PHY 121 Mechanics *S.G. Rajeev*

Lecture 14

More Collisions

Inelastic Collision in a Line

•Some energy is lost to friction or heat when two particles collide. We have so far ignored that.

•But in many cases, even when energy of the pair of particles is not conserved, momentum is conserved. This is because the loss of energy to heat is due to the transfer of energy to the randomly moving atoms.

•Unlike energy, momentum can be both positive and negative; on average as much negative momentum as positive momentum is transfered away, so momentum is conserved in those cases where the loss of energy is due to random causes.

•A collision in which *energy is not conserved* but *momentum is conserved* is said to be an *inelastic collision*. If the maximum amount of energy is lost is this way the collision is *completely inelastic*.

•If I drop a ball on the ground it doesn't bounce quite to its original height. The loss of energy during the collision to heating the Earth is the reason. Some energy is also lost to deforming the ball.

Ballistic Pendulum

•This was a device used to measure the speed of bullets in the days before fast timers were available. A bullet is shot into a block of wood hung by a string. The momentum of the bullet is tranfered to the wood. The energy of the bullet is partly transfered to the kinetic energy of the block but is also lost to heat and in deforming the wood.

•Let v be the velocity of the bullet and m its mass. Let V be the velocity of the block of wood just after the collision and M its mass. Conservation of momentum gives

$$mv = (M+m)V$$

or

$$V = \frac{m}{M+m}v.$$

•The kinetic energy of the wood after it has been hit is

$$\frac{1}{2}MV^2 = \frac{1}{2}\frac{Mm^2}{(M+m)^2}v^2.$$

•This is less than the original kinetic energy $\frac{1}{2}mv^2$ of the bullet. Indeed if some of the energy had not been lost to heat or (deforming the block) the bullet would have bounced back!. But after the collision is over the pendulum moves in a way that conserves energy.

•The maximum height to which the block of wood can rise is given by the coservation of energy:

$$(M+m)gh = \frac{1}{2}\frac{Mm^2}{(M+m)^2}v^2$$

so that

$$v = \frac{M+m}{m}\sqrt{(2gh)}.$$

Elastic Scattering of a Pair of Particles: General Case

•In general if two partcles collide, there is no reason to expect that the initial velocities will be on the same line. Even if we arrange the initial velocities were on the same line, the final velocities need not be on that line. Can we predict the angle between the initial and final velocities using the conservation laws alone?

•There are six unknowns, the three components of the final velocities of the two particles. The law of conservation of energy gives one equation and the conservation of momentum gives three more (one for each component.) Thus we have

four equations for six variables: there will be two undetermined variables.

•To predict exactly the final velocities of the particles we will need to know more about the forces during the collisions. But if somehow we know the *direction* at which one of the particles emerges, we can predict everything else. The direction gives us the two remaning variables to solve our scattering problem. The solution is quite difficult in general though.

Elastic Collisions on a Plane

•If we know the masses, the initial velocities and the angle at which one of the particles is scattered, we can find the final velocities of the particles. The laws of cosnervation of energy and momentum give

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

and

$$\frac{1}{2}m_1\mathbf{u}_1^2 + \frac{1}{2}m_2\mathbf{u}_2^2 = \frac{1}{2}m_1\mathbf{v}_1^2 + \frac{1}{2}m_2\mathbf{v}_2^2.$$

•These are three equations for four unknown quantities, so we need one extra piece of information, the scattering angle of one of the particles.

•Suppose for example $\mathbf{u}_2 = 0$: the second particle is at rest. More-over, suppose the masses are equal: $m_1 = m_2 = m$. We can then immediately prove that the after the collision, the particles fly apart at right angles to each other (except in the case of a head-on collision).

•For, conservation of momentum gives

$$\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{u_1}$$

and conservation of energy gives

$$\mathbf{v}_1^2 + \mathbf{v}_2^2 = \mathbf{u_1}^2.$$

•Then

$$(\mathbf{v}_1 + \mathbf{v}_2)^2 = \mathbf{u_1}^2;$$

i.e.,

$$\mathbf{v}_1^2 + \mathbf{v}_2^2 + 2\mathbf{v}_1 \cdot \mathbf{v}_2 = \mathbf{u_1}^2.$$

This gives

$$\mathbf{v_1} \cdot \mathbf{v}_2 = 0.$$

•Either, one of the velocities is zero or, they are at right angles. If the collision is head-on, the first particle will be brought to rest and the second one will move away in the direction it was hit.

•In any other situation, both particles will be moving and they will be moving at right angles to each other.