PHY 121 Mechanics *S.G. Rajeev*

Lecture 13

Collisions

•Consider a pair of particles of masses m_1 and m_2 and initial velocities u_1 and u_2 moving *on a line*. They are isolated from the rest of the world. They aproach each other, collide and then move away from each other. What are the *final velocities* v_1 and v_2 ?

•It is impossible to know the details of the forces acting on the particles during the short time when they are in contact. Still, we can predict the final velocities after the collision.

Elastic Collision of a Pair of Particles on a Line

•The trick is to think in terms of the conservation laws rather than in terms of Newton's laws.

•In the situation above, energy and momentum are conserved. The conservation of energy gives

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2.$$

•Moreover, conservation of momentum gives

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

These two equations can be solved to give the final velocities v_1 and v_2 .

•The general solution is rather messy, so let us consider first a special case.

Suppose the second particle was at rest initially: $u_2 = 0$. Then,

$$v_2 = \frac{m_1}{m_2}(u_1 - v_1).$$

Then,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\frac{m_1^2}{m_2^2}(u_1 - v_1)^2 - \frac{1}{2}m_1u_1^2 = 0$$

or,

$$v_1^2[1 + \frac{m_1}{m_2}] - 2u_1\frac{m_1}{m_2}v_1 + [\frac{m_1 - m_2}{m_2}]u_2 = 0.$$

•After some algebra we will get

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1.$$

(You will also get a solution $v_1 = u_1$ for the quadratic equation, but that represents the situation where there was no collision so must be discarded.)

•We can already see some interesting facts. If a body hits a heavier body at rest, it will recoil: $m_1 < m_2$ implies that v_1 has the opposite sign of u_1 .

•If instead it hits a body lighter than itself, it will slow down, but won't change its direction of motion.

•If they have equal mass, particle one will stop and transfer all its momentum to particle 2 which moves away with velocity $v_2 = u_1$.

•We can also get the velocity of the second body after the collision in the special case $u_1 = 0$:

$$v_2 = \frac{2m_1}{m_1 + m_2}u_1.$$

•The general case can be solved in the same way but with more work:

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$$

$$v_2 = \frac{m_2 - m_1}{m_1 + m_2} u_2 + \frac{2m_1}{m_1 + m_2} u_1.$$

•You can also get it from the special solution above by transforming to the reference frame in which the second particle is at rest, solving the problem there, and then transforming back.

The Slingshot Effect

•We can extract energy for a spacecraft by making it go around a planet like Jupiter. This trick makes space travel much more affordable. How does this work?

•The encounter between the spacecraft and the planet can take several *hours* but since the motion before and after takes *years*, we can treat this as a collision. This is an elastic collision, the energy that the spacecraft gains comes from the kinetic energy of the planet.

•In our previous calculation, let us choose the particle 1 to be the spacecraft and particle 2 the planet. Thus $m_2 >> m_1$. We have,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \sim -u_1 + 2u_2$$

$$v_2 = \frac{m_2 - m_1}{m_1 + m_2} u_2 + \frac{2m_1}{m_1 + m_2} u_1 \sim u_2.$$

•Thus the velocity of the planet hardly changes but the spacecraft has rebounded with a new velocity $2u_2 - u_1$.

•The orbital speed of Jupiter is $u_2 = 13\ km/s$. If we approach it with a speed of $u_1 = 12\ km/s$ in the opposite direction, we will be

swung out with a speed of $v_2 = 38\ km/s$: an almost tenfold increase in kinetic energy.