gamma^\mu T_a \psi \to \bar\psi\gamma^\mu T_a \psi + i\alpha_b \bar\psi \gamma^\mu (\underset{\uparrow\ \psi\ \text{transf}}{T_a T_b} - \underset{\bar\psi\ \text{transf}}{T_b T_a})\psi$$

$$= \bar\psi\gamma^\mu T_a\psi - f_{abc}\,\alpha_b(\bar\psi\gamma^\mu T_c\psi)$$

and this last term screws up the invariance. So we need another term in G's transformation:

$$\boxed{G_\mu^a \to G_\mu^a - \frac{1}{g}\partial_\mu \alpha_a - f_{abc}\,\alpha_b G_\mu^c}$$

That takes care of the part of $\mathcal{L}$ containing $\psi$. But now when we go back to put in the gauge field lagrangian $\mathcal{L} = -F_a^{\mu\nu} F_{\mu\nu}^a$ with $F_a^{\mu\nu} = (\partial^\mu G_a^\nu - \partial^\nu G_a^\mu)$, we find that it's not gauge invariant anymore either, with the new transformation. So we have to extend our definition of $F^{\mu\nu}$ to

$$\boxed{F_{\mu\nu}^a = (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) - g\, f_{abc}\, G_\mu^b G_\nu^c}$$

and keep

$$\mathcal{L}_{gauge} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$$

Note: Again, this is all related to geometrical things & isn't as arbitrary as it looks

note that the sum over a is a trace as promised

So for general gauge fields we have

$$\boxed{\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu}}$$

with $D_\mu = \partial_\mu + ig T_a G^a_\mu$

and $F^{\mu\nu}$ as on p. 3.38.

N.B. The eq'ns with boxes around them are correct for the general case.

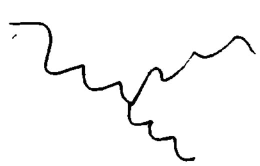
Comments

- The gauge bosons still have to be massless to satisfy gauge invariance. We'll come back to that.

- Our general result reduces to what we had for QED when the group is $U(1)$. In that case, $f_{abc} = 0$, $a = 1$ only, and $T_a = 1$. actually $Q$, the charge op.

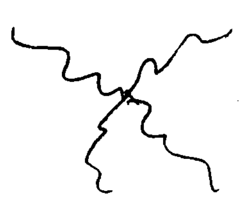
- We now have interacting gauge bosons as a direct result when the group is non abelian! It comes from the $f_{abc} G^b G^c$ term in $F_{\mu\nu}$. When we take $F^{\mu\nu}_a F^a_{\mu\nu}$ we get the usual derivative terms (that give rise to the propagator), but we also get new Interaction terms that look like

$$g f_{abc} G^b_\mu G^c_\nu \partial^\mu G^{\nu,a}$$



$\partial^\mu$ brings down factor of momentum

and $g^2 f_{abc} f_{ade} G^b_\mu G^c_\nu G^{\mu,d} G^{\nu,e}$



strength $g^2$

2. $SU(2)_L \times U(1)_Y$ : Electroweak

So that's how it works in general. Now let's look at the standard model. We have to figure out what the symmetries are, what the associated quantum numbers of the fermions are, & how we get the weak + EM interactions out of it. The problem is they're mixed together + have to be untangled. (E.g., the U(1) turns out not to be exactly the U(1) of E+M...)

In fact, we don't have to mess with the group structure much. What we have to do is write down the interactions that follow from the group structure as described above, and show that the weak + EM int's we see in expts actually fall out. So if we get the groups right, the rest is just algebra.

It's useful to switch back to the current language, where interactions with gauge fields are written as

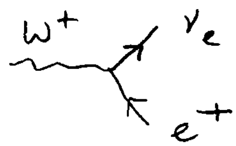
$$\left(\bar{\psi}\gamma^\mu T_a \psi\right) G_\mu^a \sim J_a^\mu G_\mu^a$$

(we'll leave out the coupling constant, and may throw in projection operators as appropriate.)

Following Halzen + Martin, chap. 13, let's see if we can combine the weak charged + neutral currents into a symmetry group of the weak interactions to get something like

Start with the charged currents that couple to W's (using leptons as example)

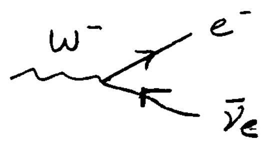
$$J_\mu \equiv J_\mu^+ = \bar{u}_\nu \gamma_\mu \tfrac{1}{2}(1-\gamma^5) u_e$$




$$\equiv \nu_L \gamma_\mu e_L \quad \text{for short}$$

$$J_\mu^\dagger \equiv J_\mu^- = \bar{u}_e \gamma_\mu \tfrac{1}{2}(1-\gamma_5) u_\nu$$

$$= \bar{e}_L \gamma_\mu \nu_L$$




Note that we put in the lefthanded projection operator $\frac{1}{2}(1-\gamma_5)$ since the W only couples to left-handed particles. The subsript $L$ on $e + \nu$ means we project out the left-handed part.

**Aside:** In case you're wondering,

$$\bar{e}_L = e_L^\dagger \gamma^0 = e^\dagger \tfrac{1}{2}(1-\gamma_5)\gamma^0 \qquad \text{since } \gamma_5^\dagger = \gamma_5$$

$$= e^\dagger \gamma^0 \tfrac{1}{2}(1+\gamma_5) \qquad \text{since } \{\gamma^0, \gamma^5\} = 0$$

$$= \bar{e}\, \tfrac{1}{2}(1+\gamma_5)$$

so

$$\bar{e}_L \gamma_\mu \nu_L = \bar{e}\, \tfrac{1}{2}(1+\gamma_5)\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\nu$$

$$= \bar{e}\, \gamma_\mu \tfrac{1}{2}(1-\gamma_5)\tfrac{1}{2}(1-\gamma_5)\nu$$

$$= \bar{e}\gamma_\mu \tfrac{1}{2}(1-\gamma_5)\nu \qquad \text{since } \left[\tfrac{1}{2}(1-\gamma_5)\right]^2 = \tfrac{1}{2}(1-\gamma_5)$$

Now, the fact that there are 3 weak bosons suggests 3 generators, which suggests $SU(2)$. We can make it even more suggestive by combining $\nu$ + $e^-$ into a doublet

$$\chi_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}_L$$

And use raising + lowering ops $\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

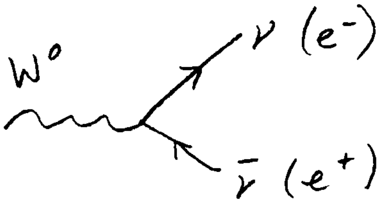
Where the $\tau_i$'s are the Pauli spin matrices (same as $\sigma_i$ on p. 3.35). Then

$$J_\mu^+(x) = \bar{\chi}_L \gamma_\mu \tau_+ \chi_L$$

$$J_\mu^-(x) = \bar{\chi}_L \gamma_\mu \tau_- \chi_L$$

Now all we need to complete $SU(2)$ is a $J^3$ neutral current

$$J_\mu^3 = \bar{\chi}_L \gamma_\mu \tfrac{1}{2} \tau_3 \chi_L = \tfrac{1}{2} \bar{\nu}_L \gamma_\mu \nu_L - \tfrac{1}{2} \bar{e}_L \gamma_\mu e_L$$




So we'd have an isospin-type triplet of currents, suggesting $J^3$ describes the weak neutral current interaction. Wrong! In another case of beautiful-theory-shot-down-by-ugly-fact, recall the warning on p. 3.32: Z's couple to right-handed fermions. But this $J^3$ has only left-handed fermions, so it cannot be the weak neutral current. Damn.

So the observed weak neutral current doesn't respect the $SU(2)_L$ symmetry. But recall that the EM current also has a righthanded piece. So we could combine the observed currents into two orthogonal combinations that have definite transf. properties

$J^3_\mu$ as above, which completes the weak isospin triplet

and $J^Y_\mu$. which is invariant under $SU(2)_L$

<u>Y</u> is called <u>weak hypercharge</u>

$$j^Y_\mu = \bar{\psi}\gamma_\mu Y \psi$$

and Y is <u>defined</u> by the requirement to get the charge right:

$T^3 \Leftrightarrow$ 3rd component of weak isospin

$$Q = T^3 + \frac{Y}{2}$$

so $$j^{EM}_\mu = j^3_\mu + \frac{1}{2} j^Y_\mu .$$

$T_3$ is $\pm\frac{1}{2}$ for $\nu_L, e_L$ respectively: $\binom{\nu}{e}_L$ form a doublet.
Y is just assigned to make Q come out right.

<u>Comments</u>

- Weak isospin and weak hypercharge are analogous to regular old isospin + hypercharge that you discussed last semester. They are <u>not</u> the same thing. Confusing, but true. [Remember how isospin symmetry helped calculate ratios of scattering rates for π's, p's, etc? That was regular old isospin – you were talking about strong interactions. This new thing, weak isospin, is similar mathematically (they're both SU(2)) but is a different quantum number.]

– Left-handed fermions form weak isospin doublets: these are the very doublets that everyone writes down when giving the matter content of the SM

| | $T$ (weak isospin) | $T_3$ | $Q$ | $Y = 2(Q-T_3)$ |
|---|---|---|---|---|
| $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ | $\frac{1}{2}$ | $+\frac{1}{2}$ | $0$ | $-1$ |
| | | $-\frac{1}{2}$ | $-1$ | $-1$ |
| $\begin{pmatrix} u \\ d \end{pmatrix}_L$ | $\frac{1}{2}$ | $+\frac{1}{2}$ | $+\frac{2}{3}$ | $\frac{1}{3}$ |
| | | $-\frac{1}{2}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ |

etc.

– Right-handed fermions, which don't couple to W's, are weak isospin singlets: they have $T=0$.
So they're written by themselves

| | $T$ | $T_3$ | $Q$ | $Y = 2(Q-T_3)$ |
|---|---|---|---|---|
| $e_R^-$ | $0$ | $0$ | $-1$ | $-2$ |
| $u_R$ | $0$ | $0$ | $\frac{2}{3}$ | $\frac{4}{3}$ |
| $d_R$ | $0$ | $0$ | $-\frac{1}{3}$ | $-\frac{2}{3}$ |

etc.

$\nu_R$'s don't exist as far as we know. But they would if $\nu$'s had mass.

— Weak hypercharge generates a $U(1)$ of its own: $U(1)_Y$.

So we're almost there. We've got $SU(2)_L \times U(1)_Y$, along with the appropriate currents and assignments for fermions. We know how to write down Lagrangians containing interactions between the fermions and the $SU(2)_L$ and $U(1)_Y$ gauge bosons. But we have to relate the latter to the actual $W^\pm$, $Z$ and photon.

So, Let the $\underline{SU(2)_L \text{ and } U(1)_Y}$ bosons be

$SU(2)_L$ $\quad W^1_\mu, W^2_\mu, W^3_\mu$ $\qquad$ They're all vector bosons.

$U(1)_Y$ $\quad B_\mu$

So the interactions look like (according to our recipe)

$$SU(2)_L: \quad -ig J^a_\mu W^\mu_a = -ig \bar{\chi}_L \gamma_\mu T^a W^\mu_a \chi_L, \quad a = 1, 2, 3$$

$$U(1)_Y: \quad -i\frac{g'}{2} j^Y_\mu B^\mu = -ig' \bar{\psi} \gamma_\mu \frac{Y}{2} \psi B^\mu$$

<u>Comments</u>

— $\chi_L$ is as above; $\psi$ contains both left-handed components $\chi_L$ and right-handed components $\psi_R$. They transform under $SU(2) \times U(1)$ as

$$\chi_L \to e^{i\alpha^a T_a + i\beta Y}\chi_L$$

$$\psi_R \to e^{i\beta Y}\psi_R \qquad \text{"isospin singlet"}$$

- $T$ and $Y$ are generators of the $SU(2)$ and $U(1)$. The $\frac{1}{2}$ in $\frac{Y}{2}$ is a convenient convention.

- $g$ and $g'$ are the $SU(2)$ and $U(1)$ couplings,

__Physical gauge bosons__:

The $W^\pm$ are just $W^\pm = \frac{1}{2}(W_1 \pm iW_2)$ as suggested above

The $Z^0$ and $\gamma$ ($Z_\mu$ + $A_\mu$) are orthogonal linear combinations of $W^3$ and $B$:

$$A_\mu = \cos\theta_W\, B_\mu + \sin\theta_W\, W^3_\mu$$

$$Z_\mu = -\sin\theta_W\, B_\mu + \cos\theta_W\, Z_\mu$$

$\theta_W$ is the Weinberg, or weak mixing angle, to be determined by experiment.

So we can write the neutral part of the Electroweak interaction in terms of the photon and $Z$:

$$-igJ_\mu^3 W^{3\mu} - i\frac{g'}{2}j_\mu^Y B^\mu = -i\left[g\sin\theta_W J_\mu^3 + g'\cos\theta_W \frac{j_\mu^Y}{2}\right]A^\mu$$

$$-i\left[g\cos\theta_W J_\mu^3 - g'\sin\theta_W \frac{j_\mu^Y}{2}\right]Z^\mu$$

Now we require that the thing make physical sense. We know that the EM interaction is

$$-ie\, j_\mu^{EM} A^\mu, \quad \text{with } j_\mu^{EM} = J_\mu^3 + \tfrac{1}{2} j_\mu^Y$$

So we require that the 1st term in square brackets (previous page) gives this, i.e.

$$g \sin\theta_W J_\mu^3 + g' \cos\theta_W \frac{j_\mu^Y}{2} = e\, j_\mu^{EM}$$

This requires $\boxed{e = g\sin\theta_W = g'\cos\theta_W}$

So the couplings $g$ & $g'$ are fixed in terms of $e$ & $\theta_W$.

Let's also rewrite the weak neutral interaction current, i.e., the $Z$ term on the previous page:

$$\mathcal{L}_{NC}^{weak} = \frac{-ig}{\cos\theta_W} \underbrace{\left(J_\mu^3 - \sin^2\theta_W\, j_\mu^{EM}\right)}_{J_\mu^{NC}} Z^\mu$$

$$= \frac{-ig}{\cos\theta_W} \bar{\psi} \gamma_\mu \left[\tfrac{1}{2}(1-\gamma^5)T^3 - \sin^2\theta_W Q\right] \psi Z^\mu$$

where we've written the LH projection op. explicitly. We'll come back to this shortly.

So this makes a nice framework (once we get W + Z masses - see next lecture). It says that

The weak interactions are universal: once you know $e$ and $\sin^2\theta_W$ (the usual way $\theta_W$ is expressed), and, say $G_F$ (or $M_W$), you can predict everything. And it works!

In fact, once the theory was in place, it was possible to predict the masses of the W and Z before they were discovered; see Cahn Chap. 12 for discovery. Actually, the prediction was used to help design the accelerator at CERN where they were discovered in 1983. Remember $G_F$, which can be measured e.g. in $\mu$ decay? It's related to $M_W$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

$g$ is related to $e$ and $\sin\theta_W$, so we can write

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F}$$

$\alpha = \frac{1}{137}$ comes fr/ EM interactions, $G_F$ from $\mu$ decay, and $\sin^2\theta_W \simeq 0.23$ from weak neutral current interactions.

We then predict $M_W \simeq 80$ GeV.

As we'll see below, electroweak symmetry breaking predicts that the masses of the W + Z are related:

At tree level (lowest order),

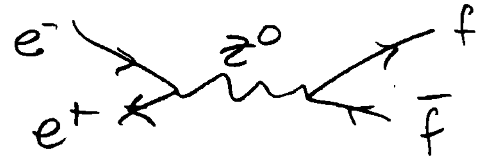
$$M_Z^2 = M_W^2 / \cos^2\theta_W$$

or $M_Z \simeq 90$ GeV. In 1983, the W + Z were found at CERN in a $\bar{p}p$ collider in the predicted mass range: a great triumph for the SM.

Physics at the $Z^0$

LEP and SLC are $e^+e^-$ colliders designed to produce $Z^0$'s for precision electroweak measurements. They sit at center of mass energy $\sqrt{s} \simeq M_Z$. LEP is a Z factory: it does precision measurements by producing lots of Z's (millions accummulated by now). SLC doesn't produce as many but in the last couple of years has achieved comparable sensitivity with polarized beams.

You can see from the form of the NC interaction at the bottom of 3.46 that there's lots of potential for measurements in Z physics. ~~At~~ LEP and SLC the basic interaction is

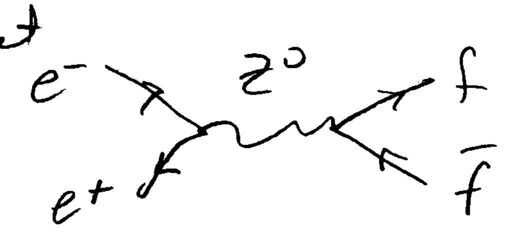



, where f is lepton or quark

Because the energy is close to the Z mass, we have to include the $Z^0$ decay width $\Gamma_Z$ in the Z propagator: it's just described by a Breit-Wigner resonance. By measuring the cross section as a function of energy (the line shape), you can get info about $m_Z$, $\Gamma_Z$, etc. From $\Gamma_Z$ you can find out how many light neutrinos there are, as discussed earlier in the semester. You can also study individual decay modes or decay distributions to get more detailed info. Ultimately, the hope is to find a deviation from the SM. So far the SM is winning, with one possible exception in $Z \to b\bar{b}$.* Let me go quickly through a couple of topics on physics at the Z.

## 1. Line Shape

When we're creating a heavy particle like the Z which decays, if we sit on or near resonance ($E_{cm}^2 = m_Z^2$), we have to be careful about what we use for the propagator. Based on what I've covered so far, you'd put




$$\sim \frac{1}{s - m_Z^2} = \frac{1}{E_{cm}^2 - m_Z^2}$$

* Summer 1996: This deviation is going away.

This has one obvious problem: it blows up for $E_{cm} \to m_Z$. While this would be exciting, it's not physical. We need to take into account the natural width of the Z. Because it's unstable, the Z doesn't always have $p^2 = m_Z^2$ exactly; $p^2$ is distributed about $m_Z^2$ w/ a Lorentzian dist. w/ FWHM (full width at half the maximum) equal to $\Gamma_Z$. This is the same $\Gamma_Z$ we can get by adding up all the partial widths for individual decays (e.g. $Z^0 \to e^- e^+$), and $1/\Gamma_Z$ is the Z lifetime.

Thus the lifetime is related to an uncertainty in the square of the momentum. This should remind you of discussions in regular old QM about lifetimes of unstable states and resonances, which should bring to mind Breit-Wigners. In this case, the correct Z propagator contains the width and is the relativistic generalization of a B-W:

$$\frac{1}{s - m_Z^2} \to \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z}$$

and when we take the absolute square (as when computing $|M|^2$ for a cross section) we get

$$\frac{1}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}.$$

So now we can write down the expression for the Z production cross section as a function of center-of-mass energy (also known as the <u>line shape</u>). It is, near resonance,

$$\sigma(E_{cm}) = \sigma_0 \frac{E_{cm}^2 \Gamma_Z^2}{(E_{cm}^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2},$$

where

- $\sigma_0$ = cross section at $E_{cm} = M_Z$; it has $G_F$ and the couplings of the electron to the Z. For more details see e.g. Barger + Phillips ~ p. 114 or so. Or better yet, calculate it yourself! Just be careful w/ $\gamma_5$'s.

- strictly speaking, I should have put $E^4\Gamma_Z^2/M_Z^2$ instead of $M_Z^2\Gamma_Z^2$, but at this level we won't worry about the difference.

Measurement of this line shape allows us to make fits to measure $\sigma_0$, $M_Z$, and $\Gamma_Z$.

We can compare to SM predictions to see if something non-standard is going on (so far, it's not). In particular, measuring $\Gamma_Z$ gives us a limit on light neutrinos (on how many there are, I mean.) Recall that

$\Gamma_Z$ = <u>total</u> width, and <u>all</u> decay modes contribute, whether you see them directly or not. This includes


So if there are more than 3 generations of light neutrino (light because we want the Z to decay to it), we would measure a $\Gamma_Z$ bigger than what we calculate in the SM. We don't.

## (2) Forward-backward asymmetry

We already saw this in our discussion of $e^+e^- \to \mu^+\mu^-$: it's the asymmetry in the angular distribution about $\cos\theta = 0$, and it's there because of the $\gamma_5$ in the $Z$ couplings.

We can define

$$A_{FB} = \frac{N(\text{forward}) - N(\text{backward})}{N(\text{forward}) + N(\text{backward})}$$

It's a prediction of the SM, and depends on energy and on the detailed nature of the weak interactions, and makes a sensitive test of SM

## (3) Left-right asymmetry

At SLC, they can polarize the initial electron and positron beams. The resulting polarized cross sections are also sensitive to weak interaction physics. One "standard" quantity to measure is the <u>left-right asymmetry</u>, which involves scattering left- and right-handed electrons off unpolarized positrons.*

* Positrons are harder to polarize than electrons

The left-right asymmetry is

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Again, everything's consistent w/ the SM.

Aside: Measurements at LEP reached such a level of precision that they could see not only tidal effects of the moon, but even effects on their magnetic fields from the passing of the TGV. Pretty amazing.