PHY 121 Mechanics *S.G. Rajeev*

Lecture 10

Conservation of Energy

•Consider a particle moving on a straight line, under the influence of a force $F(x)$. As it moves from x_{in} to x_{fi} its kinetic energy changes by

$$\int_{x_{\rm in}}^{x_{\rm fi}} F(x)dx.$$

Now let $U(x)$ be the negative of the integral of F(x). To be precise, we define

$$U(x) = -\int_0^x F(x)dx$$

so that the change in the kinetic energy is

$$\frac{1}{2}mv_{\rm fi}^2 - \frac{1}{2}mv_{\rm in}^2 = U(x_{\rm in}) - U(x_{\rm fi})$$

•In other words

$$\frac{1}{2}mv_{\rm in}^2 + U(x_{\rm in}) = \frac{1}{2}mv_{\rm fi}^2 + U(x_{\rm fi}).$$

If x is some arbitrary point, and v the velocity at that point, the quantity

$$\frac{1}{2}mv^2 + U(x)$$

is a *constant*. It has the same value at the initial point the end point and anywhere in middle. This quantity is called *energy* and we have just discovered a basic principle of physics, the *principle of conservation of energy*.

•The first term in energy is of course kinetic energy. The second term represents another kind of energy , *potential energy*. The sum is a

constant, but it is possible to convert potential energy into kinetic energy and vice versa.

•Kinetic energy is the most obvious form of energy; potential energy is not as visible but is just as real since it can be converted to kinetic energy. Kinetic energy is like money you have in your hand while potential energy is money you have in the bank. You can make a withdrawal or deposit any time, so it is really the sum of the two that determines how rich or poor you are.

Gravitational Potential Energy

•Imagine throwing a ball straight up in the air. The force is a constant Mg so the potential energy at a height h is just Mgy , y being the height.As it leaves your hand all its energy is kinetic. At the top, it has no kinetic energy but has potential energy. Indeed we can use this to find the maximum height it will reach. If v is the initial velocity and h the maximum height, conservation of energy gives

$$\frac{1}{2}mv^2 = mgh.$$

or

$$h = \frac{v^2}{2g}.$$

(Do this another by another method and verify this result!)

•What if the particle is moving in more than one direction? There is still a concept of potential energy for certain kinds of forces called *conservative forces*. Suppose the particle moves in three directions, under the influence of a force $\mathbf{F}(\mathbf{r})$ at the position

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

The Force itself can be expressed in terms of its components:

$$\mathbf{F} = F_1\mathbf{i} + F_2\mathbf{j} + F_3\mathbf{k}$$

•A *conservative force* is one for which there is a function $U(\mathbf{r})$ of

three variables (x, y, z) such that

$$F_1 = -\frac{\partial U}{\partial x}$$

$$F_2 = -\frac{\partial U}{\partial y}$$

$$F_3 = -\frac{\partial U}{\partial z}.$$

•(Aside: The symbol $\frac{\partial U}{\partial x}$ denotes the derivative of U with respect to x treating y and z as if they are constants. For example if $U(x, y, z) = x^2y + zx$,

$$\frac{\partial U}{\partial x} = 2xy + z.$$

Gravitational Potential Energy

•Many important forces are conservative. For example the force due to gravity near the Earth's surface is a constant,

$$\mathbf{F}(\mathrm{r}) = -Mgy\mathbf{j}$$

This is a conservative force because it is the negative derivative of

$$U(\mathrm{r}) = Mgy.$$

Indeed,

$$\frac{\partial U}{\partial x} = \frac{\partial U}{\partial z} = 0$$

and

$$\frac{\partial U}{\partial y} = -Mg$$

as we need. Thus for a particle moving in the gravitational field of

the Earth, (close to its surface),

$$\frac{1}{2}M|\mathbf{v}|^2 + Mgy$$

is a constant.

The Pendulum

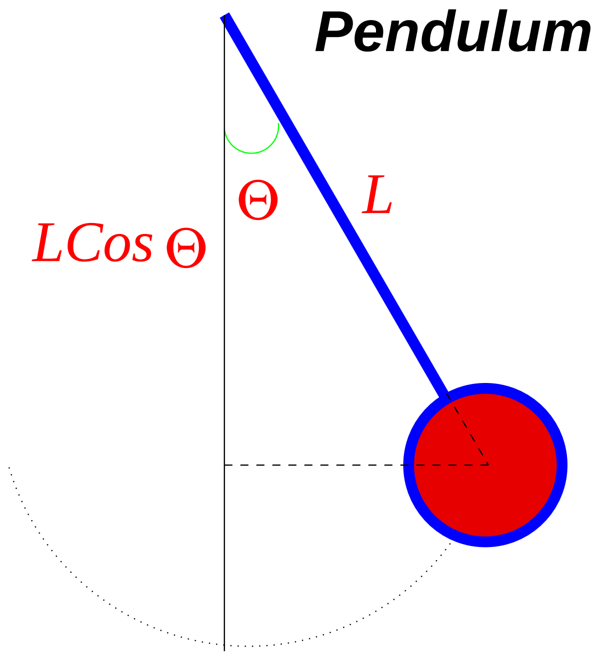
•An illustration of this is the pendulum. (See the attached figure for the definition of the variables.) For simplicity let us only study the case where it moves in a plane. The potential energy is given by the vertical distance it moves from the bottom of the arc. This distance is $L - L\cos\theta$ so that

$$U(\theta) = MgL(1 - \cos\theta).$$

•The speed of the pendulum is $L\frac{d\theta}{dt}$. (Why?). Thus

$$\frac{1}{2}ML^2(\frac{d\theta}{dt})^2 + MgL(1 - \cos\theta)$$

is the total energy of the pendulum and it is constant.



•At the height of the arc, it has no kinetic energy, only potential

energy. As it falls, it gains kinetic energy losing potential energy. At the very bottom all its energy is kinetic. We can see that the maximum height to which the bob can rise is determined by the speed at the bottom:

$$Mgh = \frac{1}{2}Mv_{\text{max}}^2.$$

•You can test this by swinging the pendulum all the way to one end, next to your nose, and letting it go. (*DON'T PUSH IT AS YOU LET GO.*) If you stand perfectly still the bob will return and will come to almost exactly the spot you released it from: your nose will be safe.