# Electroweak Symmetry Breaking



It's time to come to terms with gauge boson masses and introduce the Higgs. You will recall that a mass term for the gauge bosons violates gauge invariance, and is not renormalizable.

The Higgs mechanism (proposed 30 yrs ago independently by Higgs; Brout & Englert; and Guralnik, Hagen* & Kibble and shown to be renormalizable around '70 by 't Hooft) takes care of both problems. In the Higgs mechanism, the <u>symmetry is broken not in the Lagrangian but in the ground state of the universe (a.k.a. the vacuum) itself.</u> The upshot is that you get massive gauge bosons <u>and</u> a gauge invariant Lagrangian. The price you pay is introduction of a scalar field that interacts with the $W^\pm$ & $Z$ as well as with itself. You end up with a new particle, the Higgs boson.

This method of having the symmetry broken by the vacuum itself (it's a consequence of the self-interactions of the scalar field, as you'll see below) is called

spontaneous symmetry breaking.

("spontaneous" as opposed to "dynamical" in which some new dynamics gives rise to bound states that look like the Higgs...)

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*Yes, it's the same Hagen!

**Aside**: Suppose you don't care about gauge invariance or even renormalizability, and you just forge ahead with a plain old mass term with no Higgs. You *still* have problems, this time with unitarity.

Look at $W_L^+ W_L^- \to Z_L Z_L$, where the subsript $L$ refers to the longitudinal component. In the absence of the Higgs boson, this is a pure gauge interaction, and we have

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L)\Big|_{gauge} = \frac{s}{v^2}$$

Where $s = (\text{cm energy})^2$ and $v = 246$ GeV. This increases w/ increasing energy, and for $\sqrt{s} > 1.7$ TeV, violates unitarity.

But if we allow Higgs exchange, no problem:

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L)\Big|_{SM} = \frac{m_H^2}{v^2}$$

Now we have (tree-level) unitarity if $m_H \lesssim 1.7$ TeV. A more sophisticated analysis brings this down to the order of 1 TeV. This is the source of the upper limit of 1 TeV for the Higgs mass.

Following the very nice, lucid discussion in Halzen + Martin, we'll build up electroweak symmetry breaking in several steps:

1. Spontaneous symmetry breaking: simple example
2. Spontaneous breaking of an abelian gauge symm. ($U(1)$) + the Higgs mechanism
3. Spontaneous breaking of a nonabelian gauge symm: local $SU(2)$

   [N.B.: For once, going fr/ abelian to nonabelian will have no fundamental differences]

4. How it works in the SM
   $\Rightarrow$ how to get masses for W's + Z's but not $\gamma$

## 1. SSB: simple example

First we'll see how a symmetry can be broken by the ground state of the theory, without worrying about gauge boson masses.

Consider a real, scalar field $\phi$ that interacts only w/ itself, with Lagrangian ($\lambda > 0$) $\mathcal{L} = T - V$

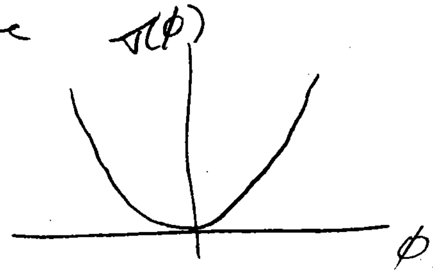
$$\mathcal{L} = \underset{\substack{\uparrow \\ \text{Kinetic term}}}{\tfrac{1}{2}(\partial_\mu \phi)^2} - \underbrace{\left(\tfrac{1}{2}\mu^2\phi^2 + \tfrac{\lambda}{4}\phi^4\right)}_{\text{potential}} \quad ,$$

Symmetry: $\mathcal{L}$ invariant under $\phi \to -\phi$

This is the symmetry that's going to break, Consider $V(\phi)$ for $\mu^2 > 0$ + $\mu^2 < 0$ :

a) $\underline{\mu^2 > 0}$ : $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{2}\lambda\phi^4$

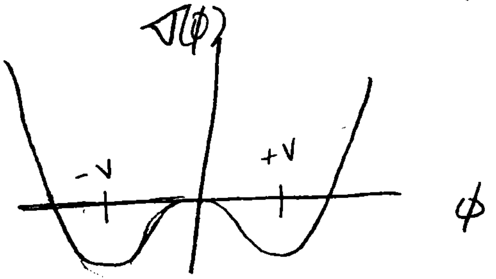
In this case, the $\mu^2$ is a mass term, the potential looks like




the ground state ($\equiv$ vacuum) has $\phi = 0$, + it's plain old $\phi^4$ theory which you get enough of in field theory class. Enough said.

b) $\underline{\mu^2 < 0}$ $\quad V(\phi) = -\frac{1}{2}|\mu^2|\phi^2 + \frac{1}{2}\lambda\phi^4$

Life is more interesting. The potential looks like




and has minima at $\underline{\phi = \pm v}$ $\quad \left(v = \sqrt{-\mu^2/\lambda}\right)$

The ground state is not at $\phi = 0$! This means that if we want to do perturbation theory, we should expand about $\pm v$, not about $\phi = 0$, because we have to expand about the lowest energy state.

So we expand about $\phi = +v$. <u>This choice breaks the symmetry</u>. The universe has to pick one of these to be the ground state and thereby breaks the symmetry. Hence the symmetry breaking is spontaneous. So we write

$$\phi(x) = \overset{\text{constant}}{v} + \underbrace{\eta(x)}_{\text{shifted field}}$$

where $\eta(x)$ is a scalar field with value 0 in the ground state. Substituting into $\mathcal{L}$, we get

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 - \lambda \eta^3 - \tfrac{1}{4}\lambda \eta^4 + \text{const}$$

and the field $\eta$ has a right sign mass term, with

$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

along w/ self-interaction terms.

<u>Comments</u>

- What if we had tried to do pert. theory about $\phi = 0$? It wouldn't converge, because we wouldn't be expanding about the min. energy state.

- <u>Nomenclature</u>: Actually $\phi = v$ is a <u>classical</u> minimum; the quantum field is not equal to $v$ in the ground state, but its expectation value is. We say $\phi$ has <u>vacuum expectation value</u>, or vev, equal to $v$. In symbols,

$$\langle \phi \rangle = v$$

Note that the new field $\eta(x)$ has $\langle \eta \rangle = 0$.

– This may seem contrived, but there are plenty of physical examples where the lagrangian has a symmetry but the ground state doesn't (the symmetry is "hidden"). The classic example is a <u>ferromagnet</u>: the Lagrangian has rotational symmetry but in the ground state, all the spins are lined up in one direction. The dir. is arbitrary, and so there is a continuum of possible ground states, but nature has to pick one. The ground states are related by rotations; in our example the two ground states $\pm v$ are related by reflection

## ② <u>Spontaneous breaking of a $U(1)$ symmetry + the Higgs mech.</u>

Now let's take a complex scalar field

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

with Lagrangian

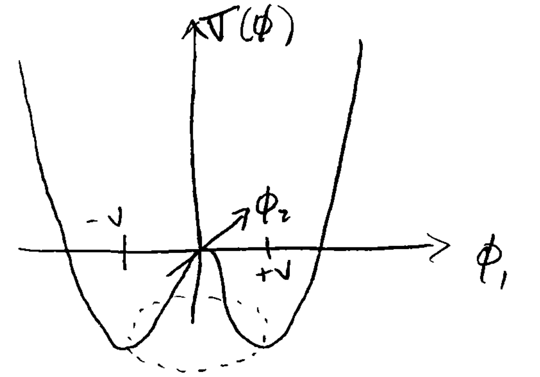
$$\mathcal{L} = (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$$

<u>Symmetry</u>: $\mathcal{L}$ invariant under global $U(1)$: $\phi \to e^{i\alpha}\phi$.

Again take $\mu^2 < 0$ + $\lambda > 0$. Now the <u>minimum of</u> $V$ is given by

$$|\phi|^2 = \phi_1^2 + \phi_2^2 = v^2, \quad (v^2 = -\mu^2/\lambda \text{ as before})$$

and the potential, which is now a function of $\phi_1 + \phi_2$, looks like a wine bottle or mexican hat; the minimum is a circle of radius $v$:




There's a continuum of possible ground states, none of which has $|\phi| = 0$, and again, we (+ nature) have to pick one to expand about. The possible ground states are related by exactly the symmetry transf. the lagrangian is invariant under. We pick $\langle\phi_1\rangle = v$, $\langle\phi_2\rangle = 0$:

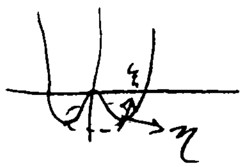
$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + \eta(x) + i\xi(x)\right] \qquad \begin{aligned}\phi_1 &= v + \eta(x)\\ \phi_2 &= \xi(x)\end{aligned}$$

We've exchanged the two degrees of freedom $\phi_1$ + $\phi_2$ for $\eta$ + $\xi$. The lagrangian becomes (just plug + chug)

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \xi)^2 + \tfrac{1}{2}(\partial_\mu \eta)^2 + \mu^2\eta^2 + \text{const.} + \text{cubic + quartic terms in } \eta, \xi$$

$\updownarrow$

So, as above, $\eta$ has a mass of $m_\eta = \sqrt{-2\mu^2}$. There's no mass term for $\xi$; it's a massless Goldstone (or Nambu-Goldstone) boson. Note that the $\xi$ direction is along the circle of min $V$:




so in the $\xi$ direction the potential is flat. Ergo, no mass.

This is an example of <u>Goldstone's</u> <u>Theorem</u> in action. It says that you get a massless scalar whenever a continuous symmetry is broken in the ground state.

<u>N.B.</u> You may be wondering if one of these scalars is ~~the Hig.~~ Higgs. Not quite, but we'll see it fall out below. The $\eta$ is almost the Higgs.

Meanwhile, let's promote the <u>U(1)</u> <u>symmetry</u> from global to <u>local</u> to show that the Goldstone boson gets "eaten" by the gauge boson, which becomes massive.

So we want invariance under <u>local</u> U(1) transf.

$$\phi \to e^{i\alpha(x)}\phi$$

We do the usual covariant derivative thing

$$D_\mu = \partial_\mu - ieA_\mu$$

with gauge field $A_\mu$ transforming as

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

and gauge invariant Lagrangian

$$\mathcal{L} = (\partial^\mu + ieA^\mu)\phi^*(\partial_\mu - ieA_\mu)\phi - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

If we had $\mu^2 > 0$, this would be a plain old charged scalar theory. But we don't, so it isn't.

This is exactly the same scalar potential as above, so we do the same expansion

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$$

and substituting into $\mathcal{L}$ we get

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2 \lambda \eta^2 + \frac{1}{2} e^2 v^2 A_\mu A^\mu$$

$$- e v A_\mu \partial^\mu \xi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \text{interaction terms.}$$

And voila, the gauge boson has a mass! $\boxed{m_A = ev}$

We have to do one more thing, though, to get the counting bookkeeping straight. There are two funny things here:

- We started out with 4 degrees of freedom
  - 2 from $\phi$
  - 2 for $A_\mu$ (transverse)

  Now we seem to have
  - 1 for $\eta$
  - 1 for $\xi$
  - 3 for $A_\mu$ (2 transverse + 1 longitudinal fr/ mass)

  can't both be right
- What about the funny $-evA_\mu\partial^\mu\xi$ term in $\mathcal{L}$?

The answer is that $\xi$ is not an independent physical particle. We have to fix up the Lagrangian so that it exhibits only the physical degrees of freedom. To do so we make use of the gauge freedom:

Note that

$$\phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi)$$

$$\simeq \frac{1}{\sqrt{2}}(v+\eta)e^{i\xi/v}$$

to lowest order in $\xi$. So let's use a different set of real fields, including a shift in $A_\mu$ by a gauge transf.

$$\phi \to \frac{1}{\sqrt{2}}(v + h(x))\, e^{i\theta(x)/v}$$

$$A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu \theta$$

$\eta$ & $\xi$ same as $h$ & $\theta$ only in lowest order

Substituting this into the original Lagrangian makes $\theta$ disappear. We have chosen $\theta$ s.t. $h(x)$ is real; it amounts to a gauge choice. Then we have

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 + e^2 v^2 A_\mu^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$
$$+ \frac{1}{2}e^2 A_\mu^2 h^2 + v e^2 A_\mu^2 h - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

with only the physical, massive particles $A$ + $h$ left. The "extra" degree of freedom wasn't real; it was only gauge freedom.

So we've got gauge invariance <u>and</u> massive gauge bosons, plus a new physical scalar field $h$. $h$ is a <u>Higgs</u> <u>particle</u> (but not <u>the</u> Higgs particle; that comes from breaking a different symmetry).

## (3) Spontaneous Breaking of a local SU(2)

In the SM we'll have to deal w/ $SU(2)\times U(1)$, so for the moment let's stick with SU(2). For once, going from an abelian to a nonabelian theory adds nothing fundamentally new. Everything works the same; there're just more terms to keep track of.

So, again, the scalar part of the lagrangian is

$$\mathcal{L} = (\partial_\mu \phi)^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

where now $\phi$ is an SU(2) doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

i.e. it has 4 degrees of freedom. We want SU(2) invariance, i.e. under

$$\phi \to e^{i\alpha_a \tau_a/2} \phi$$

For local gauge transformations $[\alpha = \alpha(x)]$, we do the usual

$$D_\mu = \partial_\mu + ig \frac{\tau_a}{2} W_\mu^a$$

where $W_\mu^a$, $a = 1, 2, 3$ are the gauge bosons assoc. w/ SU(2).

Under infinitessimal transformations

$$\phi \to \left(1 + i\vec{\alpha}\cdot\vec{\tau}/2\right)\phi$$

where we've used the shorthand $\vec{\alpha}\cdot\vec{\tau}$ for $\alpha_a \tau_a$. This is just convenient notation; $\vec{\alpha}$ + $\vec{\tau}$ are <u>not</u> spatial vectors.

Recalling that the structure constants for $SU(2)$ are $f_{abc} = \epsilon_{abc}$ and $(\vec{p}\times\vec{q})_i = \epsilon_{ijk} p_j q_k$, we can write the gauge field transf. as

$$\vec{W}_\mu \to \vec{W}_\mu - \frac{1}{g}\partial_\mu \vec{\alpha} - \vec{\alpha}\times\vec{W}_\mu$$

and the gauge invariant Lagrangian is

$$\mathcal{L} = \left(\partial_\mu \phi + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu \phi\right)^\dagger \left(\partial^\mu \phi + ig\frac{\vec{\tau}}{2}\cdot\vec{W}^\mu\phi\right) - V(\phi) - \frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu}$$

and $V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$, $\mu^2 < 0$ as usual.

Now the minimum of $V(\phi)$ is

$$\phi^\dagger\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

We have to pick a minimum to expand about, so we choose

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0 \text{ and } \langle\phi_3^2\rangle \equiv v^2,$$

or $\phi_0 \equiv \langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$,

thus breaking the $SU(2)$ symmetry. We play the expansion game, and, noting that the trick

$$\phi(x) = e^{i\vec{\tau}\cdot\vec{\theta}(x)/v}\begin{pmatrix}0\\ \frac{v+h(x)}{\sqrt{2}}\end{pmatrix}$$

along with an appropriate transf. of $\vec{W}_\mu$ works just as it did in U(1) to give mass + eliminate $\vec{\theta}$, we just substitute

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

into the Lagrangian, noting that in doing so we have thus made a gauge choice (this is called unitary gauge).

As in U(1), the W masses come from the $v$ part of $\phi$, and substituting into $\left| ig \frac{1}{2} \vec{\tau} \cdot \vec{W}_\mu \phi \right|^2$ we find for the mass terms

$$\frac{g^2 v^2}{8} \left( |W_1^\mu|^2 + |W_2^\mu|^2 + |W_3^\mu|^2 \right)$$

and since boson mass terms look like $\frac{1}{2} m W_\mu^2$, we have $m = \frac{1}{2} g v$.

That's how it works in pure SU(2). In real life we have SU(2) × U(1), so finally we move on to the SM

## ④ Electroweak Symmetry Breaking in the SM: SU(2) × U(1)

The trick in the SM is to use what we've developed above to make the $W^\pm$ + Z massive but keep the photon massless.

That means we have to pick the Higgs field with the appropriate quantum numbers such that the vacuum breaks SU(2) and $U(1)_Y$ but doesn't break $U(1)_{EM}$. The way to do that is to make sure the vacuum state is electrically neutral and thus invariant under $U(1)_{EM}$ transformations. That is we must have the charge operator $Q$ acting on $\phi_0$ give $0$:

$$Q\phi_0 = 0$$

So $\phi_0 \to e^{i\alpha(x)Q}\phi_0 = \phi_0 + \underbrace{\cancel{i\alpha(x)Q\phi_0 + \ldots}}_{0} \doteq \phi_0$

If the EM symmetry remains unbroken, the photon will remain massless.

So let's back up and specify $\phi$. We want $\mathcal{L}$ to be $SU(2) \times U(1)_Y$ invariant, so we have for the scalar part

$$\mathcal{L} = \left|\left(i\partial_\mu - g\vec{T}\cdot\vec{W}_\mu - g'\frac{Y}{2}B_\mu\right)\phi\right|^2 - V(\phi)$$

The $\phi_i$'s have to belong to $SU(2)\times U(1)_Y$ multiplets. we take a complex isospin doublet ($T_3 = \pm\frac{1}{2}$) with $Y=1$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \begin{array}{ccc} T_3 & Y & Q = T_3 + \frac{Y}{2} \\ \hline +\frac{1}{2} & 1 & +1 \\ -\frac{1}{2} & 1 & 0 \end{array}$$

where $\phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, $\phi^0 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)$. We use the usual $V(\phi)$ for symmetry breaking, and for v.e.v. $\phi_0$,

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

which has the required EM invariance (i.e. it has $Q=0$).

Expand in unitary gauge $\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix}$ and it all falls into place. We just care about the gauge boson masses for now, so we'll just look at the $\phi_0$ piece.

Substituting,

$$\left| \left(-ig \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu - i\frac{g'}{2} B_\mu\right)\phi \right|^2$$

$$= \frac{1}{8} \left| \begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \left(\tfrac{1}{2} v g\right)^2 W_\mu^+ W^{-\mu} + \frac{1}{8} v^2 \left(W_\mu^3, B_\mu\right) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

For the charged W's, $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$ and mass term should be $M_W^2 W^+ W^-$, so we have

$$\boxed{M_W = \tfrac{1}{2} v g}$$

For the neutral bosons, we have

$$\frac{1}{8} v^2 (\quad)(\quad)(\,) = \frac{1}{8} v^2 \left[g W_\mu^3 - g' B_\mu\right]^2 + 0 \left[g' W_\mu^3 + g B_\mu\right]^2$$

Comparing to the expected mass terms $\frac{1}{2} M Z^2$ for example, we conclude that the first [ ] is the Z and the 2nd [ ] is the photon, since the 2nd term is massless. Before identifying $m_Z$, though, we have to normalize:

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2+g'^2}} \quad , \quad m_A = 0$$

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}} \quad , \quad \boxed{m_Z = \tfrac{1}{2} v \sqrt{g^2+g'^2}}$$

Now we know where the mixing angle comes from, and we have (as implied in the last lecture)

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

and these are the same Z & A identified previously.

Also, at tree level,

$$\frac{m_W}{m_Z} = \cos\theta_W$$

and we see that if the $W_\mu^3$ & $B^\mu$ didn't mix, the W & Z would have the same mass, as in pure SU(2).
We can also identify v in terms of more familiar quantities:

$$\frac{1}{2v^2} = \frac{g^2}{8 m_W^2} = \frac{G_F}{\sqrt{2}} \quad \Rightarrow \quad v = 246 \text{ GeV}$$

## Fermion masses

While we're at it, we may as well use the Higgs mechanism to give mass to the fermions. It turns out that a fermion mass term $-m\bar{\psi}\psi$ violates $SU(2)_L$ invariance. We can write

$$-m\bar{\psi}\psi = -m\bar{\psi}\left[\tfrac{1}{2}(1-\gamma_5)+\tfrac{1}{2}(1+\gamma_5)\right]\psi$$
$$= -m\left(\bar{\psi}_R\psi_L - \bar{\psi}_L\psi_R\right)$$

and since $\psi_R$ is an $SU(2)$ singlet and $\psi_L$ is a member of a doublet, it isn't invariant under $SU(2)_L$. But we can solve that problem by introducing an interaction term between $\phi$ & the fermions, then let the vev give the mass. Schematically,

$$\underset{\uparrow}{g}_{ffh}\bar{f}\phi f \rightarrow \underbrace{g_{ffh}v}_{m_f}\bar{f}f + \underset{\uparrow}{g_{ffh}\bar{f}fh}^{*}$$

"Yukawa coupling" — interaction term; coupling $\propto$ mass!

So we can kill two birds with one stone by using the same scalar field to give mass to the gauge bosons & to the fermions. It doesn't *have* to be that way; that's just the simplest.

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* More carefully, we take, e.g. for the electron

$$\mathcal{L} = -g_{eeh}\left[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} e_R + \bar{e}_R(\phi^-, \bar{\phi}^0)\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right],$$

which is gauge invariant.

Finally we pull together all of this and give the Electroweak lagrangian: $L$ & $R$ stand for left-handed fermion doublets (e.g. $\binom{\nu_e}{e^-}$) and right-handed singlets, respectively. The $G_i$'s are Yukawa couplings, and $\phi_c = -i\tau_2\phi^* = \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} v+h \\ 0 \end{pmatrix}$

is necessary for giving mass to up-type quarks.

### 15.5 The Standard Model: The Final Lagrangian

To summarize the standard (Weinberg–Salam) model, we gather together all the ingredients of the Lagrangian. The complete Lagrangian is:

$$\mathcal{L} = -\tfrac{1}{4}\mathbf{W}_{\mu\nu}\cdot\mathbf{W}^{\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \qquad \left\{\begin{array}{l}\mathbf{W}^\pm, Z, \gamma \text{ kinetic} \\ \text{energies and} \\ \text{self-interactions}\end{array}\right.$$

$$\begin{aligned} &+\bar{L}\gamma^\mu\left(i\partial_\mu - g\frac{1}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu - g'\frac{Y}{2}B_\mu\right)L \\ &+\bar{R}\gamma^\mu\left(i\partial_\mu - g'\frac{Y}{2}B_\mu\right)R \end{aligned} \qquad \left\{\begin{array}{l}\text{lepton and quark} \\ \text{kinetic energies} \\ \text{and their} \\ \text{interactions with} \\ \mathbf{W}^\pm, Z, \gamma\end{array}\right. \qquad (15.40)$$

$$+\left|\left(i\partial_\mu - g\frac{1}{2}\boldsymbol{\tau}\cdot\mathbf{W}_\mu - g'\frac{Y}{2}B_\mu\right)\phi\right|^2 - V(\phi) \qquad \left\{\begin{array}{l}\mathbf{W}^\pm, Z, \gamma, \text{ and Higgs} \\ \text{masses and} \\ \text{couplings}\end{array}\right.$$

$$-(G_1\bar{L}\phi R + G_2\bar{L}\phi_c R + \text{hermitian conjugate}). \qquad \left\{\begin{array}{l}\text{lepton and quark} \\ \text{masses and} \\ \text{coupling to Higgs}\end{array}\right.$$

$L$ denotes a left-handed fermion (lepton or quark) doublet, and $R$ denotes a right-handed fermion singlet.

↑ fr/ Halzen + Martin

On to Higgs phenomenology...

# Signals for Unstable Particles:

the "bump" in the mass spectrum

invariant mass of decay products = mass of parent particle.

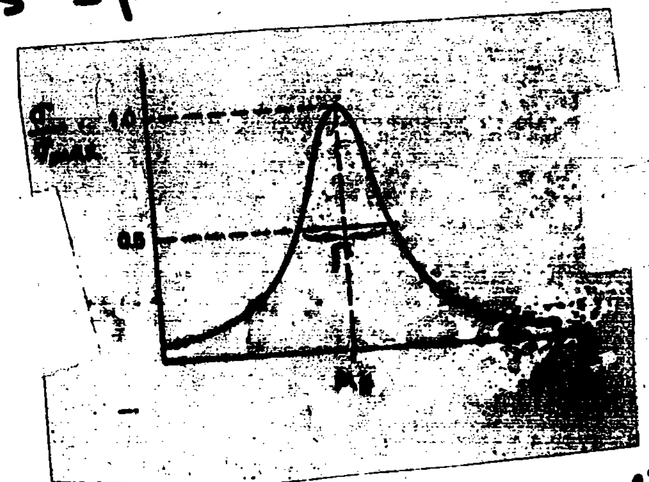
Ex: $H \to b\bar{b}$

$$p_b + p_{\bar{b}} = p_h \Rightarrow M^2_{b\bar{b}} \equiv (p_b + p_{\bar{b}})^2 = M_H^2$$

But finite width $\Rightarrow$ mass spread.

Breit-Wigner:

$$\frac{\sigma}{\sigma_{max}} = \frac{M_h^2 \Gamma^2}{(M - M_h)^2 + M_h^2 \Gamma^2}$$



$\Rightarrow$ If width small enough (+ resolution sufficient), look for peak in mass spectrum above background.

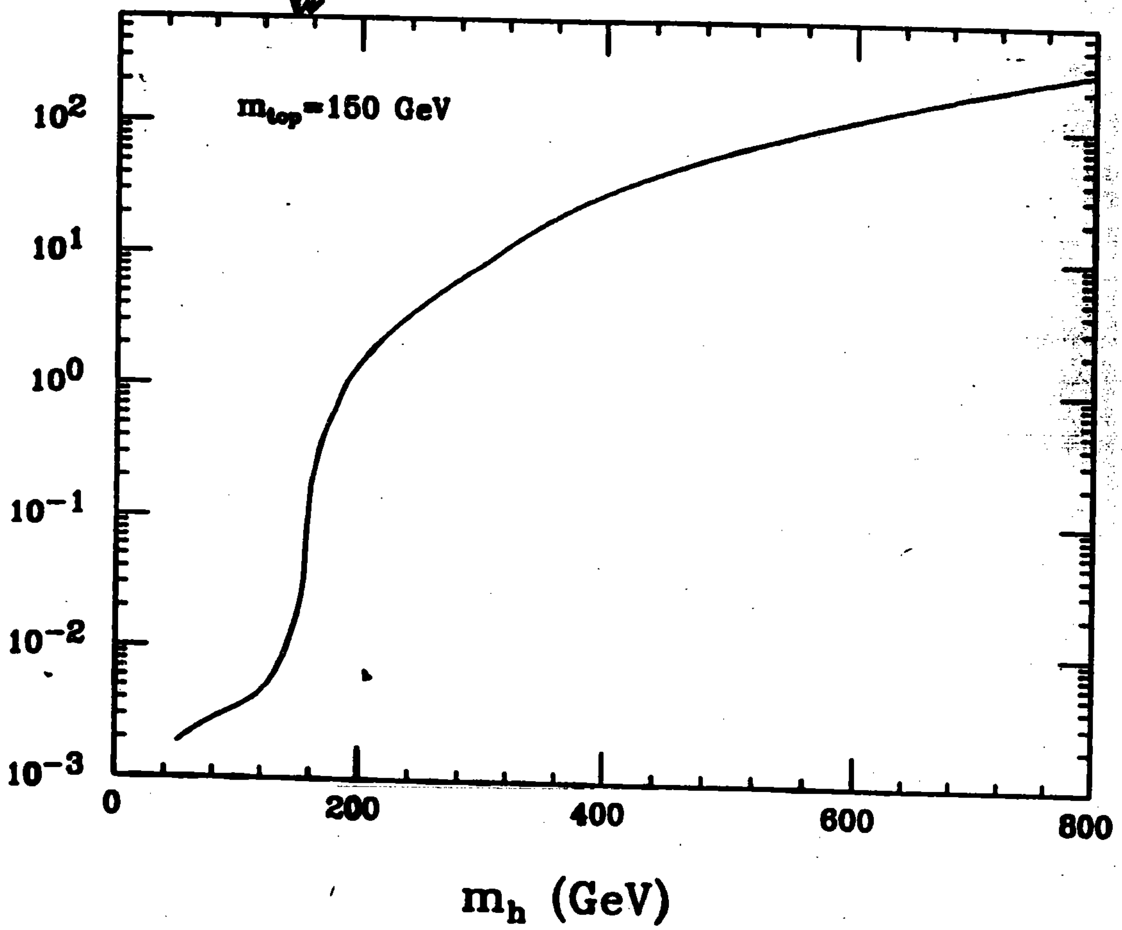
$\leftarrow$ fig.

Otherwise, more cleverness required.

(4.22)

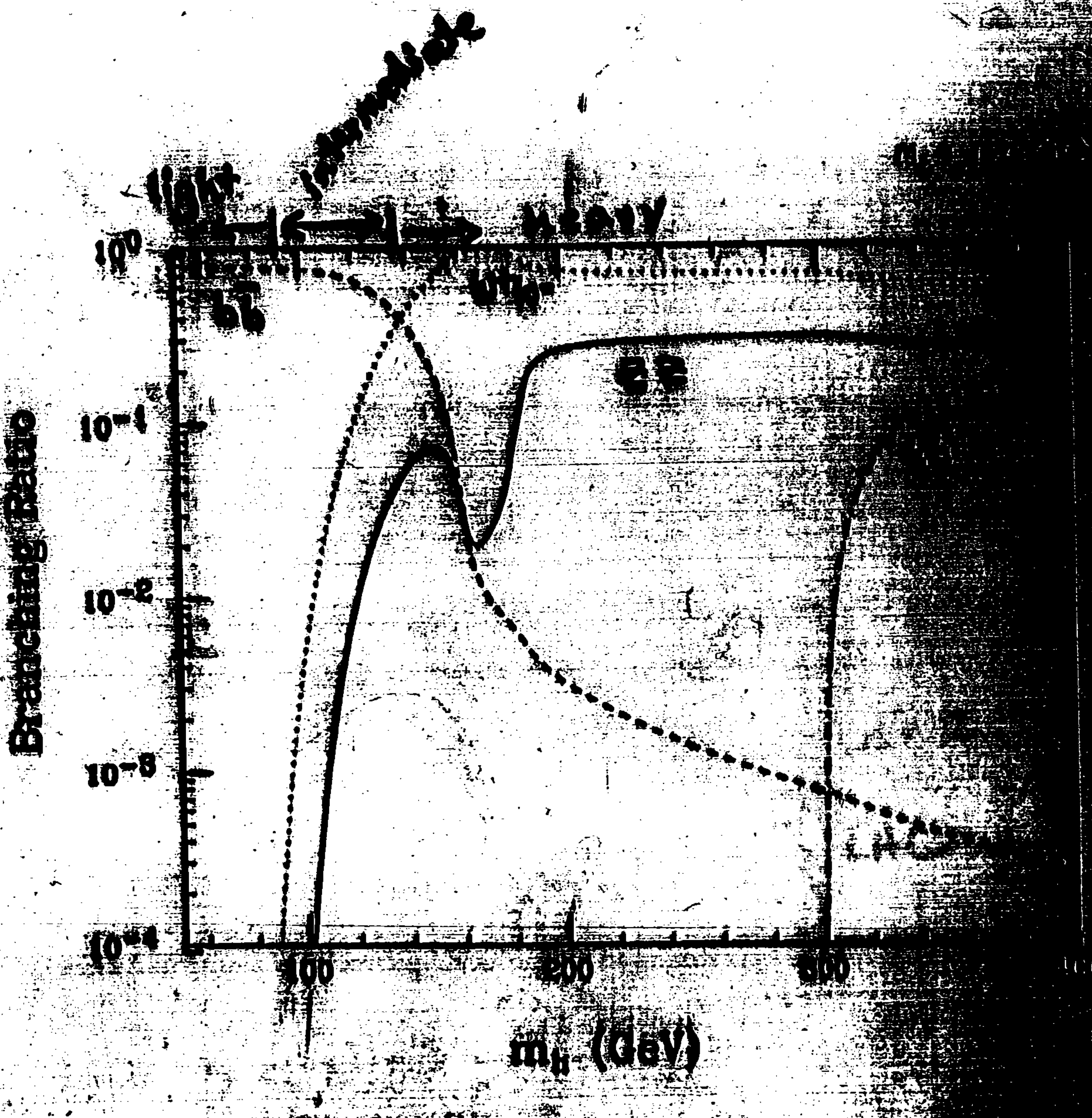
# SM Higgs Decay Width

WW, ZZ thresholds



4.23

# Higgs Decays



# How to Search for the SM Higgs

Best search strategy depends on mass:

**Light**: 50 GeV $\lesssim m_H \lesssim$ 90 GeV $(\simeq m_Z)$

LEP II: straightforward

**Intermediate**: $m_Z \lesssim m_H \lesssim$ 140 GeV

~~SSC~~/LHC: challenging, but possible

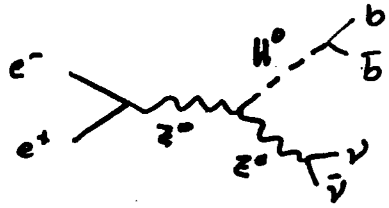
**Heavy**: 140 GeV $\lesssim m_H \lesssim$ 600 GeV

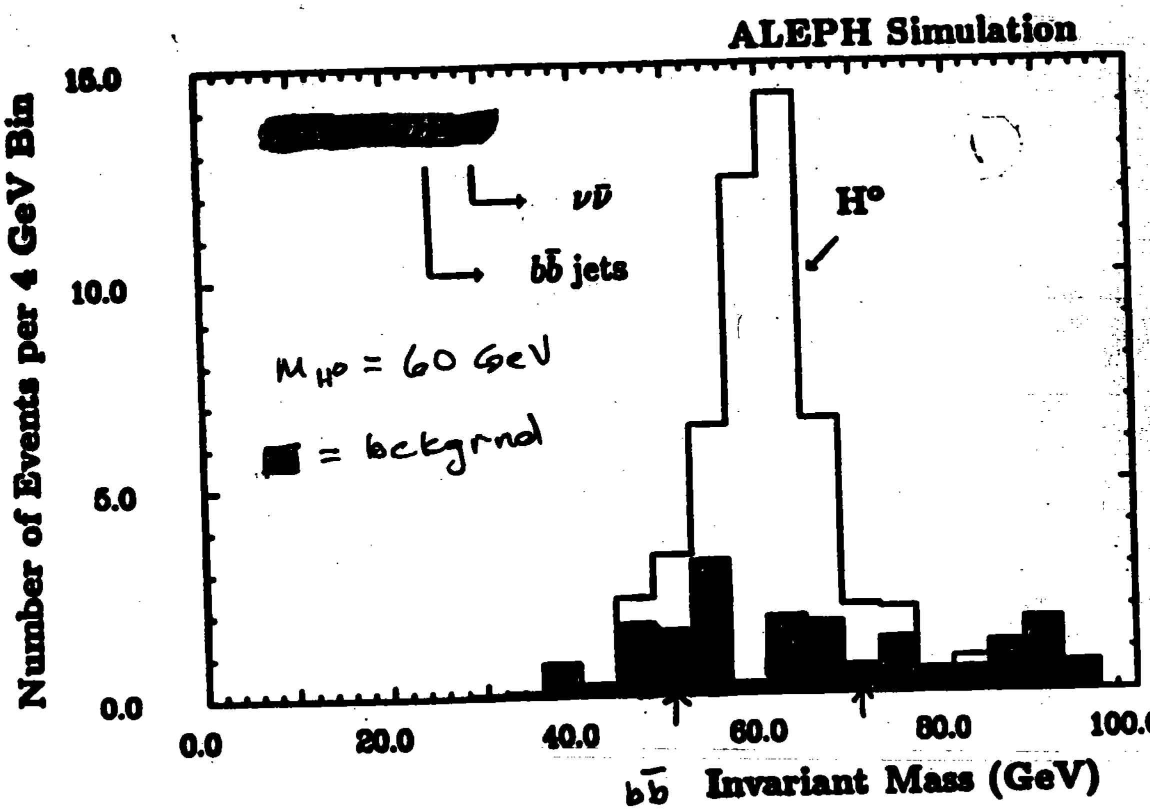
~~SSC~~/LHC: straightforward

**Obese**: ~600 GeV $\lesssim m_H$

$\left.\begin{array}{l}\text{~~SSC~~: difficult} \\ \text{LHC: very difficult (lower collision energy)}\end{array}\right\}$ indirect

# Higgs Detection at LEP II



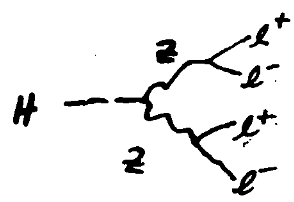

34 signal events
8 background events

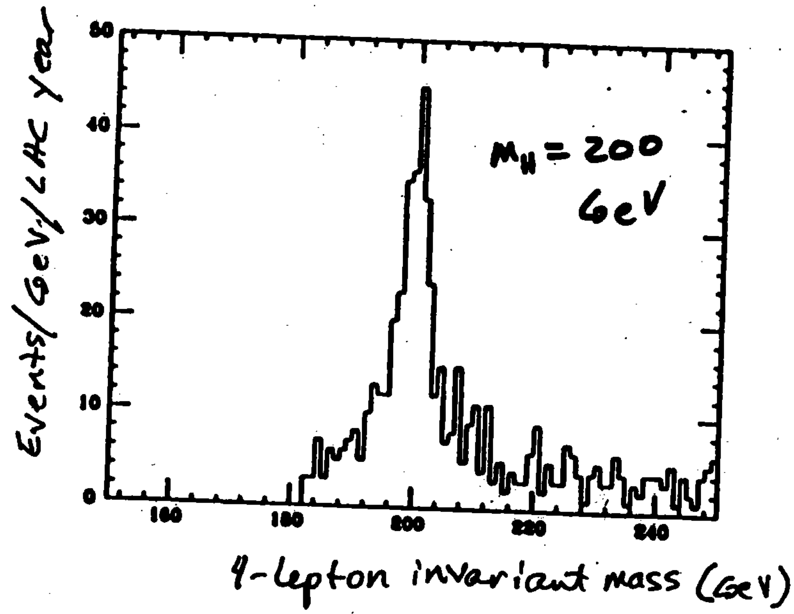
Boncret et al,
LEP200
workshop

# $H \to 4\ell$ detection at LHC

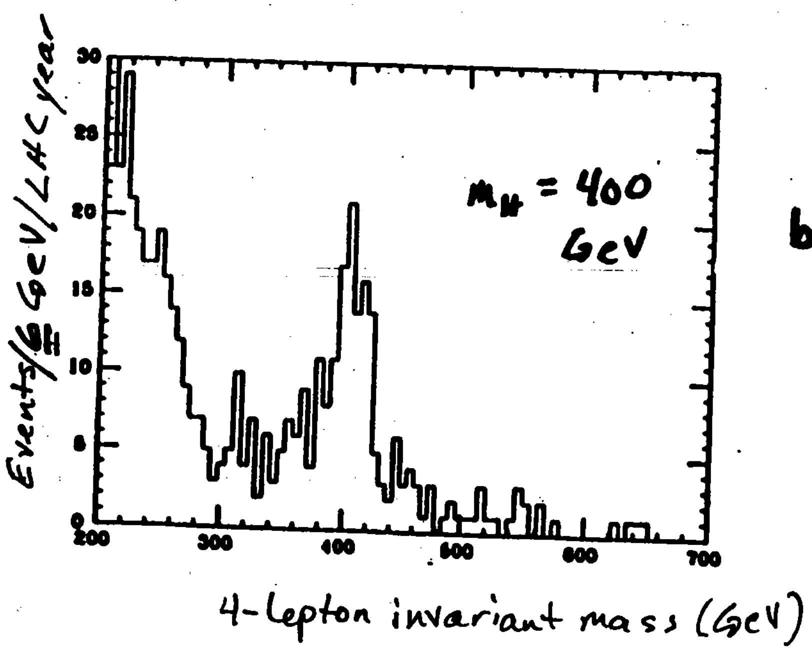


"gold plated"




bckgrnd included




bckgrnd included

Good to $m_H \sim 500$-$600$ GeV

(LHC Workshop)

# EWSB Beyond the SM

SM less than satisfying:

- naturalness/finetuning problem

$\Rightarrow$ quadratic divergences in $m_H$ radiative corrections

- triviality

  single scalar field theory doesn't exist

Alternatives:

- Extended Higgs sector: weak coupling

  e.g. Supersymmetry

- Composite Higgs: strong coupling

  e.g. Technicolor

# Supersymmetry



**Fermion-boson symmetry:**

all ordinary particles have SUSY partners

also, <u>two</u> Higgs doublets

**Advantages:**

- solves naturalness problem

  divergences cancelled by partners in loops

- coupling constant unification ← fig

- gravity incorporated naturally

- symmetry breaking induced by large $m_t$

**Phenomenology:**

5 physical Higgses : lightest $\lesssim$ 150 GeV

Light, intermediate, heavy strategies apply

**Most of Higgs parameter space accessible.**

← fig

# Strongly Interacting Higgs Sector

e.g. Technicolor

Higgs is composite of new fundamental fermions.
New physics at multi-TeV energies

Advantages:

- no elementary scalars
- we know such things happen in nature
  cf. cooper pairs in superconductivity

Phenomenology:

Difficult to construct specific models which are phenomenologically viable.

However, based on symmetry alone we get low energy theorems which allow us to make some model-indep. predictions

Obese Higgs strategies apply.

Jury still out. Need multi-~~SSC~~ years.
LHC